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CIRCUITANDNETWORK THEORY

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3rd SEMESTER,ELECTRICAL ENGINEERING

(AS PER SCTE&VT,ODISHA SYLLABUS)

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CHAPTER1

MagneticCircuits

Introduction:

Magneticfluxlinesalwaysformclosedloops.Theclosedpathfollowedbythe flux lines is calleda magnetic circuit. Thus, a magnetic circuit provides a path formagneticflux,justasanelectriccircuit providesapathfortheflowofelectric current.Ingeneral,thetermmagneticcircuitappliestoanyclosed pathinspace, but in the analysis of electro-mechanical and electronic system this term is specificallyusedforcircuitscontainingamajorportionofferromagneticmaterials. The study of magnetic circuit concepts is essential in the design, analysis and application of electromagnetic devices like transformers, rotating machines, electromagneticrelaysetc.

MagnetomotiveForce(M.M.F):

Flux is produced round any current – carrying coil. In order to produce the required flux density, the coil should have the correct number of turns. The product of the currentandthenumberofturnsisdefinedasthecoilmagnetomotiveforce(m.m.f).

IfI=Currentthroughthecoil(A) N =
Number of turns in the coil.
Magnetomotiveforce=Currentxturns

SoM.M.F=IXN
TheunitofM.M.F.isampere–turn(AT)butitistakenasAmpere(A)sinceN has no dimensions.

MagneticFieldIntensity:

MagneticFieldIntensityisdefinedasthemagneto-motiveforceperunitlengthofthe magnetic flux path. Its symbol is H.

$$\text{Magnetic field Intensity (H)} = \frac{\text{Magnetomotive force}}{\text{Mean length of the magnetic path}}$$

- $H = \frac{F}{l} = \frac{I.N.}{l}$ A/m

Where/isthemeanlengthofthemagneticcircuitinmeters.Magneticfieldintensityisalso called magnetic field strength or magnetizing force.

Permeability:-

Everysubstancepossessesacertainpowerofconductingmagneticlines of force. For example, iron is better conductor for magnetic lines of force than air (vaccum) . Permeabilityofamaterial(μ)isitsconductingpowerformagneticlinesofforce.Itistheratio of the flux density. (B) Produced in a material to the magnetic filed strength (H).

$$\text{i.e. } \mu = B/H$$

Reluctance:

Reluctance(s)isakintoresistance(whichlimitsthe electricCurrent).

Fluxinamagneticcircuitislimitedbyreluctance.Thusreluctance(s)isa measureofthe opposition offered by a magnetic circuit to the setting up of the flux.

Reluctanceistheratioofmagnetomotiveforcetotheflux.Thus

$$S = Mmf/\phi$$

Itsunitisampereturnsperwebber(orAT/wb).

Permeance:-

Thereciprocalofreluctanceiscalledthepermeance(symbolA). Permeance (A)

= 1/S wb/AT

TurnThasnounit.

Hencepermeanceisexpressedinwb/AorHenerys(H).

B.H.Curve:

Place a piece of an unmagnetised iron bar AB within the field of a solenoid to magnetise it. The field H produced by the solenoid, is called magnetisingfield,whosevaluecanbealtered(increasedordecreased)by

changing (increasing or decreasing) the current through the solenoid. If we increase slowly the value of magnetic field (H) from zero to maximum value, the value of flux density (B) varies along 1 to 2 as shown in the figure and the magnetic materials (i.e iron bar) finally attains the maximum value of flux density (Bm) at point 2 and thus becomes magnetically saturated.

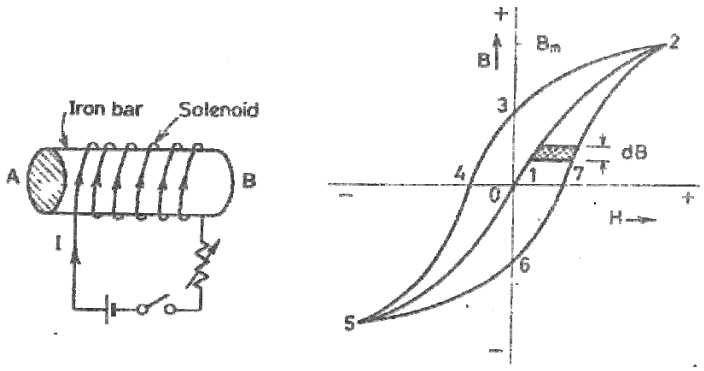


Fig.2.1

Now if value of H is decreased slowly (by decreasing the current in the solenoid) the corresponding value of flux density (B) does not decreases along 2-1 butdecreases somewhatlessrapidlyalong 2 to 3. Consequentlyduringthe reversalof magnetization,thevalue of B is notzero, but is '13' atH= 0. Inother wards, during the period of removal of magnetization force (H), the iron bar is not completely demagnetized.

In order to demagnetise the iron bar completely, we have to supply the demagnetisastion force (H) in the opposite direction (i.e. by reserving the directionofcurrentinthesolenoid).ThevalueofBisreducedtozeroat point 4, when H='14'. This value of H required to clear off the residual magnetisation, is known as coercive force i.e. the tenacity with which the material holds to its magnetism.

If after obtaining zero value of magnetism, the value of H is made more negative, the iron bar again reaches, finally a state of magnetic saturation at the point 5, which represents negative saturation. Now if the value of H is increasedfromnegativesaturation(='45')topositivesaturation(='12')a

curve '5,6,7,2' is obtained. The closed loop "2,3,4,5,6,7,2" thus represents one complete cycle of magnetisation and is known as hysteresis loop.

CHAPTER

02COUPLEDCIRCUITS

Itisdefinedastheinterconnectedloopsofanelectricnetworkthroughthe magneticcircuit.

Therearetwotypesofinducedemf.
(1)StaticallyInducedemf.
(2)DynamicallyInducedemf.

Faraday's LawsofElectro-Magnetic:
Introduction→FirstLaw:→
Whenneverthemagneticfluxlinkedwithacircuitchanges,anemfisinducedinit.
OR
Whenneveraconductorcutsmagneticfluxanemfisinducedinit.
SecondLaw:→
Itstatesthatthemagnitudeofinducedemfisequaltotherateofchangeofflux linkages.
OR
Theemfinducedisdirectlyproportionaltotherateofchangeoffluxand number of turns

Mathematically:

$$e \propto \frac{d\phi}{dt}$$

$$e \propto N$$

Or

$$e = -N\frac{d\phi}{dt}$$

Wheree=inducedemf

N=No.ofturns ϕ

= flux

'-ve'signisduetoLenz'sLaw

Inductance:→
Itisdefinedasthepropertyofthesubstancewhichopposesanychangein Current&flux.

Unit:→Henry

Fleming'sRightHandRule:→

Itstatesthat"holdyourrighthandwithfore-finger,middlefingerandthumbatrightangles to each other. If the fore-finger represents the direction of field, thumb represents the direction of motion of the conductor, then the middle finger represents the direction of induced emf."

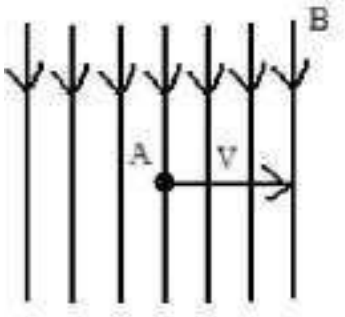
Lenz'sLaw:→

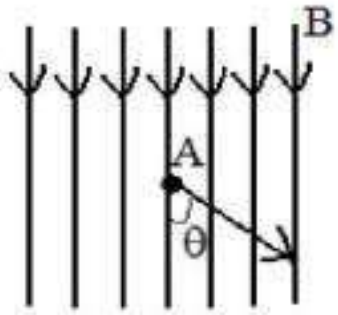
Itstatesthatelectromagneticallyinducedcurrentalwaysflowsinsucha directionthattheactionof magneticfieldsetupbyittendstoopposethevary cause which produces it.

OR

Itstatesthatthedirectionoftheinducedcurrent(emf)issuchthatit opposesthe change of magnetic flux.

(2)**DynamicallyInducedemf:→**





Inthiscasethefieldis stationaryandtheconductorsarerotatinginan uniformmagnetic fieldatfluxdensity'B"Wb/mt2andtheconductorislying perpendiculartothemagnetic field. Let '*l*'is the length of the conductor and it moves a distance of 'dx' nt in time 'dt' second.

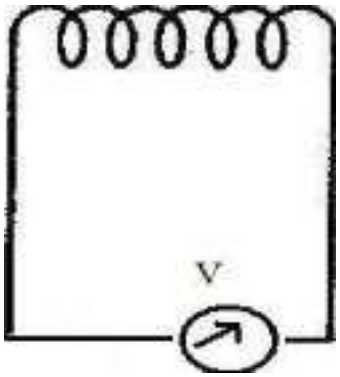
Theareasweptbytheconductor=*l.dx*
Hencethefluxcut=*ldx.B*
Changeinfluxintime'dt'second=

$$\frac{Bldx}{dt}$$

E=Blv

Iftheconductorismakinganangle'θ'withthemagneticfield,then
*e=Blvsin*θ

(1) StaticallyInducedemf:→
Heretheconductorsareremaininstationaryandfluxlinkedwithit changes by increasing or decreasing.
Itisdividedinto twotypes.
(i) Self-inducedemf.
(ii) Mutually-inducedemf.
(i) Self-inducedemf:→Itisdefinedastheemfinducedinacoilduetothe change of its own flux linked with the coil.



Ifcurrentthroughthecoilischangedthenthefluxlinkedwithitsownturn will also change which will produce an emf is called self-induced emf.
Self-Inductance:→
Itisdefinedasthepropertyofthecoilduetowhichitopposesany change (increase or decrease) of current or flux through it.
Co-efficientofSelf-Inductance(L):→
Itisdefinedastheratioofweberturnsperampereofcurrentinthecoil.
OR
Itistheratiooffluxlinkedperampereofcurrentinthecoil.

1stMethodfor'L':→

$$L=\frac{N\phi}{I}$$

WhereL=Co-efficientofself-induction N
= Number of turns
ϕ=flux
I=Current

2ndMethodforL:→ We know that

$$L = \frac{N\phi}{I}$$

$\Rightarrow LI = N\phi$

$\Rightarrow -LI = -N\phi$

$$\Rightarrow -L\frac{dI}{dt} = -N\frac{d\phi}{dt}$$

$$\Rightarrow -L\frac{dI}{dt} = -N\frac{d\phi}{dt}$$

$$\Rightarrow -L\frac{dI}{dt} = e_l$$

$$\Rightarrow L\frac{dI}{dt} = -e_L$$

$$\boxed{\Rightarrow L = \frac{-e_L}{\frac{dI}{dt}}}$$

WhereL=Inductance

$e_L = -N\frac{d\phi}{dt}$ is known as self-induced emf.

When $\frac{dI}{dt} = 1 amp/\sec.$

e=1volt

L=1Henry

Acoilissaidto beself-inductanceof1Henryif 1voltisinducedinit. When the current through it changes at the rate of 1 amp/ sec.

3rdMethodforL:→

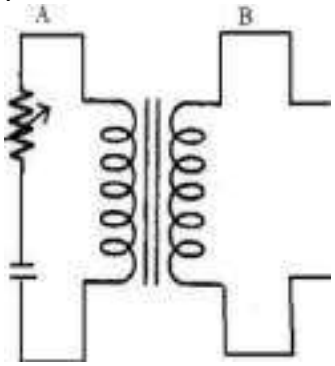
$$L = \frac{M_o M_r A N^2}{l}$$

WhereA=Areaofx-sectionofthecoil N = Number of turns

L=Lengthofthecoil

(ii) MutuallyInducedemf:→

Itis definedas theemf inducedinonecoildue tochangeincurrentin other coil. Consider two coils 'A' and 'B' lying close to each other. An emf will be induced in coil 'B' due to change of current in coil 'A' by changing the position of the rheostat.



MutualInductance:→

Itisdefinedas theemf inducedincoil 'B'duetochangeofcurrentincoil 'A' is the ratio of flux linkage incoil 'B' to 1amp. Of current incoil 'A'.

Co-efficientofMutualInductance(M):

Coefficientofmutualinductancebetweenthetwocoilsisdefinedasthe weber-turns in one coil due to one ampere current in the other.

1stMethodfor'M':→

$$M = \frac{N_2 \phi_1}{I_1}$$

N_2 = Number of turns

M=MutualInductance

ϕ_1 = flux linkage

I_1=Currentinampere

2ndMethodforM:→

We know that

$$M = \frac{N_2 \phi_1}{I_1}$$

$\Rightarrow MI_2 = N_2\phi_1$

$\Rightarrow -MI_1 = N_2\phi_1$

$$\Rightarrow -M\frac{dI_1}{dt} = -N_2\frac{d\phi_1}{dt}$$

$$\Rightarrow -M\frac{dI_1}{dt} = e_M$$

$$\Rightarrow M\frac{dI_1}{dt} = -e_M$$

$$\boxed{\Rightarrow M = \frac{-e_M}{\frac{dI_1}{dt}}}$$

Where

$$e_M = -N_2\frac{d\phi_1}{dt}$$

e_M=−1*VOLT*

ThenM= 1Henry

A coil is said tobe a mutual inductance of 1Henrywhen 1 volt isinducedwhen the current of 1 amp/sec. is changed in its neighbouring coil.

3rdMethodforM:→

$$M = \frac{M_o M_r A N_1 N_2}{l}$$

Co-efficientofCoupling:

Considertwo magneticallycoupledcoilshavingN1andN2turnsrespectively. Their individual co-efficient of self-inductances are

$$L_1 = \frac{M_o M_r A N_1^2}{l}$$

$$L_2 = \frac{M_o M_r A N_2^2}{l}$$

The flux $\phi_1$producedincoil'A'duetoacurrentofI1 ampereis

$$\phi_1 = \frac{L_1 I_1}{N_1} = \frac{M_o M_r A N_1^2}{l} \times \frac{I_1}{N_1}$$

$$\phi_1 = \frac{M_o M_r A N_1 I_1}{l}$$

Supposeafractionofthis fluxi.e.K1 ϕ_1 islinkedwithcoil'B' Similarly the

$$\text{Then } M = \frac{K_1\phi_1}{I_1} \times N_2 = \frac{K_1 N_1 N_2}{l / M_o M_r A} \text{ ----------(1)}$$

flux ϕ_2 produced in coil 'B'due to I2 amp. Is

$$\phi_2 = \frac{M_1 M_r A N_2 I_2}{l}$$

Supposeafractionofthis fluxi.e.$K_2 \phi_2$ islinkedwithcoil'A' Multiplying

Then $M = \frac{K_2 \phi_2}{I_2} \times N_1 = \frac{K_2 N_2 N_1}{l / M_o M_r A}$ ----------(2)

equation (1) & (2)

$$M^2 = \frac{K_1 K_2 N_1^2 N_2^2}{l^2 / M_o^2 M_r^2 A^2} \times N_1$$

$$= K^2 \left(\frac{M_o M_r A N_1^2}{l} \right) \left(\frac{M_o M_r A N_2^2}{l} \right)$$

$$[\because K_1 = K_2 = K]$$

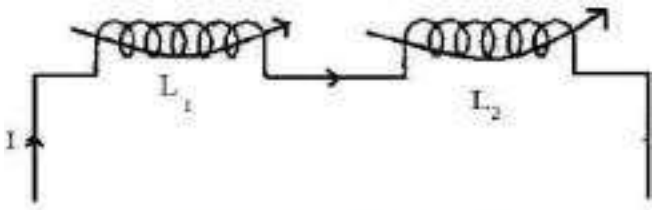
$$M^2 = K^2 L_1 L_2$$

$$K^2 = \frac{M^2}{L_1 L_2}$$

$$\Rightarrow K = \sqrt{\frac{M^2}{L_1 L_2}}$$

Where'K'isknownastheco-efficientofcoupling.
Co-efficientofcouplingisdefinedastheratioofmutualinductance between two coils to the square root of their self- inductances.

InductancesInSeries(Additive):→



Let M= Co-efficientofmutualinductance
L_1=Co-efficientofself-inductanceoffirstcoil.
L_2=Co-efficientofself-inductanceofsecondcoil. EMF induced in first coil due to self-inductance

$$e_{L_1} = -L_1 \frac{dI}{dt}$$

Mutuallyinducedemfinfirstcoil

$$e_{M_1} = -M\frac{dI}{dt}$$

EMFinducedinsecondcoilduetoselfinduction

$$e_{L_2} = -L_2\frac{dI}{dt}$$

Mutuallyinducedemfinsecondcoil

$$e_{M_2} = -M\frac{dI}{dt}$$

Totalinducedemf

$E=e_{L1}+e_{L2}+e_{m1}+e_{m2}$

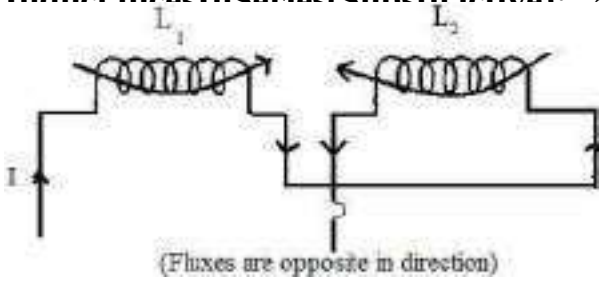
If'L'istheequivalentinductance,then

$$-L\frac{dI}{dt} = -L_1\frac{dI}{dt} - M\frac{dI}{dt} - L_2\frac{dI}{dt} - M\frac{dI}{dt}$$

$$\Rightarrow -L\frac{dI}{dt} = -\frac{dI}{dt}(L_1 - L_2 - 2M)$$

$$\Rightarrow L = L_1 + L_2 + 2M$$

InductancesInSeries(Substnactive):→



LetM=Co-efficientofmutualinductance

L_1=Co-efficientofself-inductanceoffirstcoil

L_2-=Co-efficientofself-inductanceofsecondcoil Emf induced in first coil due to self induction

$$e_{L_1} = -L_1\frac{dI}{dt}$$

Mutuallyinducedemfinfirstcoil

$$e_{M_1} = -\left(-M\frac{dI}{dt}\right) = M\frac{dI}{dt}$$

Emfinducedinsecondcoildueto self-induction

$$e_{L_2} = -L_2 \frac{dI}{dt}$$

Mutuallyinducedemfinsecondcoil

$$e_{M_2} = -\left(-M\frac{dI}{dt}\right) = M\frac{dI}{dt}$$

Totalinducedemf

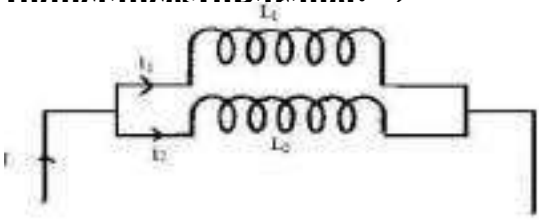
$e = e_{L1} + e_{L2} + e_{M1} + e_{M2}$

Then

$$-L\frac{dI}{dt} = -L_1\frac{dI}{dt} - L_2\frac{dI}{dt} + M\frac{dI}{dt} + M\frac{dI}{dt}$$

$$\Rightarrow -L\frac{dI}{dt} = -\frac{dI}{dt}(L_1 + L_2 - 2M) \qquad \Rightarrow L = L_1 + L_2 - 2M$$

InductancesInParallel:→



Lettwo inductancesofL_1&L_2areconnectedinparallel

Lettheco-efficentofmutualinductancebetweenthemis M. I-i_1+i_2

$$\frac{dI}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \text{ ---------- (1)}$$

$$e = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$= L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$\Rightarrow L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$\Rightarrow (L_1 - M)\frac{di_1}{dt} = (L_2 - M)\frac{di_2}{dt}$$

$$\Rightarrow \frac{di_1}{dt} = \frac{(L_2 - M)}{(L_1 - M)}\frac{di_2}{dt} \text{ ----------(2)}$$

$$\frac{dI}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$= \frac{(L_2 - M)}{(L_1 - M)}\frac{di_2}{dt} + \frac{di_2}{dt}$$

$$\Rightarrow \frac{dI}{dt} = \left(\frac{L_2 - M}{L_1 - M} + 1\right)\frac{di_2}{dt} \text{ ----------(3)}$$

If‘L’istheequivalentinductance

$$e = L\frac{di}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$L\frac{di}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$\Rightarrow \frac{di}{dt} = \frac{1}{L}\left(L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\right) \text{----------(4)}$$

Substituting the value of $\frac{di_1}{dt}$

$$\frac{di}{dt} = \frac{1}{L}\left[L_1\frac{L_2 - M}{L_1 - M} + M\right]\frac{di_2}{dt} \text{----------(5)}$$

Equatingequation(3)&(5)

$$\left[\left(\frac{L_2 - M}{L_1 - M}\right) + 1\right]\frac{di_2}{dt} = \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]\frac{di_2}{dt}$$

$$\Rightarrow \frac{L_2 - M}{L_1 - M} + 1 = \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]$$

$$\Rightarrow \frac{L_2 - M + L_1 - M}{L_1 - M} = \frac{1}{L}\left[\frac{L_1L_2 - L_1M + L_1M - M^2}{L_1 - M}\right]$$

$$\Rightarrow \frac{L_1 + L_2 - 2M}{L_1 - M} = \frac{1}{L}\left[\frac{L_1L_2 - M^2}{L_1 - M}\right]$$

$$\Rightarrow L_1 + L_2 - 2M = \frac{1}{L}\left[L_1L_2 - M^2\right]$$

$$\Rightarrow L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

Whenmutualfieldassist.

$$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

Whenmutualfieldopposes.

Exp.-01:

Twocoupledcolshaveselfinductances$L_1=10\times10^{-3}$H and $L_2=20\times10^{-3}$H. The coefficient of coupling (K) being 0.75 in the air, find voltage in the secondcoilandthefluxoffirstcoilprovidedthesecondcoilshas500turns and thecircuitcurrentisgivenby$i_1=2\sin314.1$A.

Solution:

$M=K\sqrt{L1L2}$

$M=0.75\sqrt{10\times10^{-3}\times20\times10^{-3}}$

$\Rightarrow M=10.6\times10^{-3}H$

Thevoltageinducedinsecondcoilis

$V_2=M\frac{di1}{dt}$

$=10.6\times10^{-3}\times2\times314\cos314dt$.

Themagneticcircuitbeinglinear,

$$M=\frac{N_2\phi_2}{i_1}=\frac{500\times(K\phi_1)}{i_1}$$

$$\phi=\frac{M}{500\times K}\times i_1=\frac{10.6\times10^{-3}}{500\times0.75}\times2\sin314t$$

$$=5.66\times10^{-5}\sin314t$$

Exp.02

Findthetotalinductanceofthethreeseriesconnectedcoupledcoils.Where the self and mutual inductances are

$L_1=1$H, $L_2=2$H, $L_3=5$H

$M_{12}=0.5$H, $M_{23}=1$H, $M_{13}=1$H

Solution:

$L_A=L_1+M_{12}+M_{13}$

$=1+20.5+1$

$=2.5$H

$L_B=L_2+M_{23}+M_{12}$

$=2+1+0.5$

$=3.5$H

$L_C=L_3+M_{23}+M_{13}$

$=5+1+1$

$=7$H

Totalinductancesare

$L_{ea} = L_A + L_B + L_c$

$= 2.5 + 3.5 + 7$

$= 13H$(Ans)

CHAPTER3

CircuitElementsandanalysis

1.1 Voltage

Energy is required for the movement of charge from one point to another. Let W Joules of energy be required to move positive charge Q columbs from a point a to pointb in a circuit. We say thata voltage exists between the two points. The voltageV between two points may be defined in terms of energy that would be required if a charge were transferred from one point to the other. Thus, there can be a voltage between two points even if no charge is actually moving from one to the other. Voltage between a and b is given by

$$V = \frac{W}{Q} \text{ J/C}$$

$$\text{HenceElectricPotential(V)} = \frac{\text{Workedare(W)inJoules}}{\text{Charge(Q)incolumbs}}$$

Current:

An electric currentis the movement of electric charges along a definite path. In case of a conductor the moving charges are electrons.

The unit of current is the ampere. The ampere is defined as that current which when flowing in two infinitely long parallel conductors of negligible crosssection, situated 1meter apart in Vacuum, produces between the conductors a force of 2×10^{-7}Newton per metre length.

Power : Power is defined as the work done per unit time. If a field F newton acts for t seconds through a distance dmetres alonga straightline, work done W = Fxd N.m. or J. The power P, either generated or dissipated by the circuit element.

$$P = \frac{w}{t} = \frac{Fxd}{t}$$

Power can also be written as Power = $\frac{\text{Work}}{\text{time}}$

$= \frac{\text{Work}}{\text{Charge}} \times \frac{\text{Charge}}{\text{Time}} =$ VoltagexCurrent

P=VxIwatt.

Energy: Electric energy W is defined as the Power Consumed in a given time.Hence, if currentIAflowsinanelementoveratimeperiodtsecond,whenavoltageVvoltsisapplied across it, the energy consumed is given by

W=Pxt=VxIxtJorwatt.second.

The unit of energy W isJoule(J) or watt. second.However, in practice, the unit of energy is kilowatt. hour (Kwh)

1.2 **Resistance**: Accordingto Ohm's law potential difference (V) acrossthe ends of a conductor is proportional to the current(I) flowing through the conductor at a constant temperature. Mathematically Ohm's law is expressed as

$V \alpha I$ or $V = R \times I$

OrR= $\frac{V}{I}$ whereRistheproportionalityconstantandisdesignatedastheconductor resistance and has the unit of Ohm(Ω).

Conductance : Voltage is induced in a stationary conductor when placed in a varying magnetic field. The induced voltage (e) is proportional to the time rate of change of current, di/dt producing the magnetic field.

$\text{Thereforee} \alpha \frac{di}{dt}$

$\text{Ore} = L \frac{di}{dt}$

eandiarebothfunctionoftime.TheproportionalityconstantLiscalledinductance.

The Unit of inductance is Henery (H).

Capacitance : A capacitor is a Physical device, which when polarized by an electric fieldby applying a suitable voltage across it, storesenergy in the form of a charge separation.

Theabilityofthecapacitortostorechargeismeasuredintermsofcapacitance. Capacitenceofa capacitorisdefinedasthecharge storedperVolt applied.

$$C=\frac{q}{v}=\frac{\text{Coulomb}}{\text{Volt}}=\text{Farad}$$

1.3 ActiveandpassiveBranch:

A branch is said to be active when it contains one or more energy sources. A passive branch does not contain an energy source.

Branch:Abranchisanelementofthe networkhavingonlytwoterminals.

Bilateralandunilateralelement:

A bilateral element conducts equally well in either direction. Resistors and inductors are examples of bilateralelements. When the current voltage relations are different for the two directions of current flow, the element is said to be unilateral. Diode is an unilateral element.

Linear Elements: When the current and voltage relationship in an element can be simulated by a linear equation either algebraic, differential or integral type, the element is said to be linear element.

Non Linear Elements : When the current andvoltage relationship in an element can not besimulated by a linear equation, the element is said to be non linear elements.

1.4 Kirchhoff'sVoltage Law(KVL) :

ThealgebraicsumofVoltages(orvoltagedrops)inanyclosedpathorloopisZero.

ApplicationofKVLwithseriesconnectedvoltagesource.

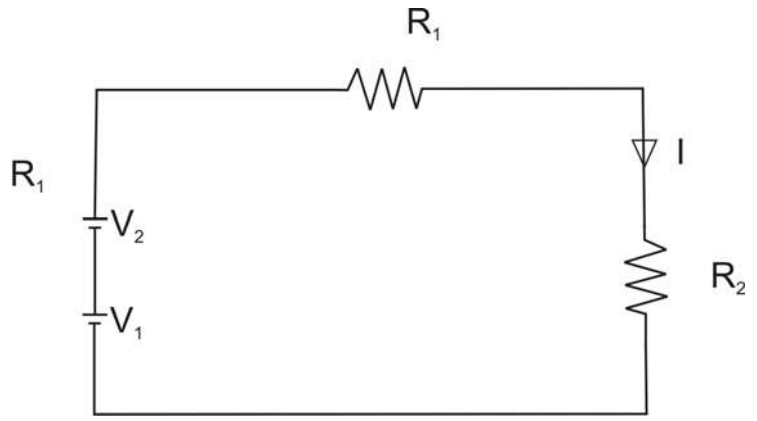


Fig. 1.1

$V_1+V_2–IR_1–IR_2=0$

$=V_1+V_2=I(R_1+R_2)$

$$I = \frac{V_1+V_2}{R_1+R_2}$$

ApplicationofKVLwhilevoltagesourcesare connectedinoppositepolarity.

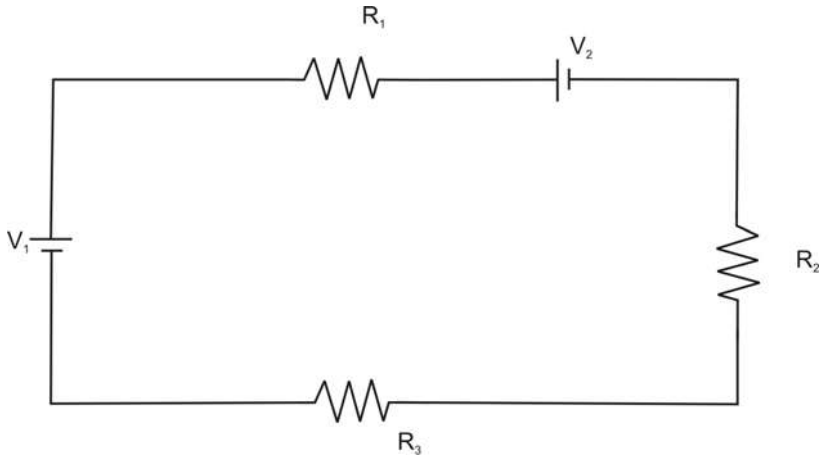


Fig. 1.2

$V_1–IR_1–V_2–IR_2–IR_3=0$

- $V_1–V_2=IR_1+IR_2+IR_3$
- $V_1–V_2=I(R_1+IR_2+IR_3)$

- $I = \frac{V_1 - V_2}{R_1 + R_2 + R_3}$

Kirchaoff'sCurrentLaw(KCL):

Thealgebraicsumof currentsmeetingatajunctionormodeiszero.

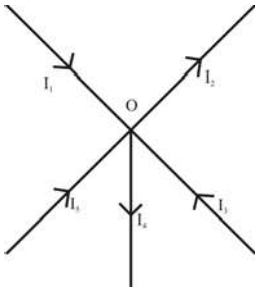


Fig. 1.3

Considering five conductors, carrying currents I_1, I_2, I_3, $I_4$and I_5meeting at a point O. Assuming the incoming currents to be positive and outgoing currents negative.

$I_1+(-I_2)+I_3+(-I_4)+I_5=0$ I_1-

$I_2+ I_3- I_4+ I_5= 0$

$I_1+I_3+I_5=I_2+I_4$

Thus above Law can also be stated as the sum of currents flowing towards any junction in an electric circuit is equal to the sum of the currents flowing away from that junction.

Voltage Division (Series Circuit)

Consideringavoltagesource (E)withresistorsR_1andR_2inseriesacrossit.

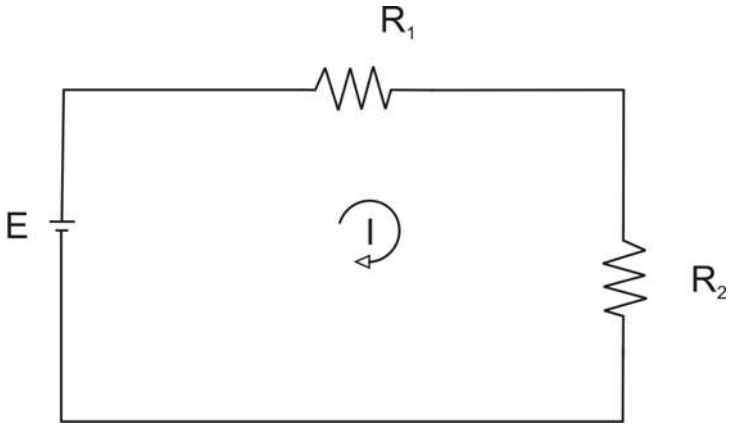


Fig. 1.4

$$I = \frac{ER_1}{+R_2}$$

Voltage drop across $R_1 = I.R_1 = \frac{E.R_1}{R_1+R_2}$

Similarly voltage drop across $R_2 = I.R_2 = \frac{E.R_1}{R_1+R_2}$

Current Division:

A parallel circuit acts as a current divider as the current divides in all branches in a parallel circuit.

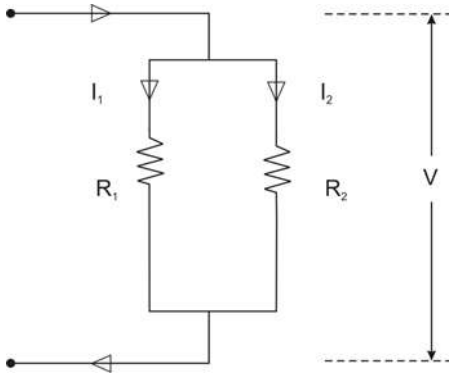


Fig. 1.5

Fig. shown the current I has been divided into I_1 and I_2 in two parallel branches with resistances R_1 and R_2 while V is the voltage drop across R_1 and R_2.

$$I_1 = \frac{V}{R_1} \text{ and } I_2 = \frac{V}{R_2}$$

Let R = Total resistance of the circuit.

Hence $\frac{1}{R} = = \frac{1}{R_1} + \frac{1}{R_2}$

- $R = \frac{R_1R_2}{R_1+R_2}$

$$I = \frac{V}{R} = \frac{V}{\frac{R_1R_2}{R_1+R_2}} = \frac{V(R_1+R_2)}{R_1R_2}$$

But=V $=I_1R_1=I_2R_2$

- $I=I_1R_1\left|\frac{(R_1R_2)}{(R_1+R_2)}\right|$
- $I = \frac{I_1(R_1+R_2)}{R_2}$

Therefore $\boxed{I_1= \frac{IR_2}{R_1+R_2}}$

Similarlyitcanbederivedthat

$$\boxed{I_2= \frac{IR_1}{R_1+R_2}}$$

NETWORKANALYSIS

Differenttermsaredefined below:

1. **Circuit:**Acircuitisaclosedconductingpaththroughwhichanelectriccurrenteither flowor is intended flow

2. **Network:** Acombinationofvariouselectricelements,connectedinany manner. Whatsoever, is called an electric network

3. **Node:**itisanequipotentialpointatwhichtwo ormore circuitelements are joined.

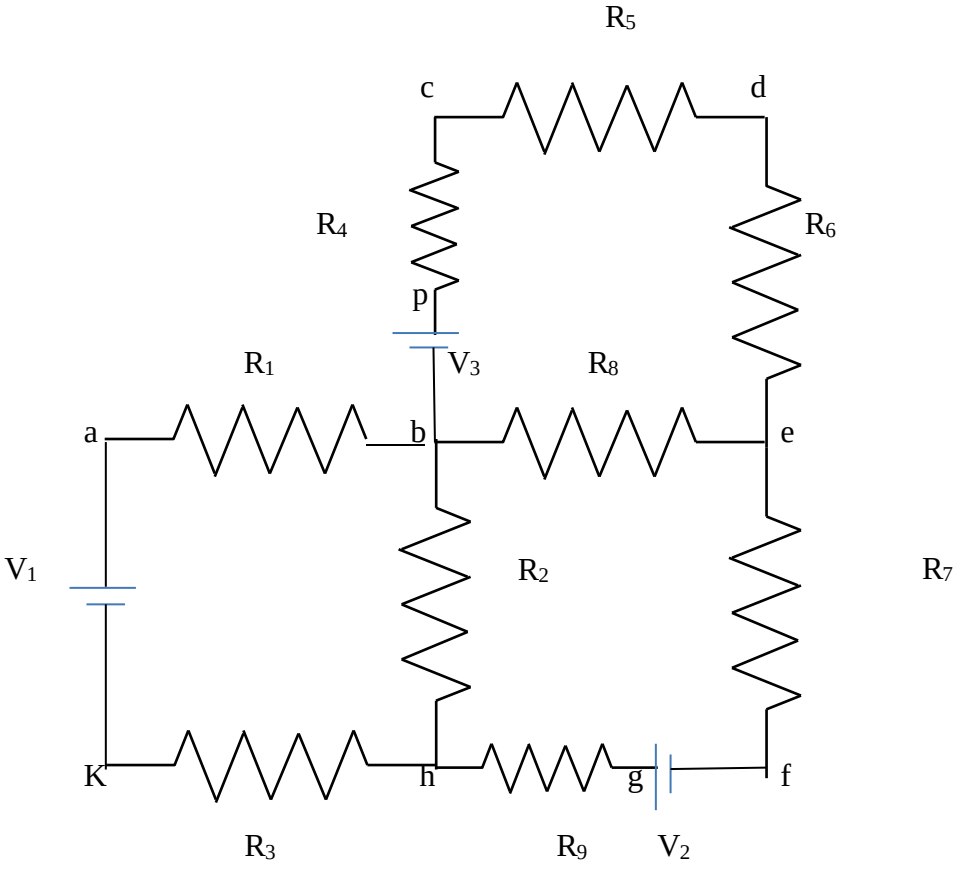
4. **Junction:**it isthat point ofa network wherethree or morecircuit elements are joined.

5. **Branch:**itisapartofanetworkwhichliesbetweenjunctionpoints.

6. **Loop:**It is a closedpath in a circuit in which no element or node is accounted more than once.

7. **Mesh:**Itisaloopthatcontainsnootherloopwithinit.

Example 3.1 In this circuit configuration of figure 3.1, obtain the no. ofi) circuit elements ii) nodes iii) junction points iv) branches and v) meshes.



Solution: i) no. of circuit elements= 12 (9 resistors + 3 voltage sources)

ii) no. of nodes =10 (a, b, c, d, e, f, g, h, k, p)

iii) no. of junction points =3 (b, e, h)

iv) no.ofbranches=5(bcde,be,bh,befgh,bakh)

v) no.of meshes = 3 (abhk, bcde, befh)

3.2 MESH ANALYSIS

Mesh and nodal analysis are two basic important techniques used in finding solutions foranetwork.Thesuitabilityofeithermeshornodalanalysistoaparticularproblemdepends mainly on the number of voltage sources or current sources .If a network has a large number of voltage sources, it is useful to use mesh analysis; as this analysis requires that all the sources in a circuit be voltage sources. Therefore, if there are any current sources in a circuit they are to be converted into equivalent voltage sources,if, onthe other hand, the network has more current sources,nodal analysis is more useful.

Mesh analysis is applicable only for planar networks. For non-planar circuits meshanalysis is not applicable .A circuit is said to be planar, if it can be drawn on a plane surface without crossovers. A non-planar circuit cannot be drawn on a plane surface without a crossover.

Figure 3.2 (a) is a planar circuit. Figure 3.2 (b) is a non-planar circuit and fig. 3.2 (c) is a planar circuit which looks like a non-planar circuit. It has already been discussed that a loopis a closed path. A mesh is defined as a loop which does not contain anyotherloopswithinit. To apply mesh analysis, our first step is to check whether the circuit is planar or not and the second is to select mesh currents. Finally, writing Kirchhoff's voltage law equations in terms of unknowns and solving them leads to the final solution.

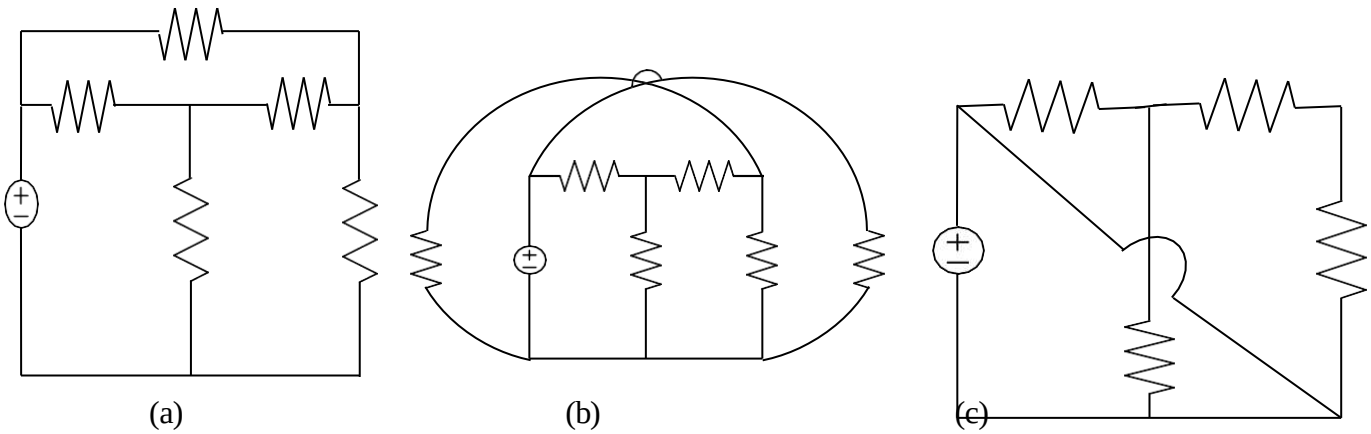


Figure 3.2

Observation of the Fig.3.2 indicates that there are two loops abefa,and bcdeb in the network.LetusassumeloopcurrentsI_1andI_2withdirectionsasindicatedinthefigure.

Considering the loop abefa alone, we observe that current $I_1$is passing through R_1,and (I_1-I_2) is passing through R_2.By applying Kirchhoff's voltage law, we can write

$$V_s=I_1R_1+R_2(I_1-I_2) \quad (3.1)$$

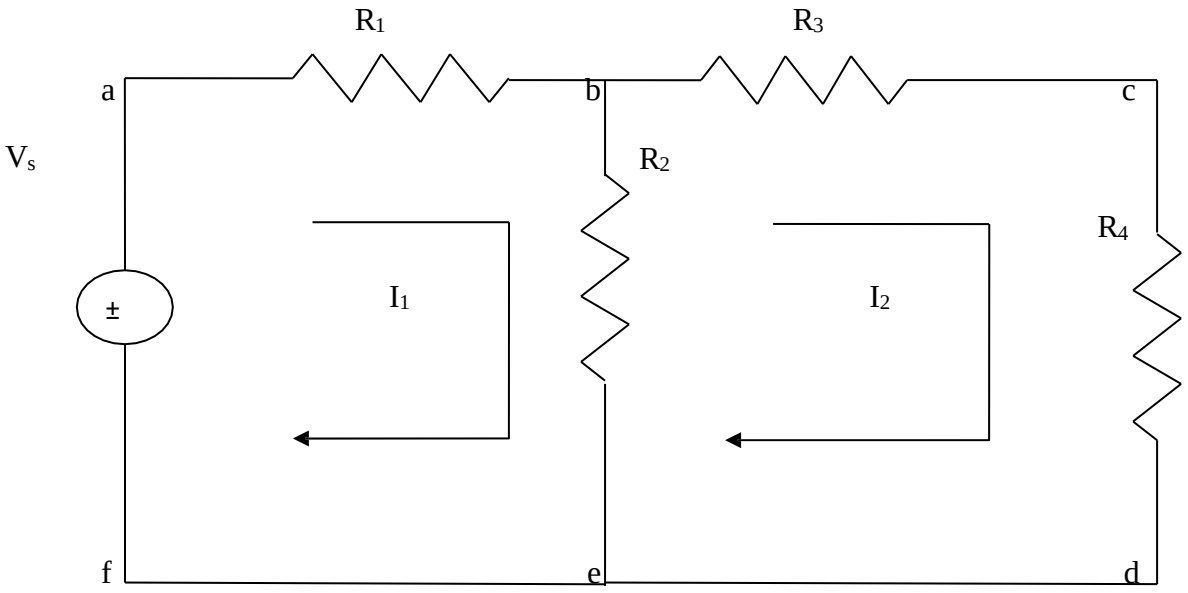


Figure 3.3

Similarly, if we consider the second mesh bcdeb, the current $I_2$is passing through $R_3$and R_4,and (I_2– I_1) is passing through R_2.By applying Kirchhoff's voltage law around the second mesh, we have

$$R_2(I_2-I_1)+R_3I_2+R_4I_2=0 \quad (3.2)$$

Byrearrangingtheaboveequations,thecorrespondingmeshcurrentequationsare

$$I_1(R_1+R_2) - I_2R_2=V_s$$

$$-I_1R_2+(R_2+R_3+R_4)I_2=0 \quad (3.3)$$

By solving the above equations, we can find the currents $I_1$and I_2,.If we observe Fig.3.3, the circuit consists of five branches and four nodes, including the reference node.The number of mesh currents is equal to the number of mesh equations.

And the number of equations=branches-(nodes-1).in Fig.3.3, the required number of mesh current would be 5-(4-1)=2.

IngeneralwehaveBnumberofbranchesandNnumberofnodesincludingthe reference node than number of linearly independent mesh equations M=B-(N-1).

Example3.2 Writethe mesh current equations in the circuit shown infig 3.4 anddetermine the currents.

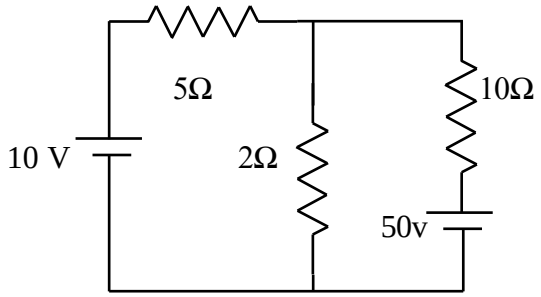


Figure 3.4

Solution: Assume two mesh currents in the direction as indicated in fig. 3.5.Themeshcurrentequationsare

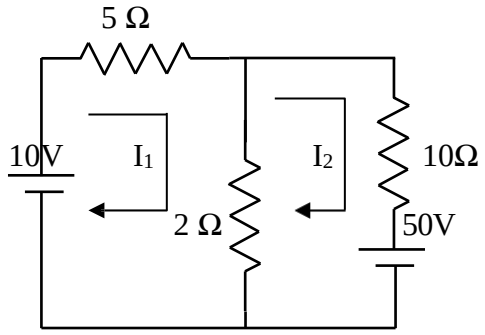


Figure 3.5

$5I_1+2(I_1-I_2)=10$

$101_2+2(1_2-1_1)+50=0$ (3.4)

Wecanrearrangetheaboveequationsas $7I_1-2I_2=10$

$-2I_1+12I_2=-50$ (3.5)

Bysolvingtheaboveequations,wehave$I_1=0.25A$,and$I_2= -4.125$

Here the current in the second mesh I_2, is negative; that is the actual current I_2 flows opposite to the assumed direction of current in the circuit of fig .3.5.

Example3.3 Determinethemeshcurrent I_1 inthecircuitshowninfig.3.6.

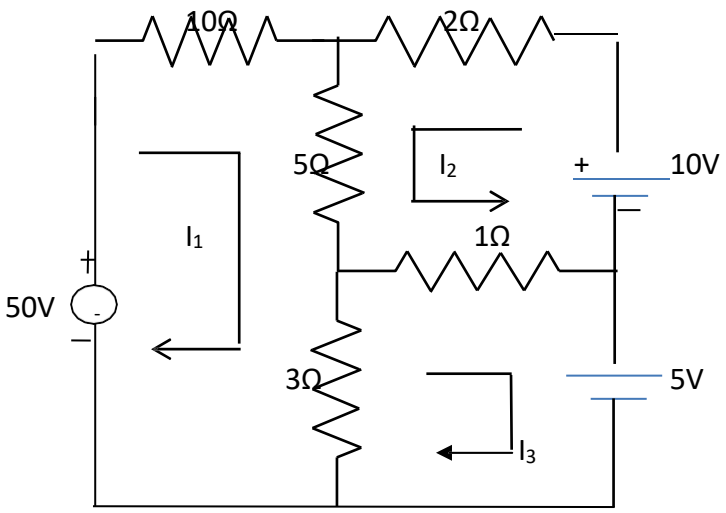


Figure 3.6

Solution: From the circuit, we can from the following three mesh equations

$$10I_1+5(I_1+I_2)+3(I_1-I_3)=50 \quad (3.6)$$

$$2I_2+5(I_2+I_1)+1(I_2+I_3)=10 \quad (3.7)$$

$$3(I_3-I_1)+1(I_3+I_2)=-5 \quad (3.8)$$

Rearrangingtheaboveequationsweget

$$18I_1+5I_2-3I_3=50 \quad (3.9)$$

$$5I_1+8I_2+I_3=10 \quad (3.10)$$

$$-3I_1+I_2+4I_3=-5 \quad (3.11)$$

Accordingto theCramer's rule

$$I_1=\frac{\begin{vmatrix}50 & 5 & -3\\ 10 & 8 & 1\\ -5 & 1 & 4\end{vmatrix}}{\begin{vmatrix}18 & 5 & -3\\ 5 & 8 & 1\\ -3 & 1 & 4\end{vmatrix}}=\frac{1175}{356}$$

OrI_1= 3.3ASimilarly,

$$I_2=\frac{\begin{vmatrix}18 & 50 & -3\\ 5 & 10 & 1\\ -3 & -5 & 4\end{vmatrix}}{\begin{vmatrix}18 & 5 & -3\\ 5 & 8 & 1\\ -3 & 1 & 4\end{vmatrix}}=\frac{-355}{356}$$

OrI_2=-0.997A (3.12)

$$I_3=\frac{\begin{vmatrix}18 & 5 & 50\\ 5 & 8 & 10\\ -3 & 1 & -5\end{vmatrix}}{\begin{vmatrix}18 & 5 & -3\\ 5 & 8 & 1\\ -3 & 1 & 4\end{vmatrix}}=\frac{525}{356}$$

OrI_3=1.47A (3.13)

$\therefore I_1$=3.3A,I_2=-0.997A,I_3=1.47A

3.3 MESH EQUATIONS BY INSPECTION METHOD The mesh equations for a general planar network can be writtenby inspection without going through the detailed steps. Consider a three mesh networks as shown in figure 3.7

The loop equation are $I_1R_1+ R_2(I_1-I_2) = V_1$

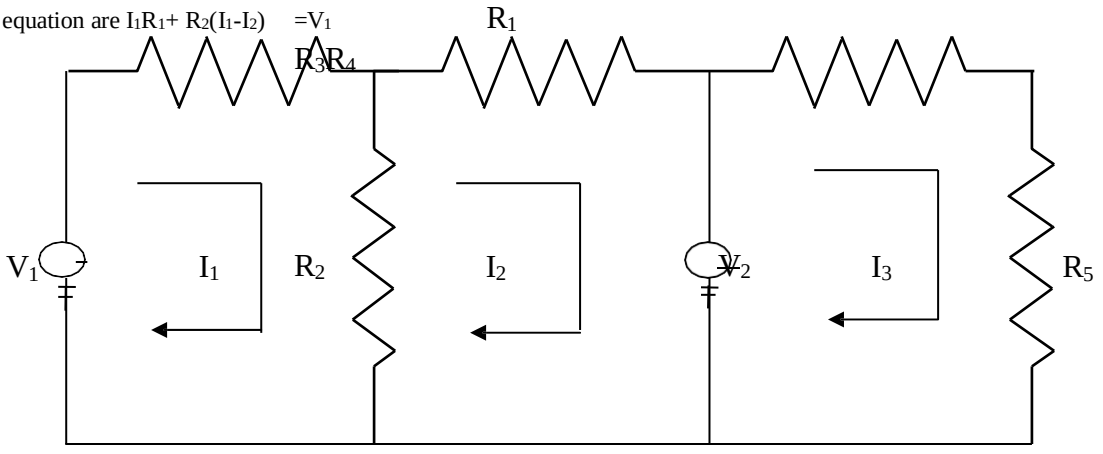


Figure 3.7

$$R_2(I_2-I_1)+I_2R_3=-V_2 \quad 3.14$$

$$R_4I_3+R_5I_3=V_2 \quad 3.15$$

Reorderingtheaboveequations,wehave

$$(R_1+R_2)I_1-R_2I_2=V_1 \quad 3.16$$

$$-R_2I_1+(R_2+R_3)I_2=-V_2 \quad 3.17$$

$$(R_4+R_5)I_3=V_2 \quad 3.18$$

Thegeneralmeshequationsforthreemeshresistivenetworkcanbewrittenas

$$R_{11}I_1\pm R_{12}I_2\pm R_{13}I_3= V_a \quad 3.19$$

$$\pm R_{21}I_1+R_{22}I_2\pm R_{23}I_3=V_b \quad 3.20$$

$$\pm R_{31}I_1\pm R_{32}I_2+R_{33}I_3=V_c \quad 3.21$$

Bycomparingtheequations3.16,3.17and3.18withequations3.19,3.20and3.21 respectively, the following observations can be taken into account.

1. The self-resistancein each mesh
2. Themutualresistancesbetweenallpairsofmeshesand
3. Thealgebraic sumof thevoltagesineachmesh.

The self-resistance of loop 1, $R_{11}=R_1+R_2$, is the sum of the resistances through which I_1passes.

The mutual resistance of loop 1, $R_{12}= -R_2$, is the sum of the resistances common to loop currents $I_1$and I_2.If the directions ofthe currents passing through the common resistances are the same, the mutual resistance will have a positive sign; and if the directions of the currents passing through the common resistance are opposite then the mutual resistance will have a negative sign.

$V_a=V_1$is the voltage which drives the loop 1. Here the positive sign is used if the direction of the currents is the same as the direction of the source. If the current direction is opposite to the direction of the source, then the negative sign is used.

Similarly $R_{22}=R_2+R_3$and $R_{33}=R_4+R_5$are the self-resistances of loops 2 and 3 respectively. The mutual resistances $R_{13}=0$, $R_{21}= -R_2$, $R_{23}=0$, $R_{31}=0$, $R_{32}=0$ are the sums of the resistances common to the mesh currents indicated in their subscripts.

$V_b=-V_2$, $V_c=V_2$arethesumofthevoltagesdrivingtheirrespectiveloops.

Example3.4writethemeshequationforthecircuitshowninfig.3.8

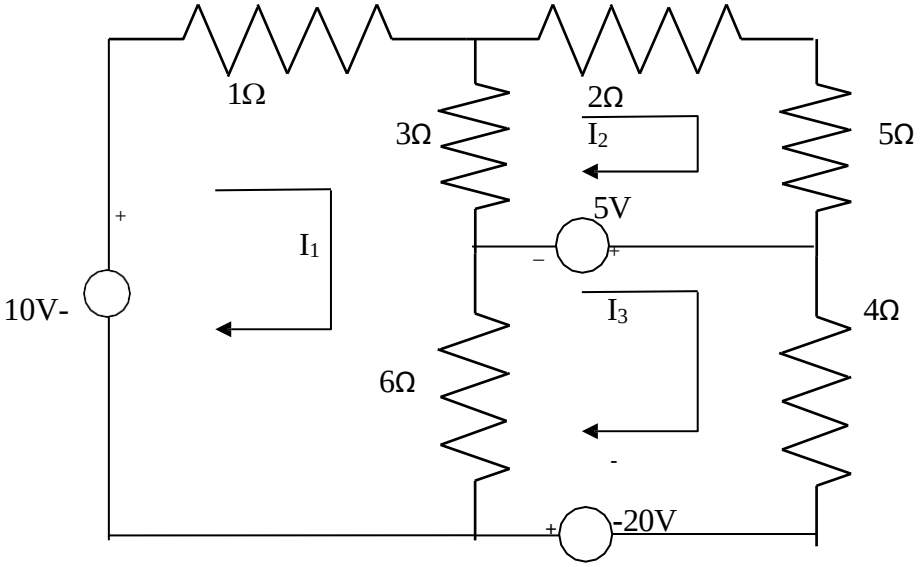


Figure3.8

Solution: thegeneralequationforthreemeshequationare

$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3 = V_a$ (3.22)

$\pm R_{21}I_1 + R_{22}I_2 \pm R_{23}I_3 = V_b$ (3.23)

$\pm R_{31}I_1 \pm R_{32}I_2 + R_{33}I_3 = V_c$ (3.24)

Considerequation3.22

R_{11}=selfresistanceof loop1=(1Ω+3Ω+6Ω)=10Ω

R_{12}=themutual resistancecommon toloop 1and loop2 =-3 Ω

Herethenegativesignindicatesthatthecurrentsareinoppositedirection. R_{13}=

the mutual resistance common to loop 1 & 3= -6 Ω

V_a=+10 V,the voltagethe driving theloop 1.

HerehepositivesignindicatestheloopcurrentI1isinthesamedirectionasthe source element.

Thereforeequation3.22canbewrittenas

$10I_1-3I_2-6I_3=10V$ (3.25)

ConsiderEq.3.23

R_{21}=themutual resistancecommon toloop 1and loop2 =-3 Ω

R_{22}= self resistance of loop 2=(3Ω+ 2 Ω +5 Ω) =10 Ω

R_{23}=0,thereisnocommonresistancebetweenloop2and3. V_b= -5 V, the voltage driving the loop 2.

ThereforeEq. 3.23 canbe written as

$-3I_1+10I_2=-5V$ (3.26)

ConsiderEq.3.24

R_{31}=themutualresistancecommontoloop1andloop3=-6Ω R_{32}= the mutual resistance common to loop 3 and loop 2 = 0 R_{33}= self resistance of loop 3=(6Ω+ 4 Ω) =10 Ω

V_c=the algebraicsumofthevoltagedrivingloop3

=(5V+20V)=25V (3.27)

Therefore,Eq3.24canbewrittenas$-6I_1+10I_3=25V$

$-6I_1-3I_2-6I_3=10V$

$-3I_1+10I_2=-5V$

$-6I_1+10I_3=25V$

3.4 SUPERMESHANALYSIS

Suppose any of the branches in the network has a current source, then it is slightly difficult to apply mesh analysis straight forward because first we should assume an unknown voltage across the current source, writing mesh equation as before, and then relate the source current totheassignedmeshcurrents.Thisisgenerallyadifficultapproach.Onwaytoovercomethis difficulty is by applying the supermesh technique. Here we have to choose the kind of supermesh. A supermesh is constituted by two adjacent loops that have a common current source. As an example, consider the network shown in the figure 3.9.

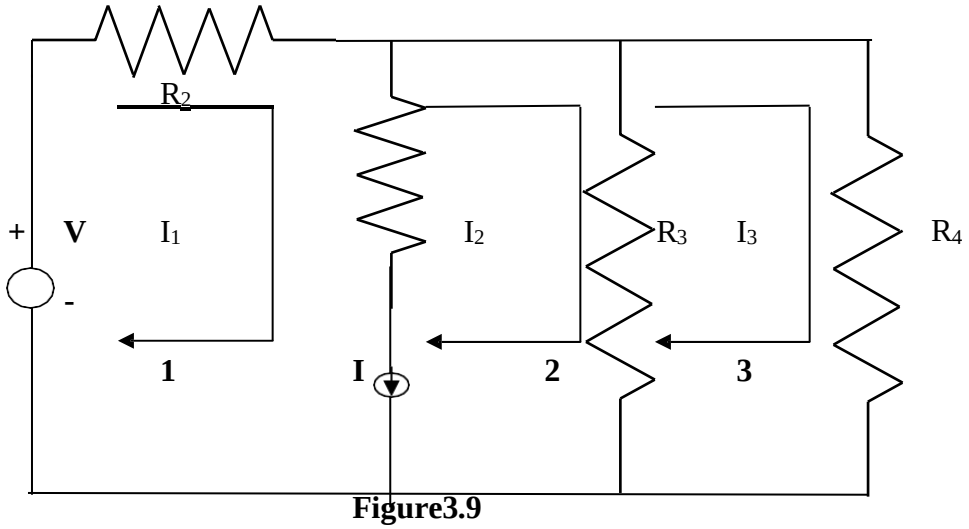


Figure3.9

HerethecurrentsourceIisinthecommonboundaryforthetwomeshes1and2.Thiscurrent source creates a supermesh, which is nothing but a combination of meshes 1 and 2.

$R_1I_1+ R_3(I_2-I_3)=V$

Or $R_1I_1+R_3I_2-R_4I_3=V$

Consideringmesh3,wehave

$R_3(I_3-I_2)+ R_4I_3=0$

Finally the current I fromcurrent source is equal to the difference between two mesh currents i.e.

$I_1-I_2=I$

wehavethusformedthreemeshequationswhichwecansolveforthethreeunknown currents in the network.

Example3.5.Determinethe currentin the5Ω resistorinthenetworkgiveninFig.3.10

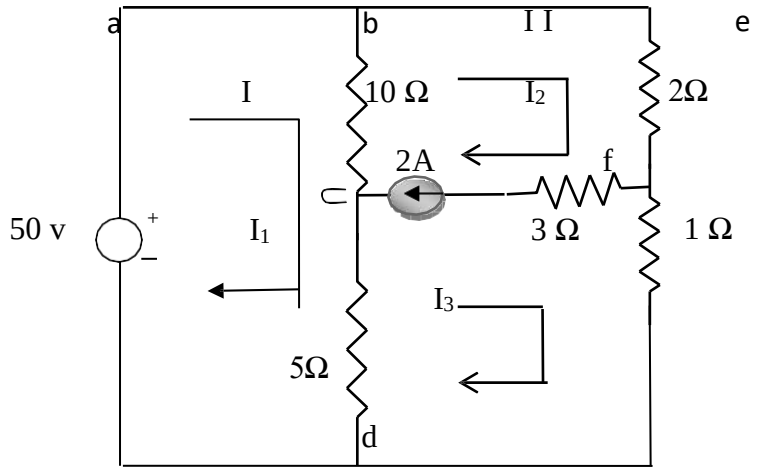


Figure 3.10

Solution:- Fromthe firstmesh, i.e.abcda, we have

$$50=10(I_1-I_2)+5(I_1-I_3)$$

$$Or15I_1-10I_2-5I_3=50 \quad (3.28)$$

Fromthe second and third meshes.we can form a super mesh

$$10(I_2-I_1)+2I_2+I_3+5(I_3-I_1)=0$$

$$Or-15I_1+12I_2+6I_3=0 \quad (3.29)$$

Thecurrentsourceisequaltothedifferencebetween II and III meshcurrents

i.e. $I_2 - I_3 = 2A$ (3.30)

Solving 3.28., 3.29 and 3.30. we have

$I_1 = 19.99A$, $I_2 = 17.33A$, and $I_3 = 15.33A$

The current in the 5Ω resistor $= I_1 - I_3$

$= 19.99 - 15.33 = 4.66A$

The current in the 5Ω resistor is 4.66A.

Example 3.6. Write the mesh equations for the circuit shown in fig. 3.11 and determine the currents, I_1, I_2 and I_3.

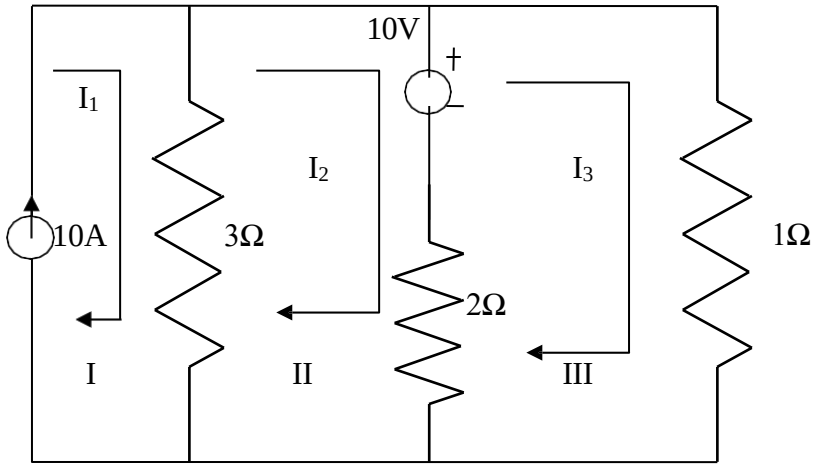


Figure 3.11

Solution ; In fig 3.11, the current source lies on the perimeter of the circuit, and the first mesh is ignored. Kirchhoff's voltage law is applied only for second and third meshes .

From the second mesh, we have

$3(I_2 - I_1) + 2(I_2 - I_3) + 10 = 0$

Or $-3I_1 + 5I_2 - 2I_3 = -10$ (3.31)

From the third mesh, we have $I_3 +$

$2(I_3 - I_2) = 10$

Or $-2I_2 + 3I_3 = 10$ (3.32)

From the first mesh, $I_1=10A$ (3.33)

From the abovethree equations, we get

$I_1=10A$, $I_2=7.27$, $I_3=8.18A$

3.5 NODALANALYSIS

In the chapter I we discussed simple circuits containing only two nodes, including the reference node. In general, in a N node circuit, one of the nodes is chosen as the reference or datum node, then it is possible to write N -1nodal equations by assuming N-1 node voltages. Forexample,a10 node circuit requires nine unknown voltages and nine equations. Each node in a circuit can be assigned a number or a letter. The node voltage is the voltage of a given node with respect to oneparticularnode,calledthereference node,whichweassumeatzeropotential.Inthecircuitshown in fig. 3.12, node 3 is assumed as theReference node. The voltage at node 1 is the voltage at thatnode with respect to node 3. Similarly, the voltage at node 2 is the voltage at that node with respect to node 3. Applying Kirchhoff's current law at node 1, the current entering is the current leaving (See Fig.3.13)

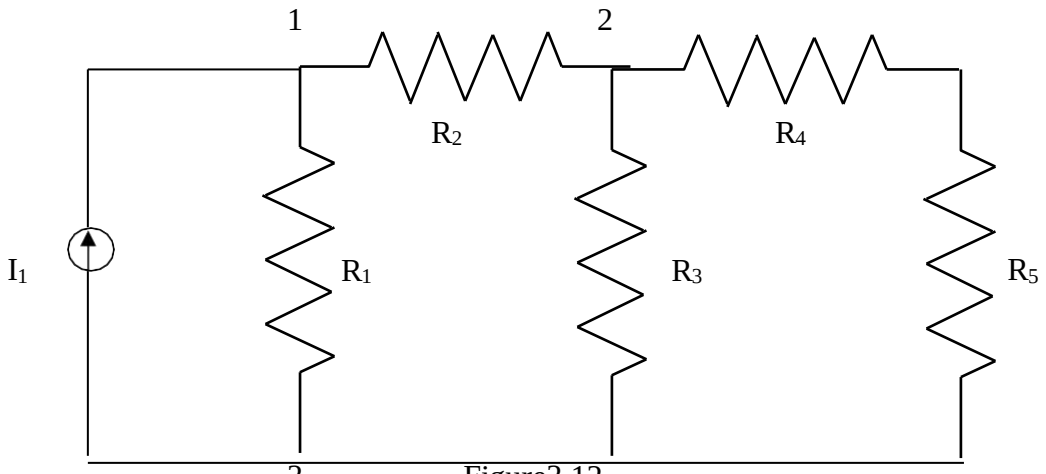


Figure3.12

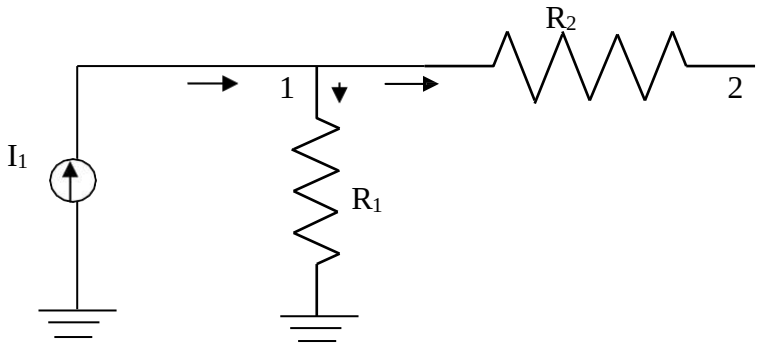


Figure3.13

$I_1= V_1/R_1+ (V_1-V_2)/R_2$

Where V_1 and V_2 are the voltages at node 1 and 2, respectively. Similarly, at node 2. the current entering is equal to the current leaving as shown in fig. 3.14

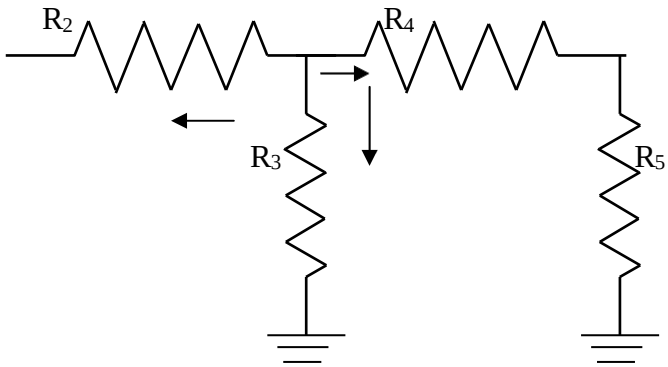


Figure 3.14

$(V_2-V_1)/R_2+V_2/R_3+V_2/(R_4+R_5)=0$

Rearranging the above equations, we have

$V_1[1/R_1+1/R_2]-V_2(1/R_2)= I_1$

$-V_1(1/R_2)+V_2[1/R_2+1/R_3+1/(R_4+R_5)]=0$

From the above equations we can find the voltages at each node.

Example 3.7 Determine the voltages at each node for the circuit shown in fig 3.15

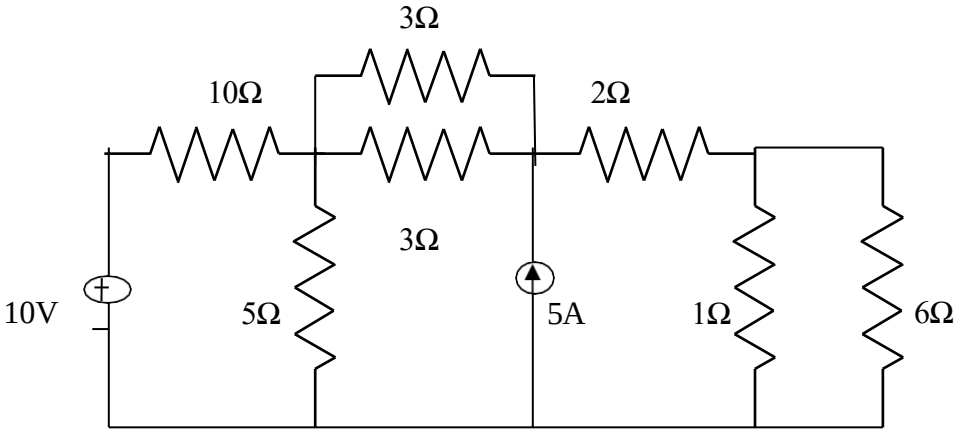


Figure 3.15

Solution: At node 1, assuming that all currents are leaving, we have $(V_1-10)/10 + (V_1-V_2)/3 + V_1/5 + (V_1-V_2)/3 = 0$

Or $V_1[1/10+1/3+1/5 +1/3]-V_2[1/3+1/3]=1$

$0.96V_1-0.66V_2=1$ (3.36)

At node 2, assuming that all currents are leaving except the current from current source, we have

$(V_2-V_1)/3+(V_2-V_1)/3+(V_2-V_3)/2=5$

$-V_1[2/3]+V_2[1/3+1/3+1/2]-V_3(1/2)=5$

$-0.66V_1+1.16V_2-0.5V_3=5$ (3.37)

Atnode3assumingallcurrentsareleaving,wehave (V_3-

$V_2)/2 + V_3/1 + V_3/6 =0$

$-0.5V_2+1.66V_3=0$ (3.38)

ApplyingCramer'sruleweget

$$V_1=\frac{\begin{vmatrix} 1 & -0.66 & 0 \\ 5 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}=\frac{7.154}{0.887}=8.06$$

Similarly,

$$V_2=\frac{\begin{vmatrix} 0.96 & 1 & 0 \\ -0.66 & 5 & -0.5 \\ 0 & 0 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}=\frac{9.06}{0.887}=10.2$$

$$V_3=\frac{\begin{vmatrix} 0.96 & -0.66 & 1 \\ -0.66 & 1.16 & 5 \\ 0 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}=\frac{2.73}{0.887}=3.07$$

3.6 NODAL EQUATIONS BY INSPECTION METHOD The nodal equations for a general planar network can also be written byinspection without going through the detailed steps. Consider a three node resistive network, including the reference node, as shown in fig3.16

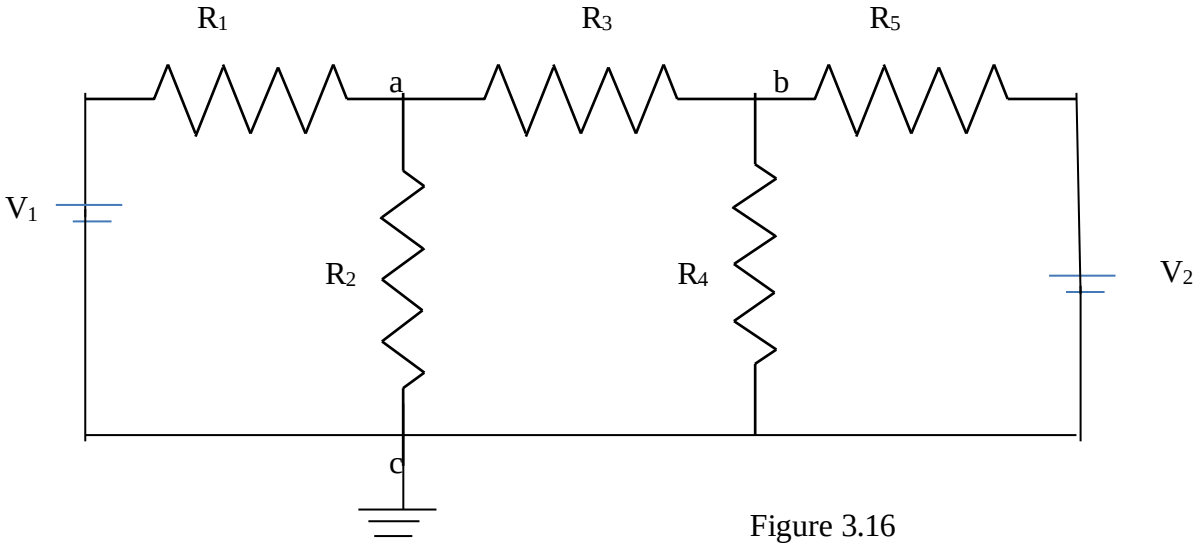


Figure 3.16

Infig.3.16thepointsaandbaretheactualnodesandcisthereferencenode. Now consider the nodes a and b separately as shown in fig 3.17(a) and (b)

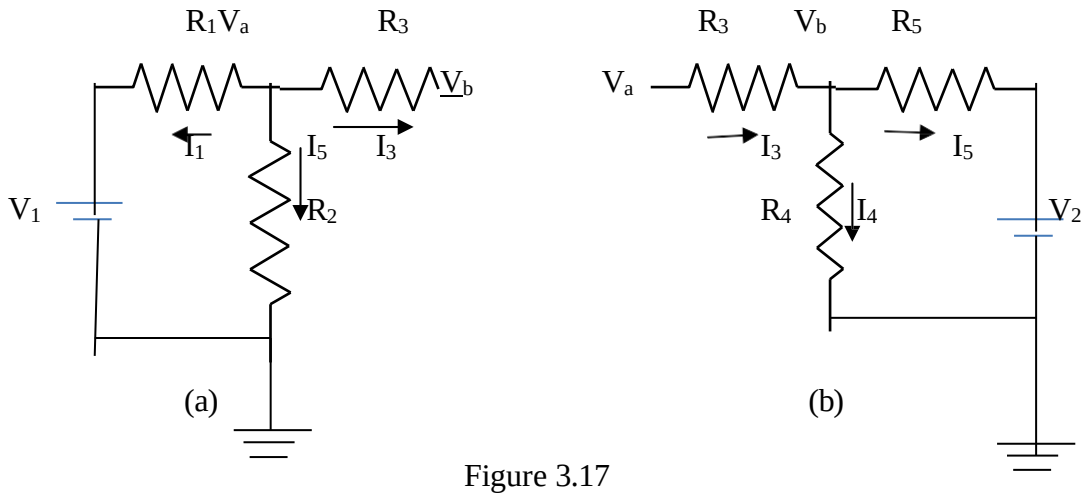


Figure 3.17

Infig3.17(a),accordingtoKirchhoff'scurrentlawwehave

$I_1+I_2+I_3=0$

$(V_a-V_1)/R_1+V_a/R_2+(V_a-V_b)/R_3=0$ (3.39)

Infig3.17(b),ifweapplyKirchhoff'scurrentlaw

$I_4+I_5=I_3$

$\therefore (V_b-V_a)/R_3+V_b/R_4+(V_b-V_2)/R_5=0$ (3.40)

Rearrangingtheaboveequationsweget

$(1/R_1+1/R_2+1/R_3)V_a-(1/R_3)V_b=(1/R_1)V_1$ (3.41)

$(-1/R_3)V_a+(1/R_3+1/R_4+1/R_5)V_b=V_2/R_5$ (3.42)

In general, the above equation can be written as

$G_{aa}V_a+G_{ab}V_b=I_1$ (3.43)

$G_{ba}V_a+G_{bb}V_b=I_2$ (3.44)

By comparing Eqs 3.41,3.42 and Eqs 3.43, 3.44 we have the self conductance at node a, $G_{aa}=(1/R_1+ 1/R_2+ 1/R_3)$ is the sum of the conductances connected to node a. Similarly, $G_{bb}= (1/R_3+ 1/R_4+1/R_5)$ is the sum of the conductances connected to node b. $G_{ab}=(-1/R_3)$ is the sum of the mutual conductances connected to node *a* and node *b*. Here all the mutual conductances have negative signs. Similarly, $G_{ba}= (-1/R_3)$ is also a mutual conductance connected between nodes *b* and *a*. $I_1$and $I_2$are the sum of the source currents at node *a* and node *b*, respectively. The current which drives into the node has positive sign, while the current that drives away from the node has negative sign.

Example3.8forthecircuitshowninthefigure3.18writethenodeequationsbythe inspection method.

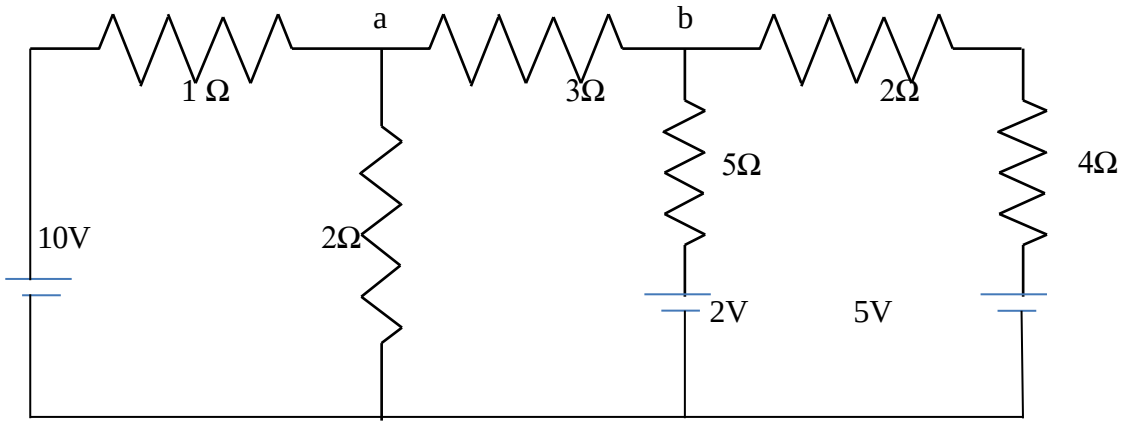


Fig 3.18

Solution:-

Thegeneralequationsare

$G_{aa}V_a+G_{ab}V_b=I_1$ (3.45)

$G_{ba}V_a+G_{bb}V_b=I_2$ (3.46)

Consider equation3.45

G_{aa}=(1+1/2+1/3)mho.Theselfconductanceatnode*a*isthesumoftheconductances connected to node *a*.

G_{bb}=(1/6+1/5+1/3)mhotheselfconductanceatnode*b*isthesumofconductances connected to node *b*.

G_{ab}=-(1/3)mho,themutualconductancesbetweennodes*a*and*b*isthesumofthe conductances connected between node *a* and *b*.

SimilarlyG_{ba}=-(1/3),thesumofthemutualconductancesbetweennodes*b*and*a*. I_1=10/1 =10

A, the source current at node *a*,

I_2=(2/5+5/6)=1.23A,thesourcecurrentatnode*b*. Therefore, the

nodal equations are

$1.83V_a-0.33V_b=10$ (3.47)

$-0.33V_a+0.7V_b=1.23$ (3.48)

3.7 SUPERNODE ANALYSIS

Suppose any of the branches in the network has a voltage source, thenitisslightlydifficult to apply nodal analysis. One way to overcome this difficulty is to apply the supernodetechnique.In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node and then the equations are formed by applying Kirchhoff's current law as usual. This is explained with the help of fig. 3.19

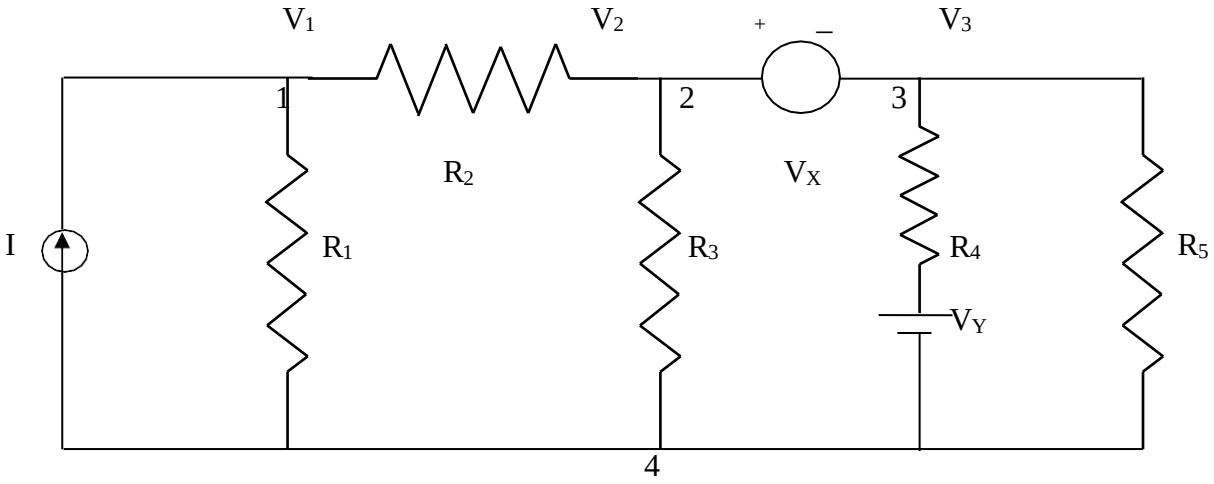


FIG3.19

Itisclearfromthefig.3.19,thatnode4isthereferencenode.ApplyingKirchhoff'scurrent law at node 1, we get

$I=(V_1/R_1)+(V_1-V_2)/R_2$

Due to the presence of voltage source $V_\chi$in between nodes 2 and 3 , it is slightly difficult to find out the current. The supernode technique can be conveniently applied in this case.

Accordingly,wecanwritethecombinedequationfornodes2and3asunder.

$(V_2-V_1)/R_2+V_2/R_3+(V_3-V_y)/R_4+V_3/R_5=0$

Theotherequationis

$V_2-V_3=V_x$

Fromthe above three equations, wecanfindthethreeunknownvoltages.

Example3.9 Determinethecurrentinthe5Ωresistorforthecircuitshowninfig. 3.20

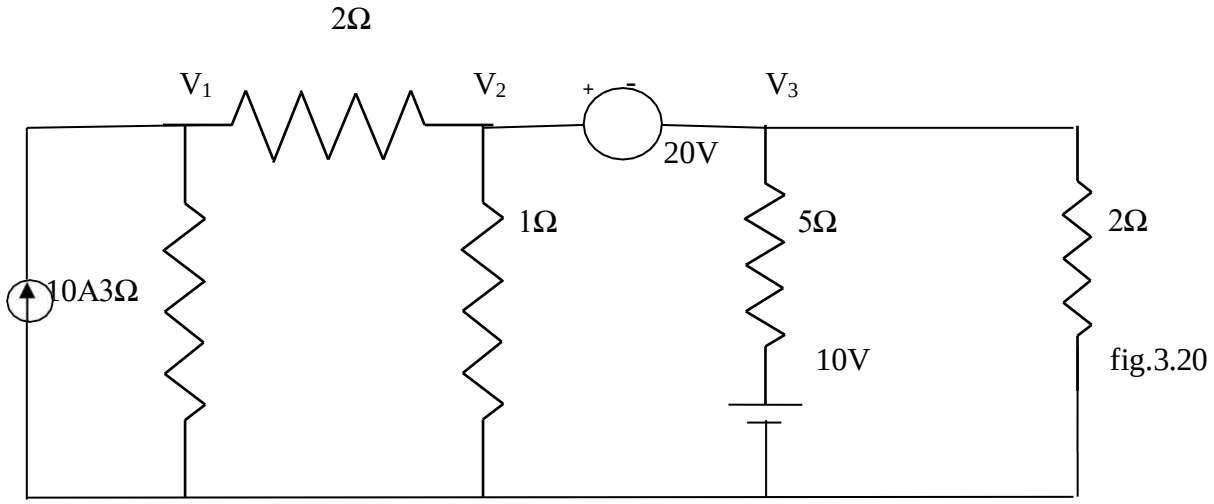


fig.3.20

Solution. Atnode1

$10=V_1/3+(V_1-V_2)/2$

Or $V_1[1/3+1/2]-(V_2/2)-10=0$

$0.83V_1-0.5V_2-10=0$ (3.49)

Atnode2and3,thesupernodeequationis

$(V_2-V_1)/2+V_2/1+(V_3-10)/5+V_3/2=0$

Or $-V_1/2+V_2[(1/2)+1]+V_3[1/5+1/2]=2$

Or $-0.5V_1+1.5V_2+0.7V_3-2=0$ (2.50)

Thevoltagebetweennodes2and3isgivenby

$V_2-V_3=20$ (3.51)

Chapter-

04NETWORKTHEOR

INTRODUCTION M

This chapter introduces a number of theorems that have application throughout the field of electricity and electronics. Not only can they be used to solve networks such as encountered in the previous chapter, but they also provide an opportunity to determine the impact of a particular source or element on the response of the entire system. In most cases, the network to be analyzed and the mathematics required to find the solution are simplified. All of the theorems appear again in the analysis of ac networks. In fact, the application of each theorem to ac networks is very similar in content to that found in this chapter.

The first theorem to be introduced is the superposition theorem, followed by Thévenin's theorem, Norton's theorem, and the maximum power transfer theorem. The chapterconcludes with a brief introduction to Millman's theorem and the substitution and reciprocity theorems.

SUPERPOSITIONTHEOREM

The superposition theorem states that "The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source."

In other words, this theorem allows us to find a solution for a current or voltage using only one source at a time. Once we have the solution for each source, we can combine the results to obtain the total solution.The termalgebraic appears in the abovetheoremstatement because the currents resulting from the sources of the network can have different directions, just as the resulting voltages can have opposite polarities.

If we are to consider the effects of each source, the other sources obviously must be removed. Setting a voltage source to zero volts is like placing a short circuit across its terminals. Therefore, when removing a voltage source from a network schematic, replace it with a direct connection (short circuit) of zero ohms. Any internal resistance associated with the source must remain in the network.

Setting a current source to zero amperes is like replacing it with an open circuit. Therefore, when removing a current source from a network schematic, replace it by an open circuit of infinite ohms. Any internal resistance associated with the source must remain in the network.

Theabovestatementsareillustratedin Fig.

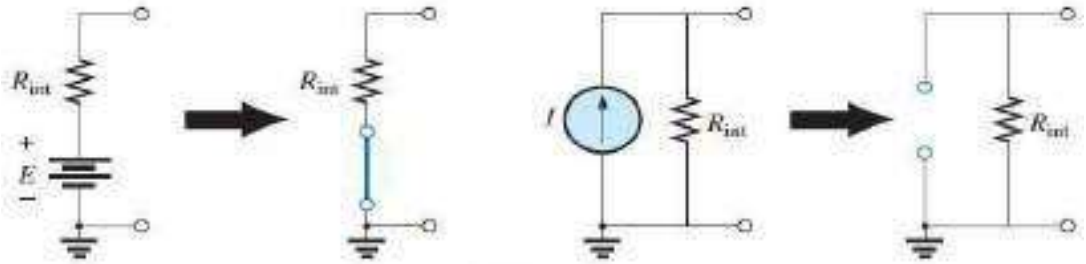


FIG. 9.1

Removing a voltage source and a current source to permit the application of the superposition theorem.

EXAMPLE9.1

a.Usingthesuperpositiontheorem,determinethecurrent through resistor *R*2 for the network in Fig. 9.2.

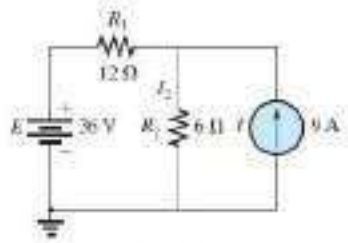


FIG. 9.2

Network to be analyzed in Example 9.1 using the superposition theorem.

Solutions:

In order to determine the effect of the 36 V voltage source,the current source must be replaced by an open-circuit equivalent as shown in Fig. 9.3. The result is a simple series circuit with a current equal to

$$I_2' = \frac{E}{R_T} = \frac{E}{R_1 + R_2} = \frac{36\ \text{V}}{12\ \Omega + 6\ \Omega} = \frac{36\ \text{V}}{18\ \Omega} = 2\ \text{A}$$

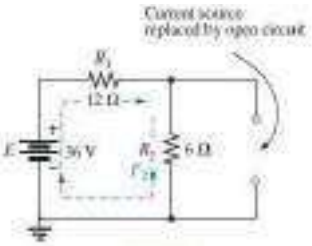


FIG. 9.3

Replacing the 9 A current source in Fig. 9.2 by an open circuit to determine the effect of the 36 V voltage source on current I_2.

Examining the effect of the 9 A current source requires replacing the 36 V voltage source by a short-circuit equivalent as shown in Fig.9.4.Theresultisaparallelcombinationofresistors*R*1and2. Applying the current divider rule results in

$$I_2'' = \frac{R_1(I)}{R_1 + R_2} = \frac{(12\ \Omega)(9\ \text{A})}{12\ \Omega + 6\ \Omega} = 6\ \text{A}$$

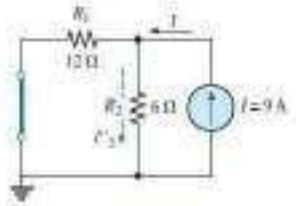


FIG. 9.4

Replacing the 36 V voltage source by a short-circuit equivalent to determine the effect of the 9 A current source on current I_2.

Since the contribution to current *I*2 has the same direction for each source, as shown in Fig. 9.5, the total solution for current *I*2 is the sum of the currents established by the two sources. That is,

$$I_2 = I_2' + I_2'' = 2\ \text{A} + 6\ \text{A} = 8\ \text{A}$$

EXAMPLE9.2 Usingthesuperpositiontheorem,determinethe current through the 12 Ω resistors in Fig. 9.8. Note that this is a two-source network of the type examined in the previous chapter when we applied branch-current analysis and mesh analysis.

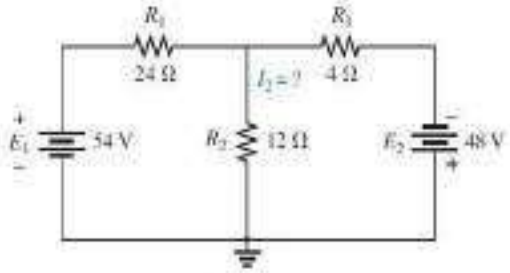


FIG. 9.8

Using the superposition theorem to determine the current through the 12 Ω resistor (Example 9.2).

Solution: Considering the effects of the 54 V source requires replacing the 48 V source by a short-circuit equivalent as shown in Fig. 9.9. The result is that the 12 Ω and 4 Ω resistors are in parallel. The total resistance seen by the source is therefore,

$$R_T = R_1 + R_2 \| R_3 = 24\ \Omega + 12\ \Omega \| 4\ \Omega = 24\ \Omega + 3\ \Omega = 27\ \Omega$$

and the source current is

$$I_s = \frac{E_1}{R_T} = \frac{54\ \text{V}}{27\ \Omega} = 2\ \text{A}$$

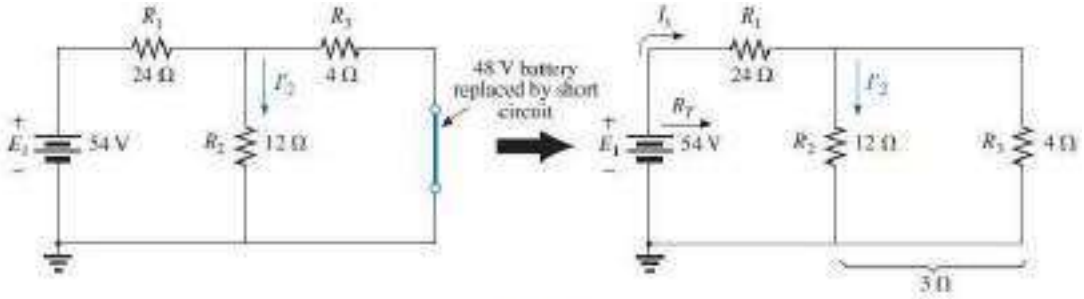


FIG. 9.9
Using the superposition theorem to determine the effect of the 54 V voltage source on current I_2 in Fig. 9.8.

Using the current divider rule results in the contribution to I_2 due to the 54 V source:

$$I_2' = \frac{R_3 I_s}{R_3 + R_2} = \frac{(4\ \Omega)(2\ \text{A})}{4\ \Omega + 12\ \Omega} = 0.5\ \text{A}$$

If we now replace the 54 V source by a short-circuit equivalent, the network in Fig. 9.10 results. The result is a parallel connection for the 12 Ω and 24 Ω resistors.

Therefore, the total resistance seen by the 48 V source is

$$R_T = R_3 + R_2 \| R_1 = 4\ \Omega + 12\ \Omega \| 24\ \Omega = 4\ \Omega + 8\ \Omega = 12\ \Omega$$

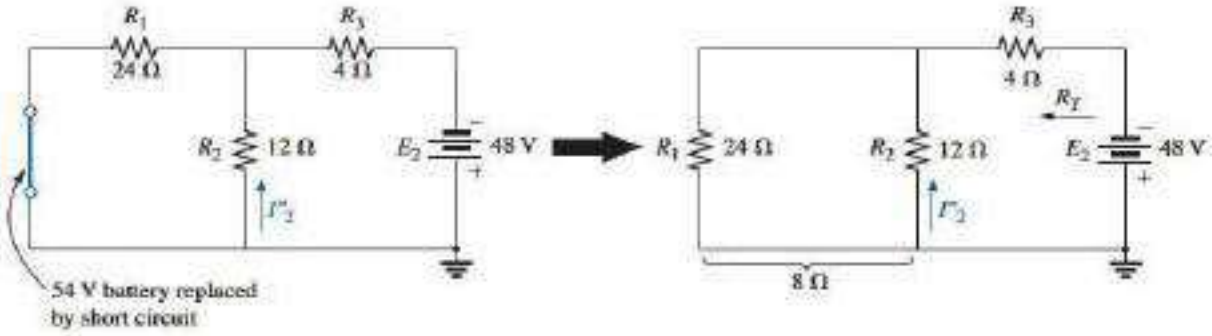


FIG. 9.10
Using the superposition theorem to determine the effect of the 48 V voltage source on current I_2 in Fig. 9.8.

and the source current is

$$I_s = \frac{E_2}{R_T} = \frac{48\ \text{V}}{12\ \Omega} = 4\ \text{A}$$

Applying the current divider rule results in

$$I_2'' = \frac{R_1(I_s)}{R_1 + R_2} = \frac{(24\ \Omega)(4\ \text{A})}{24\ \Omega + 12\ \Omega} = 2.67\ \text{A}$$

It is now important to realize that current I_2 due to each source has a different direction, as shown in Fig. 9.11. The net current therefore is the difference of the two and in the direction of the larger as follows:

$$I_2 = I_2'' - I_2' = 2.67\ \text{A} - 0.5\ \text{A} = \mathbf{2.17\ A}$$

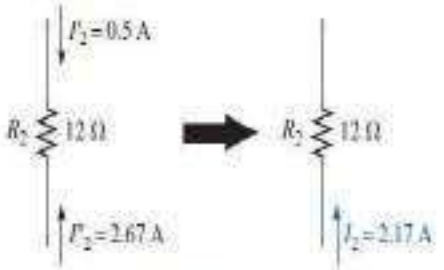


FIG. 9.11
Using the results of Figs. 9.9 and 9.10 to determine current I_2 for the network in Fig. 9.8.

EXAMPLE9.3 Usingthesuperpositiontheorem,determinecurrent*I*1forthenetworkin

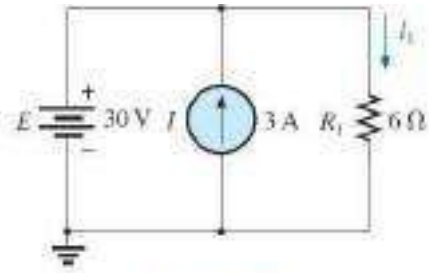


FIG. 9.12
Two-source network to be analyzed using the superposition theorem in Example 9.3.

Solution:Sincetwosourcesarepresent,thereareetwonetworks to be analyzed. First let us determine the effects of the voltage source by setting the current source to zero amperes as shown in Fig. 9.13. Note that the resulting current is defined as I_1' because it is the current through resistor *R*1 due to the voltage source only.

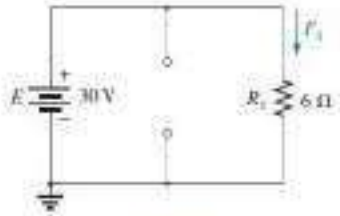


FIG. 9.13
Determining the effect of the 30 V supply on the current I_1 in Fig. 9.12.

Due to the open circuit, resistor *R*1 is in series (and, in fact, in parallel) with the voltage source *E*. The voltage across the resistor is the applied voltage, and current I_1' is determined by

$$I_1' = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{30\text{ V}}{6\ \Omega} = 5\text{ A}$$

Now for the contribution due to the current source. Setting the voltage source to zero volts results in the network in Fig. 9.14, this presents us with an interesting situation. The current source has been replaced with a short-circuit equivalent that is directly across the current source and resistor *R*1. Since the source current takes the path of least resistance, it
chooses the zero ohm path of the inserted short-circuitequivalent,andthecurrentthrough*R*1iszero amperes. This is clearly demonstrated by an application of the current divider rule as follows:

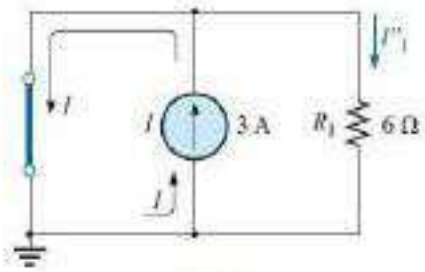


FIG. 9.14
Determining the effect of the 3 A current source on the current I_1 in Fig. 9.12.

$$I_1'' = \frac{R_{sc}I}{R_{sc} + R_1} = \frac{(0\ \Omega)I}{0\ \Omega + 6\ \Omega} = 0\text{ A}$$

Since I_1' and I_1'' have the same defined direction in Figs. 9.13 and 9.14, the total current is defined by

$$I_1 = I_1' + I_1'' = 5\text{ A} + 0\text{ A} = \mathbf{5\text{ A}}$$

Although this has been an excellent introduction to the application of the superposition theorem, it should be immediately clear in Fig. 9.12 that the voltage source is in parallel with the current source and load resistor R_1, so the voltage across each must be 30 V. The result is that I_1 must be determined solely by

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{30\text{ V}}{6\ \Omega} = 5\text{ A}$$

EXAMPLE 9.4 Using the principle of superposition, find the current I_2 through the 12 kΩ resistor in Fig. 9.15.

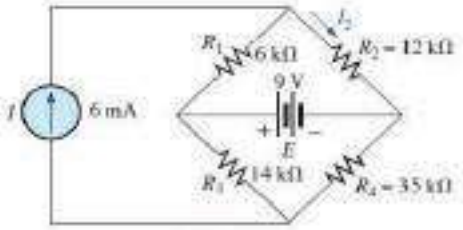


FIG. 9.15
Example 9.4.

Solution: Consider the effect of the 6 mA current source (Fig. 9.16).

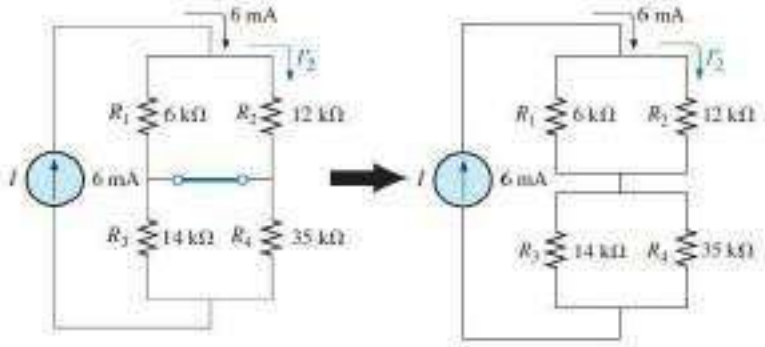


FIG. 9.16
The effect of the current source I on the current I_2.

The current divider rule gives

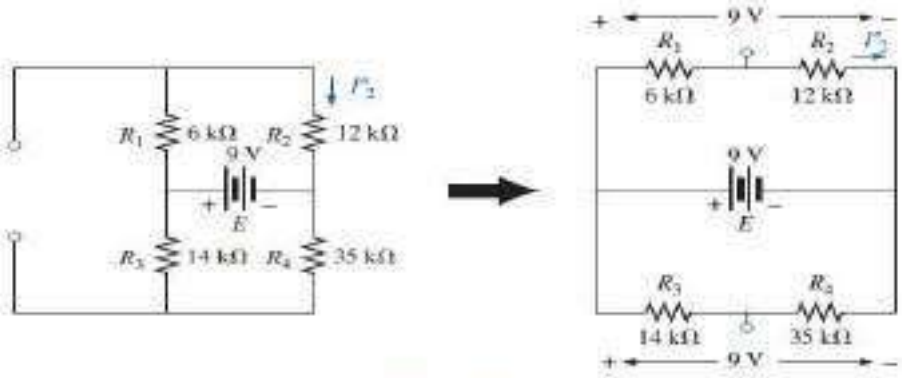
$$I_2' = \frac{R_3 I}{R_1 + R_2} = \frac{(6\ \text{k}\Omega)(6\ \text{mA})}{6\ \text{k}\Omega + 12\ \text{k}\Omega} = 2\ \text{mA}$$

Considering the effect of the 9 V voltage source (Fig. 9.17) gives

$$I_2'' = \frac{E}{R_1 + R_2} = \frac{9\ \text{V}}{6\ \text{k}\Omega + 12\ \text{k}\Omega} = 0.5\ \text{mA}$$

Since I_2' and I_2'' have the same direction through R_2, the desired current is the sum of the two:

$$\begin{aligned} I_2 &= I_2' + I_2'' \\ &= 2\ \text{mA} + 0.5\ \text{mA} \\ &= \mathbf{2.5\ mA} \end{aligned}$$



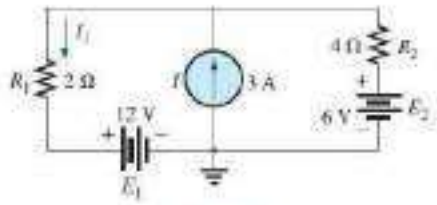


FIG. 9.18
Example 9.5.

EXAMPLE 9.5 Find the current through the 2 Ω resistor of the network in Fig. 9.18. The presence of three sources results in three different networks to be analyzed.

Solution: Consider the effect of the 12 V source (Fig. 9.19).

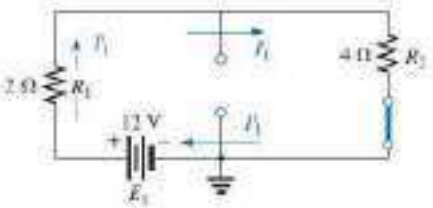


FIG. 9.19
The effect of E_1 on the current I_1.

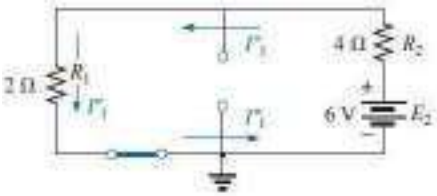


FIG. 9.20
The effect of E_2 on the current I_1.

$$I'_1 = \frac{E_1}{R_1 + R_2} = \frac{12\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{12\text{ V}}{6\ \Omega} = 2\text{ A}$$

Consider the effect of the 6 V source (Fig. 9.20):

$$I''_1 = \frac{E_2}{R_1 + R_2} = \frac{6\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{6\text{ V}}{6\ \Omega} = 1\text{ A}$$

Consider the effect of the 3 A source (Fig. 9.21): Applying the current divider rule gives

$$I'''_1 = \frac{R_2 I}{R_1 + R_2} = \frac{(4\ \Omega)(3\text{ A})}{2\ \Omega + 4\ \Omega} = \frac{12\text{ A}}{6} = 2\text{ A}$$

The total current through the 2 Ω resistor appears in Fig. 9.22, and

Same direction as I_1 in Fig. 9.18 — Opposite direction to I_1 in Fig. 9.18

$$I_1 = I'_1 + I''_1 - I'''_1$$
$$= 1\text{ A} + 2\text{ A} - 2\text{ A} = 1\text{ A}$$

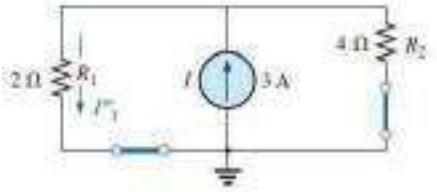


FIG. 9.21
The effect of I on the current I_1.

9.3 THÉVENIN'STHEOREM

The next theorem to be introduced, **Thévenin's theorem,** is probably one of the most interesting in that it permits the reduction of complex networks to a simpler form for analysis and design.

In general, the theoremcan be used to do the following:

- ***Analyzenetworks withsources that arenot inseries or parallel.***
- ***Reduce the number of components required to establish the same characteristics at the output terminals.***
- ***Investigate the effect of changing a particular component on the behaviour of a network without having to analyze the entire network after each change.***

Allthreeareasofapplicationaredemonstratedintheexamples to follow.

Thévenin'stheoremstatesthefollowing:

Anytwo-terminaldcnetworkcanbereplacedbyan equivalent circuit consisting solely of a voltage source and a series resistor as shown in Fig.9.23.

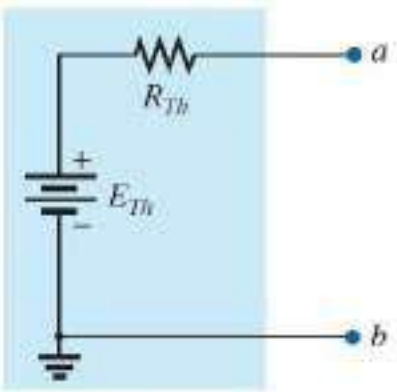


FIG. 9.23
Thévenin equivalent circuit.

ThetheoremwasdevelopedbyCommandantLeon-Charles Thévenin in 1883 as described in Fig. 9.24.

FIG. 9.24
Leon-Charles Thévenin.
Courtesy of the Bibliothèque École Polytechnique, Paris, France.

To demonstrate the power of the theorem, consider the fairly complex network of Fig. 9.25(a) with its two sources and series-parallel connections.

The theorem states that the entire network inside the blue shaded area can be replaced by one voltage source and one resistor as shown in Fig. 9.25(b). If the replacement is done properly, the voltage across, and the current through, the resistor *RL* will be the sameforeachnetwork.Thevalueof*RL*canbechangedtoany

value, and the voltage, current, or power to the load resistor is the same for each configuration.

Now,thisisaverypowerfulstatement—onethatisverifiedintheexamples tofollow.

The question then is, How can you determine the proper value of Thévenin voltage and resistance?Ingeneral,findingtheThévenin *resistance*valueisquitestraightforward.Finding the Thévenin *voltage* can be more of a challenge and, in fact, may require using the superposition theorem.

Fortunately, there is a series of steps that will lead to the proper value of each parameter. Although a few of the steps may seem trivial at first, they can become quite important when the network becomes complex.

Thévenin's Theorem Procedure

Preliminary:

1. Remove that portion of the network where the Thévenin equivalent circuit is found. In Fig. 9.25(a), this requires that the load resistor RL be temporarily removed from the network.

2. Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.) RTh:

3. Calculate RTh by first setting all sources to zero (voltage sources are replaced by short circuits and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) ETh:

4. Calculate ETh by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that causes most confusion and errors. In all cases, keep in mind that it is the open-circuit potential between the two terminals marked in step 2.)Conclusion:

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor RL between the terminals of the Thévenin equivalent circuit as shown in Fig. 9.25(b).

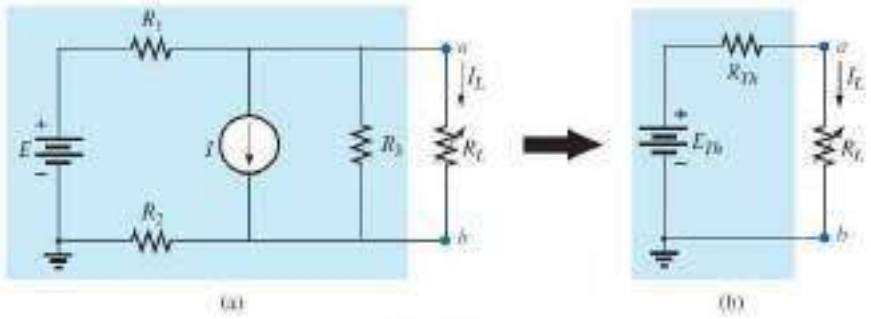


FIG. 9.25
Substituting the Thévenin equivalent circuit for a complex network.

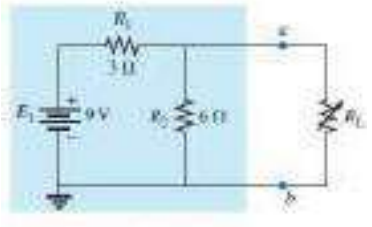


FIG. 9.26
Example 9.6

EXAMPLE 9.6 Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.26. Then find the current through R_L for values of 2 Ω, 10 Ω, and 100 Ω.

Solution:

Steps 1 and 2: These produce the network in Fig. 9.27. Note that the load resistor R_L has been removed and the two "holding" terminals have been defined as *a* and *b*.

Step 3: Replacing the voltage source E_1 with a short-circuit equivalent yields the network in Fig. 9.28(a), where

$$R_{Th} = R_1 \| R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = 2\ \Omega$$

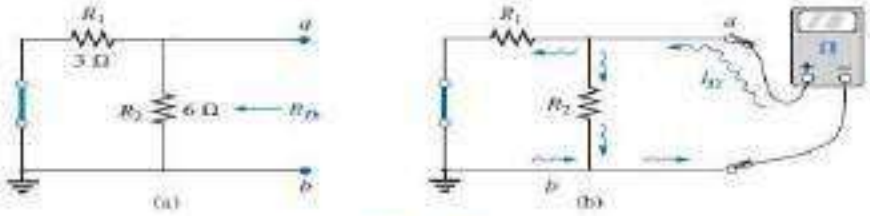


FIG. 9.28
Determining R_{Th} for the network in Fig. 9.27.

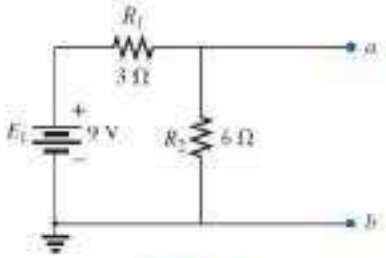


FIG. 9.27
Identifying the terminals of particular importance when applying Thévenin's theorem.

The importance of the two marked terminals now begins to surface. They are the two terminals across which the Thévenin resistance is measured. It is no longer the total resistance as seen by the source, as determined in the majority of problems of Chapter 7. If some difficulty develops when determining R_{Th} with regard to whether the resistive elements are in series or parallel, consider recalling that the ohmmeter sends out a trickle current into a resistive combination and senses the level of the resulting voltage to establish the measured resistancelevel. In Fig. 9.28(b), the trickle current of the ohmmeter approaches thenetworkthroughterminal*a*,andwhenitreachesthejunctionof*R*1 and *R*2, it splits as shown. The fact that the trickle current splits and then recombines at the lower node reveals that the resistors are in parallel as far as the ohmmeter reading is concerned. In essence, the path of the sensing current of the ohmmeter has revealed how the resistors are connected to the two terminals of interest and how the Thévenin resistance should be determined. Remember this as you work through the various examples in this section.

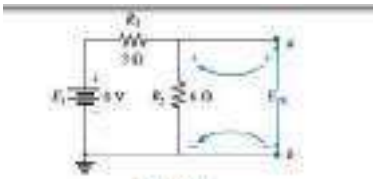


FIG. 9.29
Determining E_{Th} for the network in Fig. 9.27.

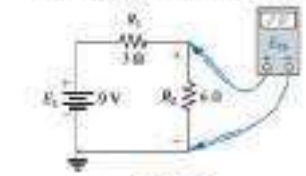


FIG. 9.30
Measuring E_{Th} for the network in Fig. 9.27.

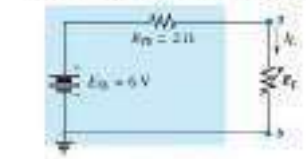


FIG. 9.31
Substituting the Thévenin equivalent circuit for the network external to R_L in Fig. 9.26.

Step 4: Replace the voltage source (Fig. 9.29). For this case, the opencircuitvoltage *ETh* is the same as the voltage drop across the 6 Ω resistor.

Applyingthevoltagedividerrulegives

$$E_{Th} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = 6\ \text{V}$$

It is particularly important to recognize that E_{Th} is the open-circuit potential between points *a* and *b*. Remember that an open circuit can have any voltage across it, but the current must be zero. In fact, the current through any element in series with the open circuit must be zero also. The use of a voltmeter to measure E_{Th} appears in Fig. 9.30. Note that it is placed directly across the resistor R_2 since E_{Th} and V_{R_2} are in parallel.

Step 5: (Fig. 9.31):

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

$R_L = 2\ \Omega$: $\quad I_L = \dfrac{6\ \text{V}}{2\ \Omega + 2\ \Omega} = 1.5\ \text{A}$

$R_L = 10\ \Omega$: $\quad I_L = \dfrac{6\ \text{V}}{2\ \Omega + 10\ \Omega} = 0.5\ \text{A}$

$R_L = 100\ \Omega$: $\quad I_L = \dfrac{6\ \text{V}}{2\ \Omega + 100\ \Omega} = 0.06\ \text{A}$

If Thévenin's theorem were unavailable, each change in R_L would require that the entire network in Fig. 9.26 be reexamined to find the new value of R_L.

EXAMPLE 9.7 Find the Thévenin equivalent circuit for the network in the shaded area ofthe network in Fig. 9.32.

Solution:

*Steps1and2:*SeeFig.9.33.

Step 3: See Fig. 9.34. The current source has been replaced with an open-circuit equivalent and the resistance determined between terminals *a* and *b*.

In this case, an ohmmeter connected between terminals *a* and *b* sends out a sensing current that flows directly through *R*1 and *R*2 (at the same level). The result is that *R*1 and *R*2 are in series and the Thévenin resistance is the sum of the two,

$$R_{Th} = R_1 + R_2 = 4\,\Omega + 2\,\Omega = \mathbf{6\,\Omega}$$

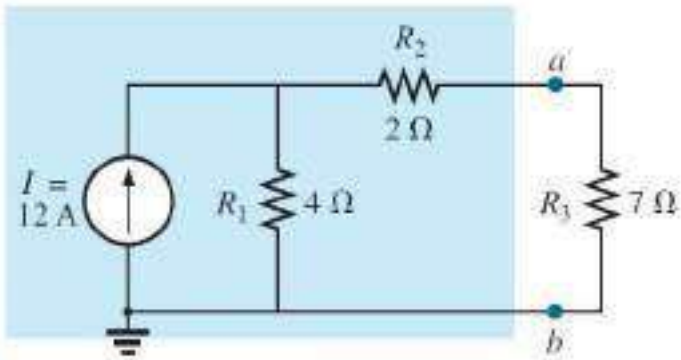


FIG. 9.32
Example 9.7.

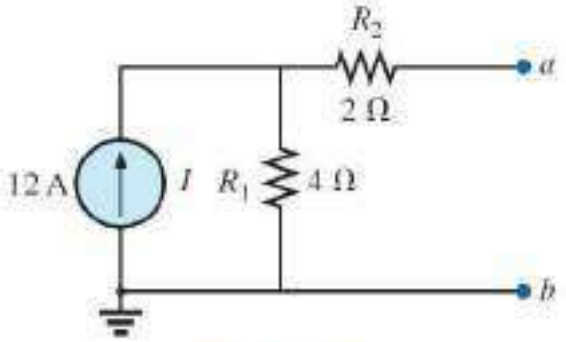


FIG. 9.33
Establishing the terminals of particular interest for the network in Fig. 9.32.

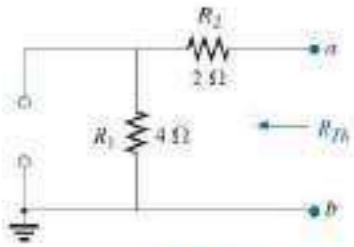


FIG. 9.34
Determining R_{Th} for the network in Fig. 9.33.

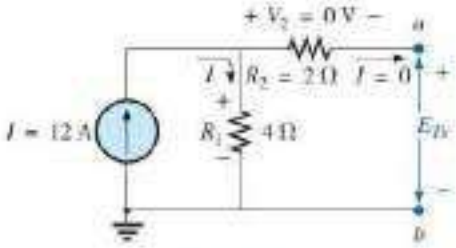


FIG. 9.35
Determining E_{Th} for the network in Fig. 9.33.

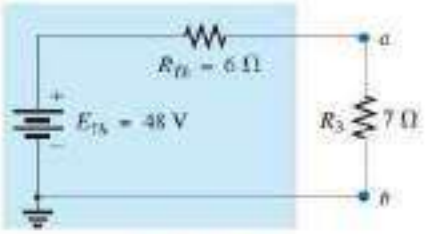


FIG. 9.36
Substituting the Thévenin equivalent circuit in the network external to the resistor R_3 in Fig. 9.32.

Step 4: See Fig. 9.35. In this case, since an open circuit exists between the two marked terminals, the current is zero between these terminals and through the 2Ω resistor. Thevoltage drop across *R*2 is, therefore,

$$V2 = I2R2 = (0)R2 = 0\text{V}$$

and

$$ETh = V1 = I1R1 = IR1 = (12\text{A})(4\_) = \mathbf{48V}$$

Step 5: SeeFig.9.36.

EXAMPLE 9.8 Find the Thévenin equivalent circuit for the network in theshadedareaofthenetworkinFig. 9.37.Noteinthisexamplethatthereis no need for the section of the network to be preserved to be at the "end" of the configuration.

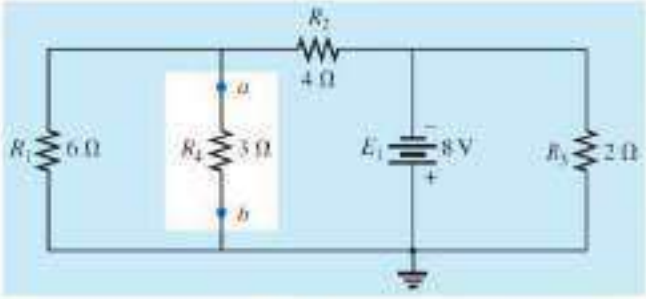


FIG. 9.37

Solution:

*Steps1and2:*SeeFig.9.38

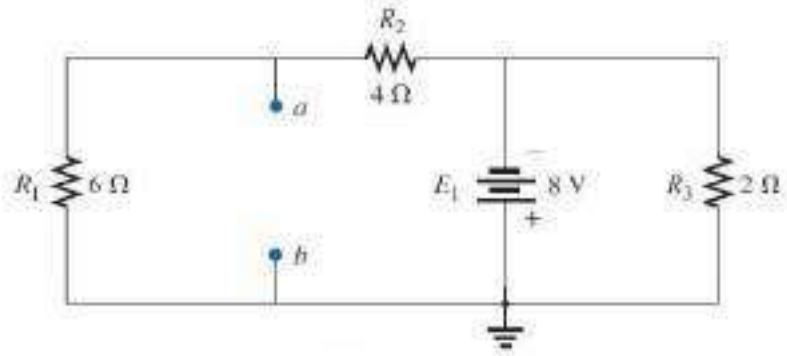


FIG. 9.38

Identifying the terminals of particular interest for the network in Fig. 9.37.

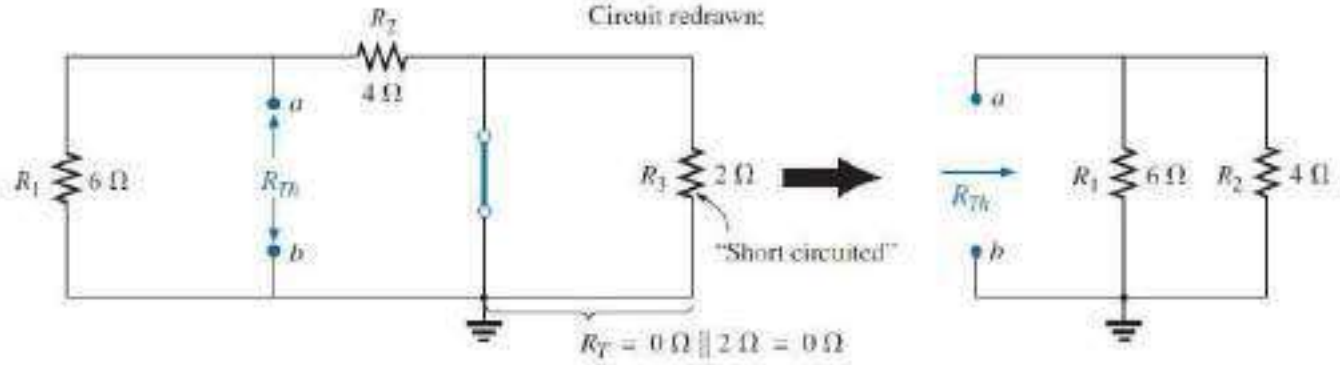


FIG. 9.39

Determining R_{Th} for the network in Fig. 9.38.

Step 3: See Fig. 9.39. Steps 1 and 2 are relatively easy to apply, but now we must be careful to"hold"ontotheterminals*a*and*b*astheThéveninresistanceandvoltageareDetermined.In Fig. 9.39, all the remaining elements turn out to be in parallel, and the network can be redrawn as shown. We have

$$R_{Th} = R_1 \| R_2 = \frac{(6\ \Omega)(4\ \Omega)}{6\ \Omega + 4\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$

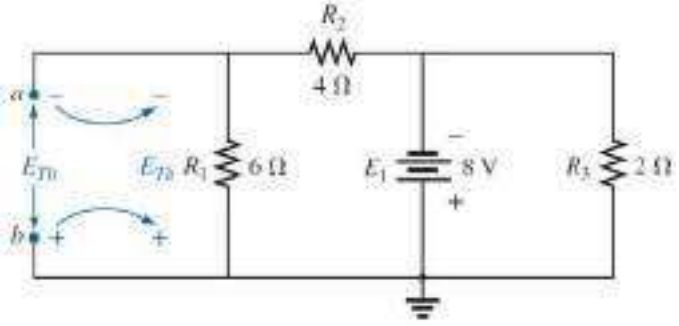


FIG. 9.40

Determining E_{Th} for the network in Fig. 9.38.

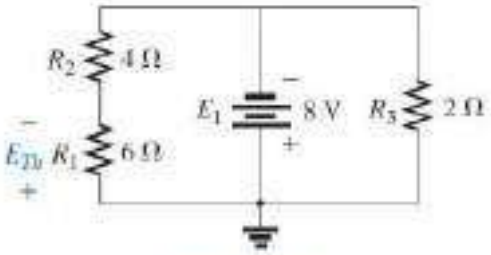


FIG. 9.41

Network of Fig. 9.40 redrawn.

Step 4: See Fig. 9.40. In this case, the network can be redrawn as shown in Fig. 9.41. Since the voltage is the same across parallel elements, the voltage across the series resistors *R*1 and *R*2 is *E*1, or 8 V. Applying the voltage divider rule gives

$$E_{Th} = \frac{R_1 E_1}{R_1 + R_2} = \frac{(6\ \Omega)(8\ \text{V})}{6\ \Omega + 4\ \Omega} = \frac{48\ \text{V}}{10} = \mathbf{4.8\ V}$$

Step 5: SeeFig.9.42.

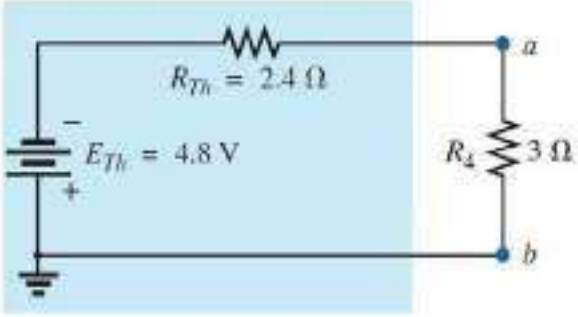


FIG. 9.42
Substituting the Thévenin equivalent circuit for the network external to the resistor R_4 in Fig. 9.37.

EXAMPLE 9.9 Find the Théveninequivalentcircuitforthe network in the shaded area of the bridge network in Fig. 9.43.

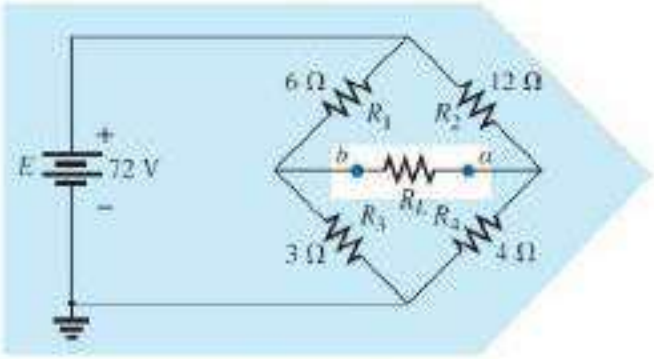


FIG. 9.43
Example 9.9.

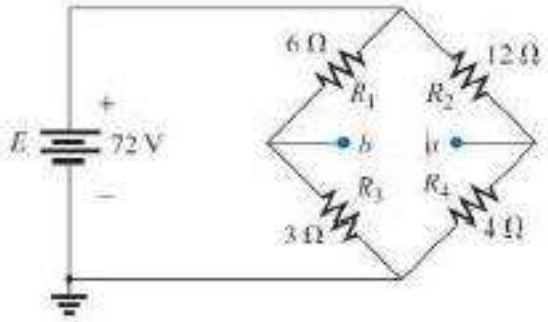


FIG. 9.44
Identifying the terminals of particular interest for the network in Fig. 9.43.

Solution:

*Steps1and2:*SeeFig.9.44.

Step 3: See Fig. 9.45. In this case, the short-circuit replacement of the voltage source *E* provides a direct connection between *c* and *c*_ in Fig. 9.45(a), permitting a "folding" of the network around the horizontal line of *a-b* to produce the configuration in Fig. 9.45(b).

$$R_{Th} = R_{a-b} = R_1 \| R_3 + R_2 \| R_4$$
$$= 6\,\Omega \| 3\,\Omega + 4\,\Omega \| 12\,\Omega$$
$$= 2\,\Omega + 3\,\Omega = \mathbf{5\,\Omega}$$

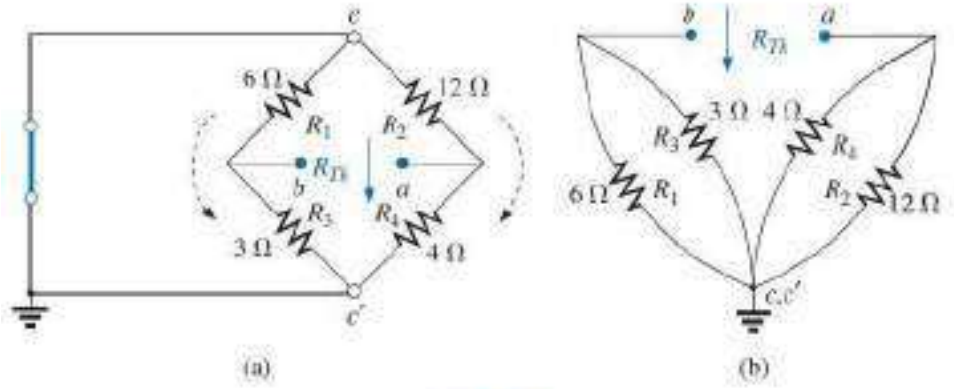


FIG. 9.45
Solving for R_{Th} for the network in Fig. 9.44.

Step 4: The circuit is redrawn in Fig. 9.46. The absence of a direct connection between *a* and *b* results in a network with three parallel branches. The voltages V_1 and V_2 can therefore be determined using the voltage divider rule:

$$V_1 = \frac{R_1 E}{R_1 + R_3} = \frac{(6\ \Omega)(72\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{432\ \text{V}}{9} = 48\ \text{V}$$

$$V_2 = \frac{R_2 E}{R_2 + R_4} = \frac{(12\ \Omega)(72\ \text{V})}{12\ \Omega + 4\ \Omega} = \frac{864\ \text{V}}{16} = 54\ \text{V}$$

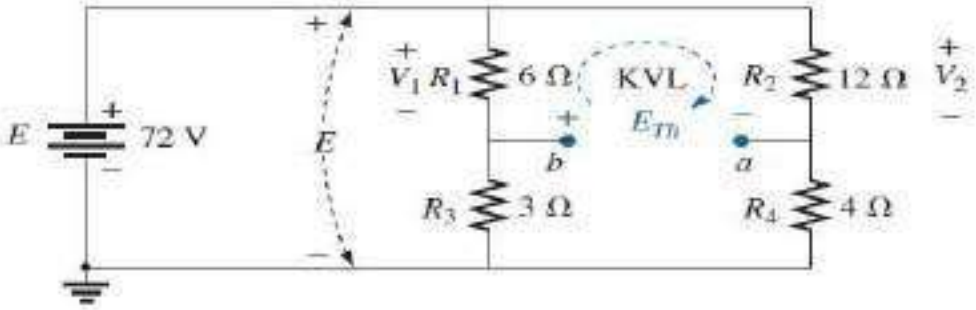


FIG. 9.46
Determining E_{Th} for the network in Fig. 9.44.

Assuming the polarity shown for E_{Th} and applying Kirchhoff's voltage law to the top loop in the clockwise direction results in

$$\Sigma_c V = +E_{Th} + V_1 - V_2 = 0$$

and

$$E_{Th} = V_2 - V_1 = 54\ \text{V} - 48\ \text{V} = \mathbf{6\ V}$$

Step 5: See Fig. 9.47.

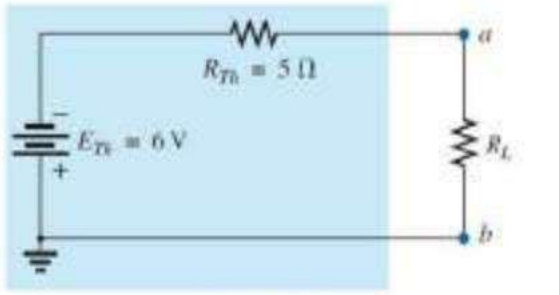


FIG. 9.47

EXAMPLE 9.11 For the network of Fig. 9.54,
a. Find the Thévenin equivalent circuit for the portion of the network in the shaded area.
b. Reconstruct the network of Fig. 9.54 with the Thévenin equivalent network in place.
c. Using the resulting network of part (b) find the voltage *Va*.

Solutions:

a. *Steps 1 and 2:* See Fig. 9.55.
Step 3: See Fig. 9.56.

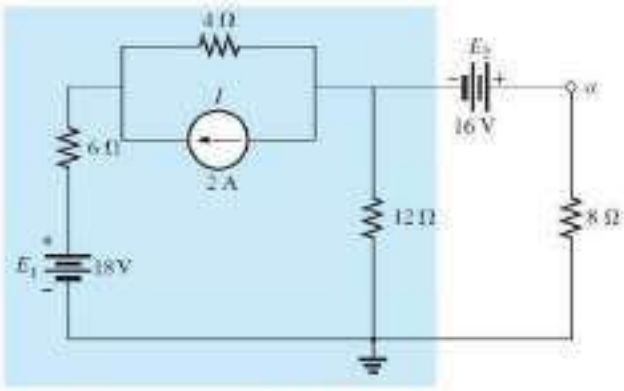


FIG. 9.54
Example 9.11

$$R_{Th} = 12\ \Omega \,\|\, (4\ \Omega + 6\ \Omega) = 12\ \Omega \,\|\, 10\ \Omega = 5$$

Step 4: Applying the superposition theorem, we will first find the effect of the voltage source on the Thévenin voltage using the network of Fig. 9.57. Applying the voltage divider rule:

$$E'_{Th} = \frac{12\ \Omega\ (18\ \text{V})}{6\ \Omega + 4\ \Omega + 12\ \Omega} = \frac{216}{22}\text{V} = 9.82\ \text{V}$$

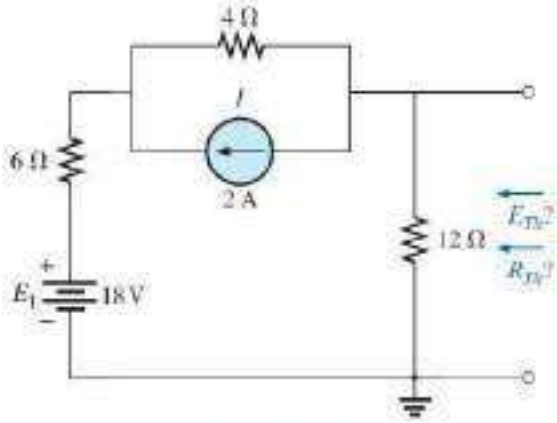


FIG. 9.55
Establishing the terminals of interest for the network of Fig. 9.54.

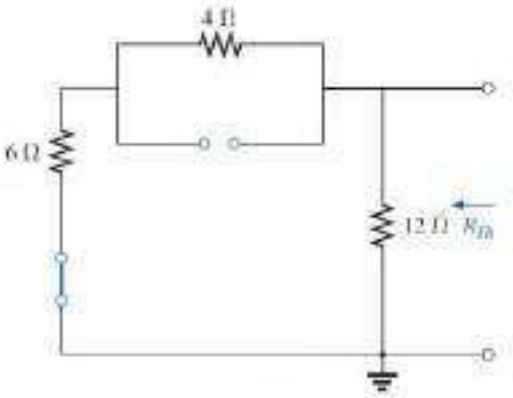


FIG. 9.56
Determining R_{Th}.

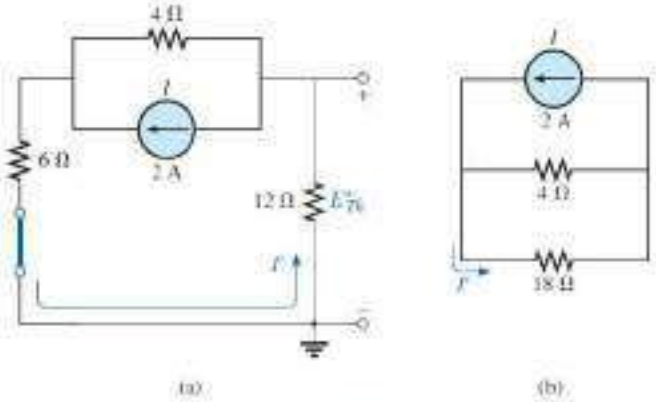


FIG. 9.58
Determining the contribution of I to E_{Th}.

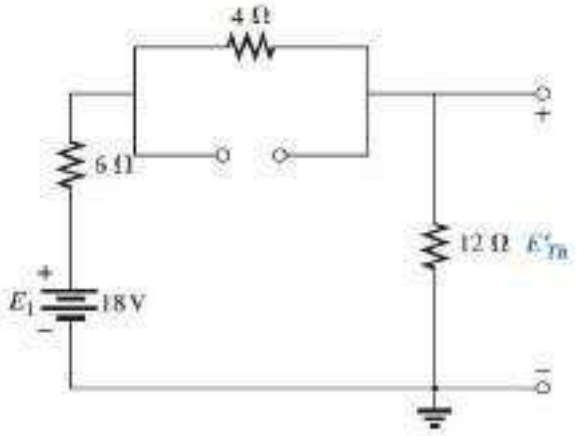


FIG. 9.57
Determining the contribution of E_1 to E_{Th}.

The contribution due to the current source is determined using the network of Fig. 9.58(a) redrawn as shown in Fig. 9.58(b). Applying the current divider rule:

$$I' = \frac{4\,\Omega\,(2\,\text{A})}{4\,\Omega + 18\,\Omega} = \frac{8}{22}\,\text{A} = 0.364\,\text{A}$$

and $E'_{Th} = -I'(12\,\Omega) = -(0.364\,\text{A})(12\,\Omega) = -4.37\,\text{V}$

so that $E_{Th} = E'_{Th} + E''_{Th} = 9.82\,\text{V} - 4.37\,\text{V} = \mathbf{5.45\,V}$

b. The reconstructed network is shown in Fig. 9.59.

c. Using the voltage divider rule:

$$V_a = \frac{8\,\Omega\,(5.45\,\text{V} + 16\,\text{V})}{5.45\,\Omega + 8\,\Omega} = \frac{8\,(21.45)}{13.45}\,\text{V} = \frac{171.6}{13.45}\,\text{V} = \mathbf{12.76\,V}$$

Instead of using the superposition theorem, the current source could first have been converted to a voltage source and the series elements combined to determine the Thévenin voltage. In any event both approaches would have yielded the same results.

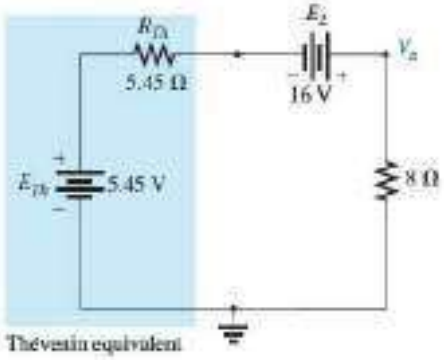


FIG. 9.59
Applying the Thévenin equivalent network to the network of Fig. 9.54.

9.4 NORTON'STHEOREM

Anytwo-terminallinearbilateraldcnetworkcanbe replaced by an equivalentcircuitconsistingofacurrentsourceanda parallel resistor, as shown in Fig. 9.65.

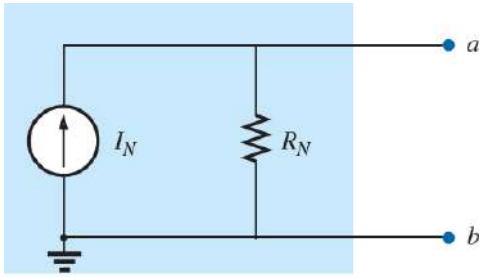


FIG. 9.65
Norton equivalent circuit.

FIG. 9.64
Edward L. Norton.
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The discussion of Thévenin's theorem with respect to the equivalent circuit can also be applied to the Norton equivalent circuit. The steps leading to the proper values of *IN* and *RN* are now listed.

Norton'sTheoremProcedure

Preliminary:

1. Removethatportionofthenetworkacrosswhichthe Nortonequivalentcircuitisfound.

2. Marktheterminalsoftheremainingtwo-terminalnetwork.

RN:

3. Calculate RN by first setting all sources to zero (voltage sources are replaced with short circuits and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since RN _ RTh, the procedure and value obtained using the approach described for Thévenin's theorem will determine the proper value of RN.

IN:

4. Calculate IN by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.

Conclusion:

5. DrawtheNortonequivalentcircuitwiththeportionofthecircuitpreviouslyremoved replaced between the terminals of the equivalent circuit.

The Norton and Thévenin equivalent circuits can also be found from each other by using the source transformation discussed earlier in this chapter and reproduced in Fig. 9.66.

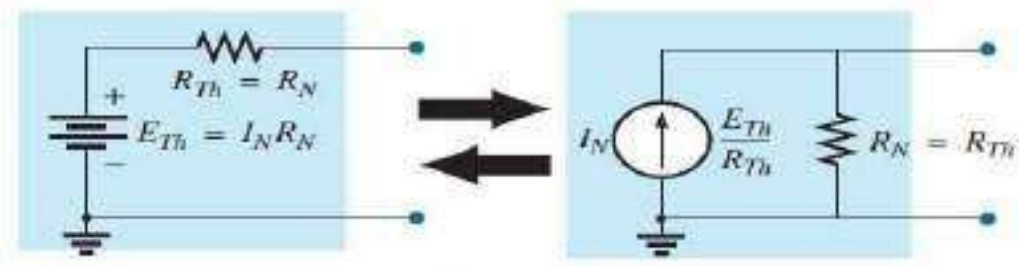


FIG. 9.66
Converting between Thévenin and Norton equivalent circuits.

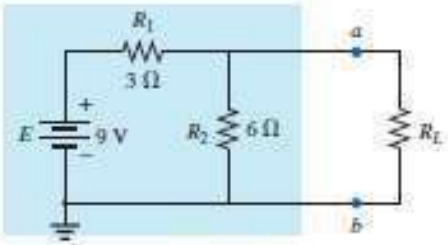


FIG. 9.67
Example 9.12.

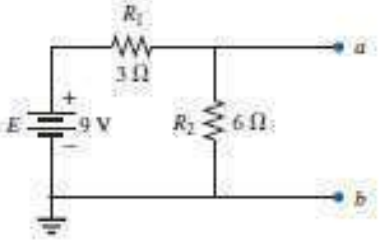


FIG. 9.68
Identifying the terminals of particular interest for the network in Fig. 9.67.

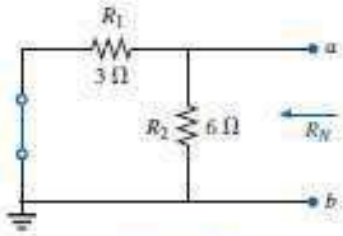


FIG. 9.69
Determining R_N for the network in Fig. 9.68.

EXAMPLE 9.12 Find the Norton equivalent circuit for the network in the shaded area in Fig. 9.67.

Solution:

Steps 1 and 2: See Fig. 9.68.

Step 3: See Fig. 9.69, and

$$R_N = R_1 \| R_2 = 3\ \Omega \| 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = 2\ \Omega$$

Step 4: See Fig. 9.70, which clearly indicates that the short-circuit connection between terminals *a* and *b* is in parallel with R_2 and eliminates its effect. I_N is therefore the same as through R_1, and the full battery voltage appears across R_1 since

$$V_2 = I_2 R_2 = (0)6\ \Omega = 0\ \text{V}$$

Therefore,

$$I_N = \frac{E}{R_1} = \frac{9\ \text{V}}{3\ \Omega} = 3\ \text{A}$$

Step 5: See Fig. 9.71. This circuit is the same as the first one considered in the development of Thévenin's theorem. A simple conversion indicates that the Thévenin circuits are, in fact, the same (Fig. 9.72).

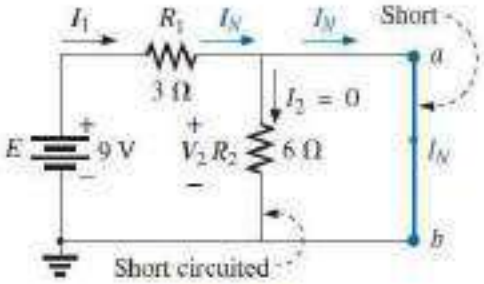


FIG. 9.70
Determining I_N for the network in Fig. 9.68.

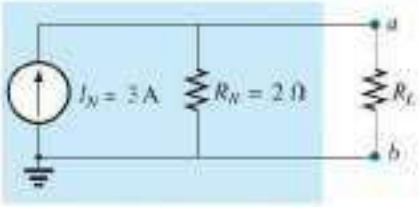


FIG. 9.71
Substituting the Norton equivalent circuit for the network external to the resistor R_L in Fig. 9.67.

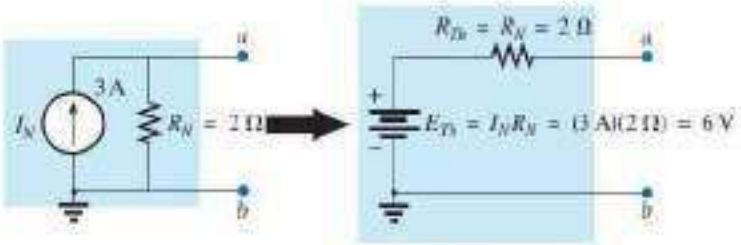


FIG. 9.72
Converting the Norton equivalent circuit in Fig. 9.71 to a Thévenin equivalent circuit.

EXAMPLE 9.13 Find the Norton equivalent circuit for the network external to the 9 _ resistor in Fig. 9.73.

Solution:

Steps 1 and 2: See Fig. 9.74.

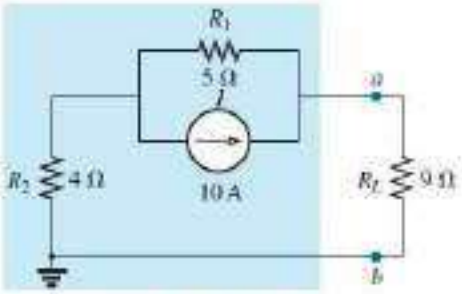


FIG. 9.73
Example 9.13.

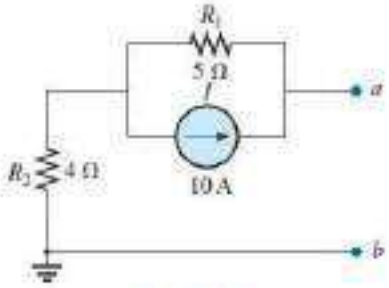


FIG. 9.74
Identifying the terminals of particular interest for the network in Fig. 9.73.

Step 3: See Fig. 9.75, and

$$R_N = R_1 + R_2 = 5\ \Omega + 4\ \Omega = 9\ \Omega$$

Step 4: As shown in Fig. 9.76, the Norton current is the same as the current through the 4 Ω resistor. Applying the current divider rule gives

$$I_N = \frac{R_1 I}{R_1 + R_2} = \frac{(5\ \Omega)(10\ \text{A})}{5\ \Omega + 4\ \Omega} = \frac{50\ \text{A}}{9} = 5.56\ \text{A}$$

Step 5: See Fig. 9.77.

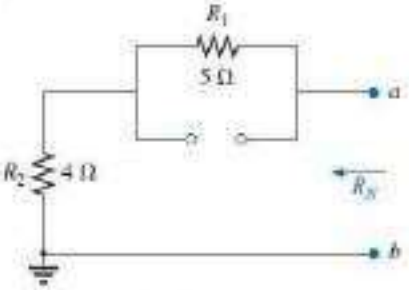


FIG. 9.75
Determining R_N for the network in Fig. 9.74.

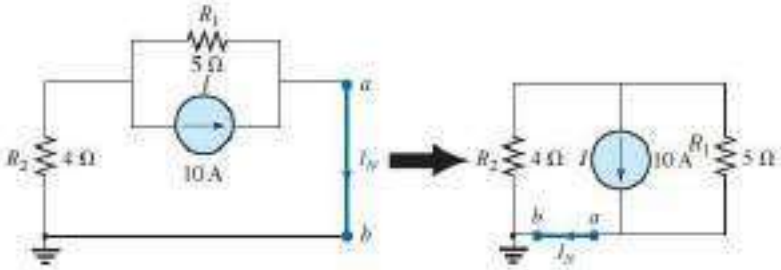


FIG. 9.76
Determining I_N for the network in Fig. 9.74.

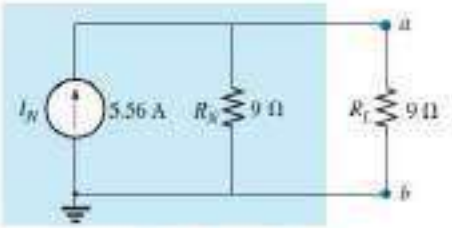


FIG. 9.77
Substituting the Norton equivalent circuit for the network external to the resistor R_L in Fig. 9.73.

EXAMPLE 9.14 (Two sources) Find the Norton equivalent circuit for the portion of the network to the left of *a-b* in Fig. 9.78.

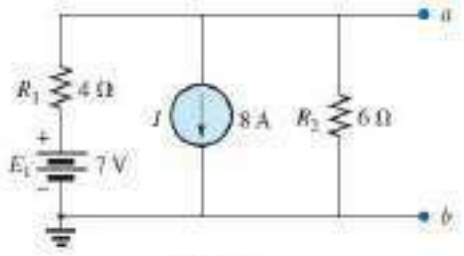


FIG. 9.79
Identifying the terminals of particular interest for the network in Fig. 9.78.

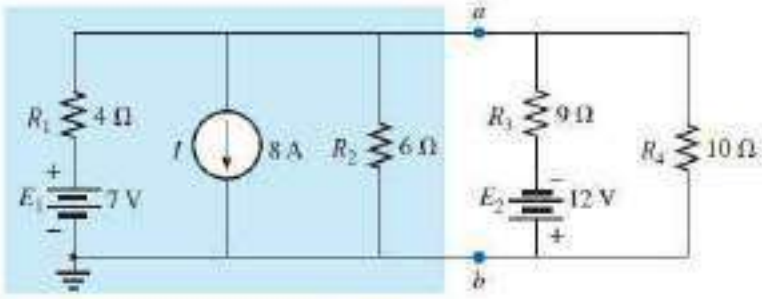


FIG. 9.78
Example 9.14.

Solution:

Steps 1 and 2: See Fig. 9.79.

Step 3: See Fig. 9.80, and

$$R_N = R_1 \| R_2 = 4\ \Omega \| 6\ \Omega = \frac{(4\ \Omega)(6\ \Omega)}{4\ \Omega + 6\ \Omega} = \frac{24\ \Omega}{10} = 2.4\ \Omega$$

Step 4: (Using superposition) For the 7 V battery (Fig. 9.81),

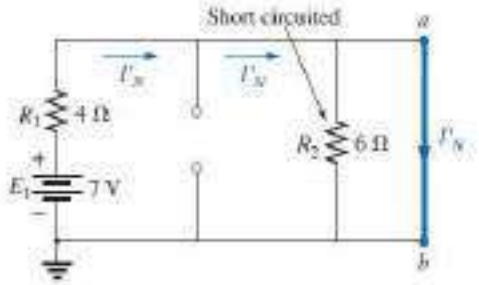


FIG. 9.81
Determining the contribution to I_N from the voltage source E_1.

$$I'_N = \frac{E_1}{R_1} = \frac{7\text{ V}}{4\ \Omega} = 1.75\text{ A}$$

For the 8 A source (Fig. 9.82), we find that both R_1 and R_2 have been "short circuited" by the direct connection between a and b, and

$$I''_N = I = 8\text{ A}$$

The result is

$$I_N = I''_N - I'_N = 8\text{ A} - 1.75\text{ A} = 6.25\text{ A}$$

Step 5: See Fig. 9.83.

9.5 MAXIMUMPOWERTRANSFER

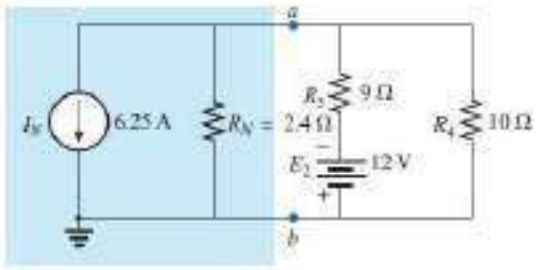


FIG. 9.83
Substituting the Norton equivalent circuit for the network to the left of terminals a-b in Fig. 9.78.

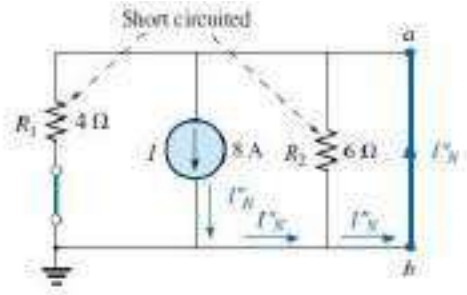


FIG. 9.82
Determining the contribution to I_N from the current source I.

THEOREM

Whendesigningacircuit,itisoftenimportanttobeabletoansweroneofthefollowing questions:

Whatloadshouldbeappliedtoasystemtoensurethattheloadisreceivingmaximum power from the system?

Conversely:

Foraparticularload,whatconditionsshouldbeimposedonthesourcetoensurethatit will deliver the maximum power available?

Even if a load cannot be set at the value that would result in maximum power transfer, it is oftenhelpfultohavesomeideaofthevaluethatwilldrawmaximumpowersothatyoucan compare it to the load at hand. For instance, if a design calls for a load of 100 Ω, to ensure that the load receives maximum power, using a resistor of 1 Ω or 1 k Ω results in a power transfer that is much less than the maximum possible.

However, using a load of 82 Ω or 120 Ω probably results in a fairly good level of power transfer.Fortunately,theprocessoffindingtheloadthatwillreceivemaximumpowerfroma particular system is quite straightforward due to the **maximum power transfer theorem,** which states the following:

Aloadwillreceivemaximumpowerfromanetworkwhenitsresistanceisexactlyequalto the Thévenin resistance of the network applied to the load. That is,

$$\boxed{R_L = R_{Th}} \qquad (9.2)$$

In other words, for the Thévenin equivalent circuit in Fig. 9.84, when the load is set equal to the Thévenin resistance, the load will receive maximum power from the network.

Using Fig. 9.84 with $R_L = R_{Th}$, we can determine the maximum power delivered to the load by first finding the current:

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{E_{Th}}{R_{Th} + R_{Th}} = \frac{E_{Th}}{2R_{Th}}$$

Then we substitute into the power equation:

$$P_L = I_L^2 R_L = \left(\frac{E_{Th}}{2R_{Th}}\right)^2 (R_{Th}) = \frac{E_{Th}^2 R_{Th}}{4R_{Th}^2}$$

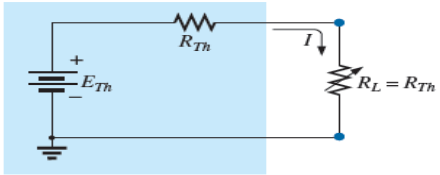


FIG. 9.84
Defining the conditions for maximum power to a load using the Thévenin equivalent circuit.

and $$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} \qquad (9.3)$$

The total power delivered by a supply such as ETh is absorbed by both the Thévenin equivalentresistanceandtheloadresistance.Anypowerdeliveredbythesourcethatdoes not get to the load is lost to the Thévenin resistance.

Under maximum power conditions, only half the power delivered by the source gets to the load. Now, that sounds disastrous, but remember that we are starting out with a fixed Thévenin voltage and resistance, and the above simply tells us that we must make the two resistancelevelsequalifwewantmaximumpowertotheload.Onanefficiencybasis,weare working at only a 50% level, but we are content because *we are getting maximum power out of our system*.

Thedcoperatingefficiencyisdefinedastheratioofthepowerdeliveredtotheload(*PL*)to the power delivered by the source (*Ps*). That is,

$$\eta\% = \frac{P_L}{P_s} \times 100\% \qquad (9.4)$$

For the situation where $R_L = R_{Th}$,

$$\eta\% = \frac{I_L^2 R_L}{I_L^2 R_T} \times 100\% = \frac{R_L}{R_T} \times 100\% = \frac{R_{Th}}{R_{Th} + R_{Th}} \times 100\%$$

$$= \frac{R_{Th}}{2R_{Th}} \times 100\% = \frac{1}{2} \times 100\% = \mathbf{50\%}$$

Ifefficiencyistheoverridingfactor,thentheloadshouldbemuchlargerthantheinternal resistance of the supply. If maximum power transfer is desired and efficiency less of a concern, then the conditions dictated by the maximum power transfer theorem should be applied.

A relatively low efficiency of 50% can be tolerated in situations where power levels are relatively low, such as in a wide variety of electronic systems, where maximum power transfer for the given system is usually more important. However, when large power levels are involved, such as at generating plants, efficiencies of 50% cannot be tolerated. In fact, a great deal of expense and research is dedicated to raising power generating and transmission efficiencies a few percentage points. Raising an efficiency level of a 10 MkW power plant from 94% to 95% (a 1% increase) can save 0.1 MkW, or 100 million watts, of power—an enormous saving. In all of the above discussions, the effect of changing the load was discussedforafixedThéveninresistance.Lookingatthesituationfromadifferentviewpoint, we can say

if the load resistance is fixed and does not match the applied Thévenin equivalent resistance,thensomeeffortshouldbemade(ifpossible)toredesignthesystemsothatthe Thévenin equivalent resistance is closer to the fixed applied load.

In other words, if a designer faces a situation where the load resistance is fixed, he or she shouldinvestigatewhetherthesupplysectionshouldbereplacedorredesignedtocreatea closer match of resistance levels to produce higher levels of power to the load.

FortheNortonequivalentcircuitinFig.9.90,maximumpowerwillbedeliveredtotheload when ,

$$R_L = R_N \quad \text{..... (9.5)}$$

This result [Eq. (9.5)] will be used to its fullestadvantageintheanalysisoftransistor networks,wherethemostfrequentlyapplied transistorcircuitmodelusesacurrentsource rather than a voltage source.

FortheNortoncircuitinFig.9.90,

$$P_{L_{max}} = \frac{I_N^2 R_N}{4} \quad \text{(W)} \qquad \text{(9.6}$$

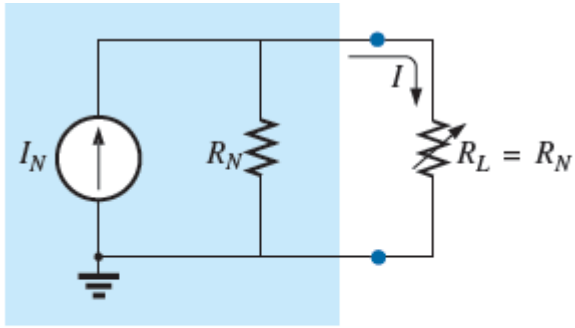


FIG. 9.90

Defining the conditions for maximum power to a load using the Norton equivalent circuit.

Todemonstratethatmaximumpoweris indeed transferred to the load undertheconditionsdefinedabove,consider the Thévenin equivalent circuit in Fig. 9.85.

Beforegettingintodetail,however,ifyouwere to guess what value of *RL* would result inmaximum power transfer to *RL*, you might think thatthesmallerthevalueof*RL*,thebetteritis because the current reaches a maximum when itis squared in the power equation. The problem is,however,thatintheequation*PL= I* 2 *LRL*, the load resistance is a multiplier. As it gets smaller, it forms a smaller product. Then again, you might suggest larger values of *RL* becausethe output voltage increases, and power is determinedby*PL=V*2*L*/*RL*.Thistime, however, the load resistance is in the denominator of the equation and causes the resulting power to decrease. A balance must obviously be made between the load resistance and the resulting current or voltage. The following discussion shows that

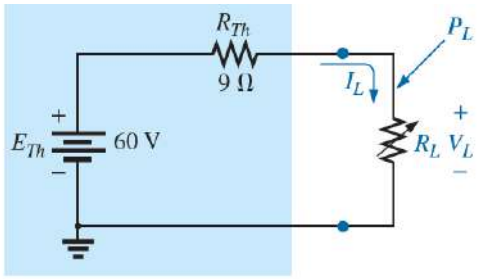


FIG. 9.85

Thévenin equivalent network to be used to validate the maximum power transfer theorem.

maximum power transfer occurs when the load voltage and current are one-half their maximum possible values.

Forthe circuitin Fig.9.85, thecurrentthroughtheloadisdeterminedby

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{60\ \text{V}}{9\ \Omega + R_L}$$

The voltage is determined by

$$V_L = \frac{R_L E_{Th}}{R_L + R_{Th}} = \frac{R_L(60\ \text{V})}{R_L + R_{Th}}$$

and the power by

$$P_L = I_L^2 R_L = \left(\frac{60\ \text{V}}{9\ \Omega + R_L}\right)^2 (R_L) = \frac{3600 R_L}{(9\ \Omega + R_L)^2}$$

If we tabulate the three quantities versus a range of values for R_L from 0.1 Ω to 30 Ω, we obtain the results appearing in Table 9.1. Note in particular that when R_L is equal to the Thévenin resistance of 9 Ω, the

power has a maximum value of 100 W, the current is 3.33 A, or one-half its maximum value of 6.67 A (as would result with a short circuit across the output terminals), and the voltage across the load is 30 V, or one-half
its maximum value of 60 V (as would result with an open circuit across its output terminals). As you can see, there is no question that maximum power is transferred to the load when the load equals the Thévenin value.

The power to the load versus the range of resistor values is provided in Fig. 9.86. Note in particular that for values of load resistance less than the Thévenin value, the change is dramatic as it approaches the peak value. However, for values greater than the Thévenin value, the drop is a great deal more gradual. This is important because it tells us thefollowing:

If the load applied is less than the Thévenin resistance, the power to the load will drop off rapidly as it gets smaller. However, if the applied load is greater than the Thévenin resistance, the power to the load will not drop off as rapidly as it increases.

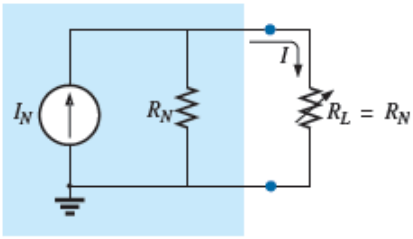


FIG. 9.90
Defining the conditions for maximum power to a load using the Norton equivalent circuit.

In all of the above discussions, the effect of changing the load was discussed for a fixed Thévenin resistance. Looking at the situation from a different viewpoint, we can say

if the load resistance is fixed and does not match the applied Thévenin equivalent resistance, then some effort should be made (if possible) to redesign the system so that the Thévenin equivalent resistance is closer to the fixed applied load.

In other words, if a designer faces a situation where the load resistance is fixed, he or she should investigate whether the supply section should be replaced or redesigned to create a closer match of resistance levels to produce higher levels of power to the load.

For the Norton equivalent circuit in Fig. 9.90, maximum power will be delivered to the load when

$$\boxed{R_L = R_N} \tag{9.5}$$

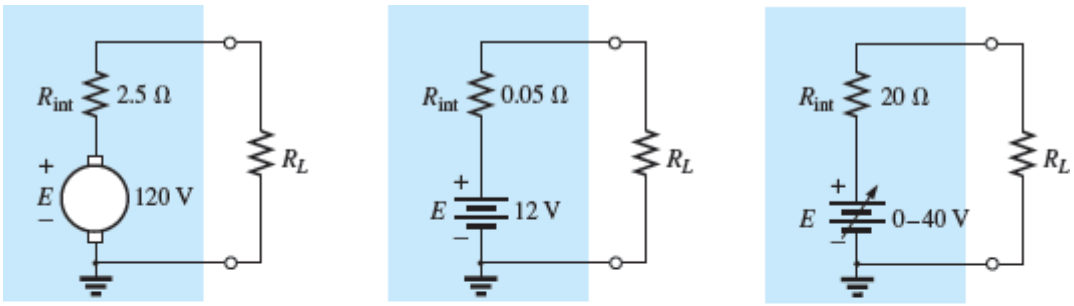
This result [Eq. (9.5)] will be used to its fullest advantage in the analysis of transistor networks, where the most frequently applied transistor circuit model uses a current source rather than a voltage source.

For the Norton circuit in Fig. 9.90,

$$\boxed{P_{L_{max}} = \frac{I_N^2 R_N}{4}} \quad \text{(W)} \tag{9.6}$$

EXAMPLE 9.15 A dc generator, battery, and laboratory supply are connected to resistive load R_L in Fig. 9.91.

a. For each, determine the value of R_L for maximum power transfer to R_L.
b. Under maximum power conditions, what are the current level and the power to the load for each configuration?
c. What is the efficiency of operation for each supply in part (b)?
d. If a load of 1 kΩ were applied to the laboratory supply, what would the power delivered to the load be? Compare your answer to the level of part (b). What is the level of efficiency?
e. For each supply, determine the value of R_L for 75% efficiency.



(a) dc generator (b) Battery (c) Laboratory supply

FIG. 9.91
Example 9.15.

Solutions:

a. For the dc generator,

$$R_L = R_{Th} = R_{int} = \mathbf{2.5\ \Omega}$$

For the 12 V car battery,

$$R_L = R_{Th} = R_{\text{int}} = \mathbf{0.05\ \Omega}$$

For the dc laboratory supply,

$$R_L = R_{Th} = R_{\text{int}} = \mathbf{20\ \Omega}$$

b. For the dc generator,

$$P_{L_{\max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{E^2}{4R_{\text{int}}} = \frac{(120\text{ V})^2}{4(2.5\ \Omega)} = \mathbf{1.44\ kW}$$

For the 12 V car battery,

$$P_{L_{\max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{E^2}{4R_{\text{int}}} = \frac{(12\text{ V})^2}{4(0.05\ \Omega)} = \mathbf{720\ W}$$

For the dc laboratory supply,

$$P_{L_{\max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{E^2}{4R_{\text{int}}} = \frac{(40\text{ V})^2}{4(20\ \Omega)} = \mathbf{20\ W}$$

c. They are all operating under a 50% efficiency level because $R_L = R_{Th}$.

d. The power to the load is determined as follows:

$$I_L = \frac{E}{R_{\text{int}} + R_L} = \frac{40\text{ V}}{20\ \Omega + 1000\ \Omega} = \frac{40\text{ V}}{1020\ \Omega} = 39.22\text{ mA}$$

and $$P_L = I_L^2 R_L = (39.22\text{ mA})^2(1000\ \Omega) = \mathbf{1.54\ W}$$

The power level is significantly less than the 20 W achieved in part (b). The efficiency level is

$$\eta\% = \frac{P_L}{P_s} \times 100\% = \frac{1.54\text{ W}}{EI_s} \times 100\% = \frac{1.54\text{ W}}{(40\text{ V})(39.22\text{ mA})} \times 100\%$$

$$= \frac{1.54\text{ W}}{1.57\text{ W}} \times 100\% = \mathbf{98.09\%}$$

which is markedly higher than achieved under maximum power conditions—albeit at the expense of the power level.

e. For the dc generator,

$$\eta = \frac{P_o}{P_s} = \frac{R_L}{R_{Th} + R_L} (\eta \text{ in decimal form})$$

and
$$\eta = \frac{R_L}{R_{Th} + R_L}$$

$$\eta(R_{Th} + R_L) = R_L$$

$$\eta R_{Th} + \eta R_L = R_L$$

$$R_L(1 - \eta) = \eta R_{Th}$$

and
$$\boxed{R_L = \frac{\eta R_{Th}}{1 - \eta}} \qquad \textbf{(9.7)}$$

$$R_L = \frac{0.75(2.5\ \Omega)}{1 - 0.75} = \mathbf{7.5\ \Omega}$$

For the battery,

$$R_L = \frac{0.75(0.05\ \Omega)}{1 - 0.75} = \mathbf{0.15\ \Omega}$$

For the laboratory supply,

$$R_L = \frac{0.75(20\ \Omega)}{1 - 0.75} = \mathbf{60\ \Omega}$$

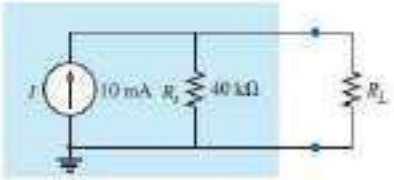


FIG. 9.92
Example 9.16.

EXAMPLE 9.16 The analysis of a transistor network resulted in the reduced equivalent in Fig. 9.92.

a. Find the load resistance that will result in maximum power transfer to the load, and find the maximum power delivered.
b. If the load were changed to 68 kΩ, would you expect a fairly high level of power transfer to the load based on the results of part (a)? What would the new power level be? Is your initial assumption verified?
c. If the load were changed to 8.2 kΩ, would you expect a fairly high level of power transfer to the load based on the results of part (a)? What would the new power level be? Is your initial assumption verified?

Solutions:

a. Replacing the current source by an open-circuit equivalent results in

$$R_{Th} = R_s = 40\ \text{k}\Omega$$

Restoring the current source and finding the open-circuit voltage at the output terminals results in

$$E_{Th} = V_{oc} = IR_s = (10\ \text{mA})(40\ \text{k}\Omega) = 400\ \text{V}$$

For maximum power transfer to the load,

$$R_L = R_{Th} = 40\ \text{k}\Omega$$

with a maximum power level of

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(400\ \text{V})^2}{4(40\ \text{k}\Omega)} = 1\ \text{W}$$

b. Yes, because the 68 kΩ load is greater (note Fig. 9.86) than the 40 kΩ load, but relatively close in magnitude.

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{400\text{ V}}{40\text{ k}\Omega + 68\text{ k}\Omega} = \frac{400}{108\text{ k}\Omega} \cong 3.7\text{ mA}$$

$$P_L = I_L^2 R_L = (3.7\text{ mA})^2(68\text{ k}\Omega) \cong \mathbf{0.93\text{ W}}$$

Yes, the power level of 0.93 W compared to the 1 W level of part (a) verifies the assumption.

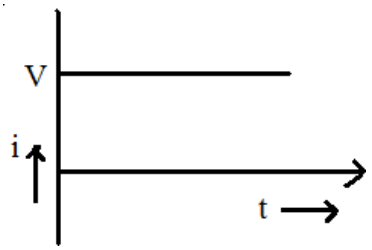
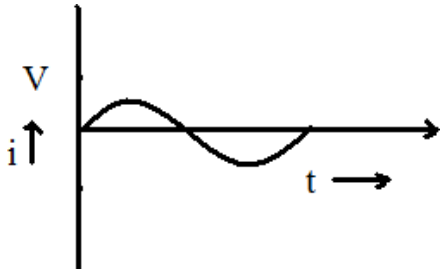
c. No, 8.2 kΩ is quite a bit less (note Fig. 9.86) than the 40 kΩ value.

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{400\text{ V}}{40\text{ k}\Omega + 8.2\text{ k}\Omega} = \frac{400\text{ V}}{48.2\text{ k}\Omega} \cong 8.3\text{ mA}$$

$$P_L = I_L^2 R_L = (8.3\text{ mA})^2(8.2\text{ k}\Omega) \cong \mathbf{0.57\text{ W}}$$

Yes, the power level of 0.57 W compared to the 1 W level of part (a) verifies the assumption.

CHAPTER-05
ACCIRCUITANDRESONANCE

DirectCurrent		**AlternatingCurrent**	
			
(1)	D.C. always flow in onedirection and whose magnitude remains constant.	(1)	A.C. is one which reverseperiodically in direction and whose magnitude undergoes a definite cycle changes in definite intervals of time.
(2)	Highcostofproduction.	(2)	Lowcostofproduction
(3)	It is not possible by D.C.Because D.C.is dangerous to the transformer.	(3)	Byusingtransformer A.C.voltage can be decreased or increased.
(4)	Itstransmissioncostistoohigh.	(4)	A.C.canbetransmittedtoalong distance economically.

DefinitionofA.C.terms:-

Cycle: Itisonecompletesetof+veand–vevaluesofalternatingquality spread over 360°or 2∏radan.

TimePeriod: Itisdefinedasthetimerequiredtocompleteonecycle.

Frequency: Itisdefinedasthereciprocaloftimeperiod.i.e.f=1/T

Or

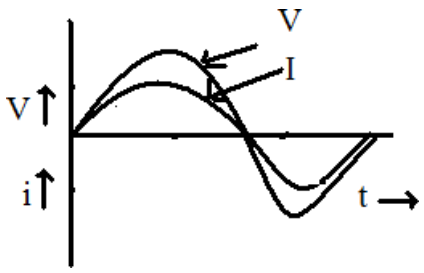
Itisdefinedasthenumberofcyclescompletedpersecond.

Amplitude : It is defined as the maximum value ofeither +ve halfcycle or –ve half cycle.

Phase: Itisdefinedastheangulardisplacementbetweentwohavesiszero.

OR

Two alternating quantity are inphase when each pass through their zero value at the same instant and also attain their maximum value at the same instant in a given cycle.



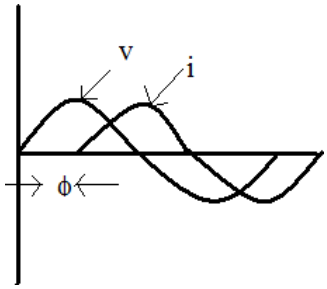
$V = V_m sinwt\ i$

$= I_m sin\ wt$

PhaseDifference:-Itisdefinedastheangulardisplacementbetweentwoalternating quantities.

OR

If the angular displacement between two waves are not zero, then that is known as phase difference. i.e. at a particular time they attain unequal distance.



OR

Two quantities are out of phase if they reach their maximum value or minimumvalueatdifferenttimesbutalwayshaveanequalphaseanglebetween them.

Here $V = V_m sinwt$

$i = I_m sin(wt-\phi)$

Inthiscasecurrentlagsvoltagebyanangle'ϕ'.

PhasorDiagram:

GenerationofAlternatingemf:-

Consider a rectangular coil of 'N" turns, area of cross-section is 'A' $nt^2$is placed in x-axis in an uniform magnetic field of maximum flux density *Bm web/nt*^{2}. The coil is rotating in the magnetic field with a velocity of w radian / second. Attime t = 0, the coil is in x-axis. After interval of time 'dt' second the coil make rotating in anti-clockwise direction and makes an angle 'θ' with x-direction.The perpendicular component of the magnetic field is $\phi = \phi n\ cos\ wt$

AccordingtoFaraday'sLawsofelectro-magneticInduction

$$e = -N\frac{d\phi}{dt}$$

$$= -N\frac{d}{dt}(\phi_m \cos wt)$$

$$= -N(-\phi_m w \cos wt)$$

$$= N w \phi_m \sin wt$$

$$= 2\pi f N \phi_m \sin wt \quad (\because w = 2\pi f)$$

$$= 2\pi f N B_m A \sin wt \; e$$

$$= E_m \sin wt$$

Where $E_m = 2\pi f N B_m A$

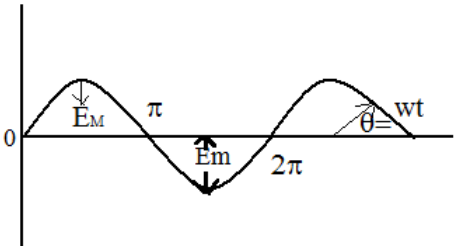
f → frequency in Hz

B_m → Maximum flux density in Wb/mt^2

Now when θ or wt = 90° e =

E_m

i.e. $E_m = 2\pi f N B_m A$



Root Mean Square (R.M.S) Value:→

The r.m.s. value of an a.c. is defined by that steady (d.c.) current which when flowing through a given circuit for a given time produces same heat as produced by the alternating current when flowing through the same circuit for the same time.

Sinuscdialalternatingcurrentis i

$= I_m \sin wt = I_m \sin \theta$

The mean of squares of the instantaneous values of current over one complete cycle

$$= \frac{\int_0^{2\pi} i^2 . d\theta}{(2\pi - 0)}$$

Thesquarerootofthisvalueis

$$= \sqrt{\frac{\int_0^{2\pi} i^2 . d\theta}{2\pi}}$$

$$= \sqrt{\int_0^{2\pi} \frac{(I_m \sin\theta)^2}{2\pi} d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta . d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi} (1-\cos 2\theta)\ d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\frac{\theta - \sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left(\int_0^{2\pi} \left(2\pi - \frac{\sin 4\pi}{2}\right) d\theta\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi} (2\pi - 0)}$$

$$= \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$I_{r.m.s} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

AverageValue:→

The average value of an alternating current is expressed by that steady current (d.c.) which transfers across any circuit the same charge as it transferred by that alternating current during the sae time.

Theequationofthealternatingcurrentisi=$I_m \sin\theta$

$$I_{av} = \frac{\int_0^{\pi} i.d\theta}{(\pi - 0)}$$

$$= \frac{\int_0^{\pi} I_m . \sin\theta\, d\theta}{\pi} = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta . d\theta$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{\pi}\left[-\cos\pi - (\cos 0^0\right]$$

$$= \frac{I_m}{\pi}\left[1 - 0(-1)\right]$$

$$I_{av} = \frac{2I_m}{\pi}$$

$$I_{av} = \frac{2 \times MaximumCurrent}{\pi}$$

Hence,$I_{av} = 0.637 I_m$

Theaveragevalueoveracompletecycleiszero

Amplitude factor/ Peak factor/ Crest factor :- It is defined as the ratio of maximum value to r.m.s value.

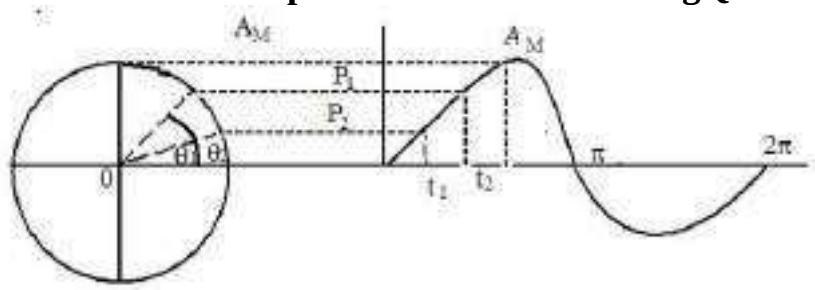
$$Ka=\frac{MaximumValue}{R.M.S.Value}=\frac{I_m}{\frac{I_m}{\sqrt{2}}}=\sqrt{2}=1.414$$

Formfactor:-Itisdefinedastheratioofr.m.svaluetoaveragevalue.

$$Kf=\frac{r.m.s.Value}{Average.Value}=\frac{0.707I_m}{0.637I_m}=\sqrt{2}=1.414$$

$$Kf=1.11$$

PhasororVectorRepresentationofAlternatingQuantity:→



An alternating current or voltage, (quantity) in a vector quantity whichhas magnitude as well as direction. Let the alternating value of current be represented by theequation e = E_mSin wt. The projection of $E_m$on Y-axis at any instant gives the instantaneous value of alternating current. Since the instantaneous values are continuously changing, so they are represented by a rotating vector or phasor. A phasor is a vector rotating at a constant angular velocity

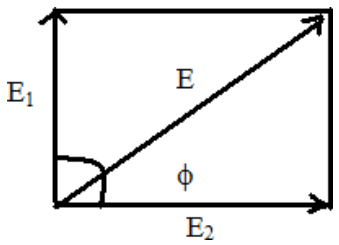
Att_1,$e_1=E_m\sin_1 wt_1$

Att_2,$e_2=E_m\sin_2 wt_2$

AdditionoftwoalternatingCurrent:→

Let$e_1=E_m\sin_1 wt$

$e_2=E_m\sin_2(wt-\phi)$



The sum of two sine waves of the samefrequency is another sine wave of samefrequency but of a different maximum value and Phase.

$$e=\sqrt{e_1^2+e+2_2^2 ee\cos\phi_{12}}$$

PhasorAlgebra:→

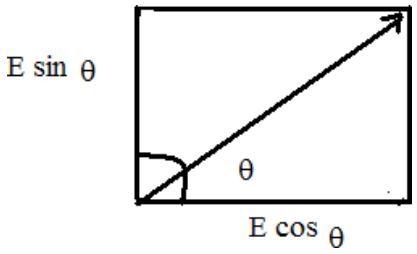
Avectorquantitycanbeexpressedintermsof

(i) RectangularorCartesianform

(ii) Trigonometricform

(iii) Exponentialform

(iv) Polarform



$E=a+jb$

$=E(\cos\theta+j\sin\theta)$

Where a = E cos θis the active part

b=Esinθisthereactivepart

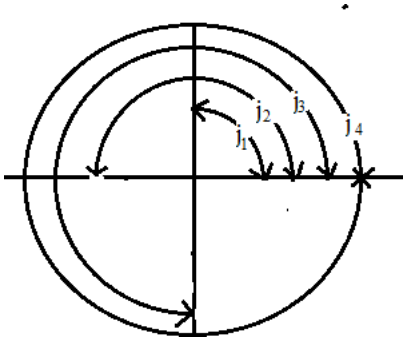
$\theta=\tan^{-1}\left(\frac{b}{a}\right)$=Phaseangle

$j=\sqrt{-1}(90^o)$

$j^2=-1(180^o)$

$j^3=-j(270^o)$

$j^4=1(360^o)$



(i) Rectangularfor:-

$E=a\pm jb$

$\tan\theta=b/a$

(ii) Trigonometricform:-

$E=E(\cos\theta\pm j\sin\theta)$

(iii) Exponentialform:-

$E=Ee^{\pm j\theta}$

(iv) Polarform:-

$E=E/\pm e$ $\quad (E=\sqrt{a^2+b^2}\,)$

AdditionorSubtration:-

$E_1=a_1+jb_1E_2=$

a_2+jb_2

$E_1\pm E_2=(a_1+a_2)\pm(b_1+b_2$

$\phi=\tan^{-1}\left(\frac{b_1+b_2}{a+a}\right)$

Multiplication:-

$E_1\times E_2=(a_1+ja_1)\pm(a_1+jb_2)$

$=(a_1a_2-b_1b_2)+j(a_1a_2+b_1b_2)$

$$\phi=\tan^{-1}\left(\frac{a_1b_2+b_1a_2}{a_1a_2-b_1b_2}\right)$$

$$E_1=E_1\angle\theta_1$$

$$E_2=E_2\angle\theta_2$$

$$E_1\times E_2=E_1E_2\angle\phi_1+\phi_2$$

Division:-

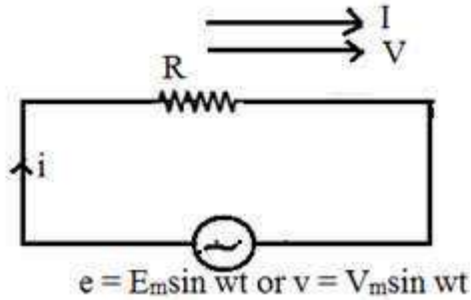
$$E_1=E_1\angle\theta_1$$

$$E_2=E_2\angle\theta_2$$

$$\frac{E_1}{E_2}=\frac{E_1\angle\theta_1}{E_2\angle\theta_2}=\frac{E_1}{E_2}\angle\theta_1-\theta_2$$

A.C.throughPureResistance:→

LettheresistanceofRohmisconnectedacrosstoA.Csupplyofapplied voltage



$$e=E_m\sin wt \text{-----------------------------} (1)$$

Let'I'istheinstantaneouscurrent.

Here $e = iR$

$$\Rightarrow i=e/R$$

$$i=E_m\sin wt/R \text{----------------------------} (2)$$

Bycomparingequation(1)andequation(2)wegetalternatingvoltage and current in a pure resistive circuit are in phase

Instantaneouspowerisgivenby P

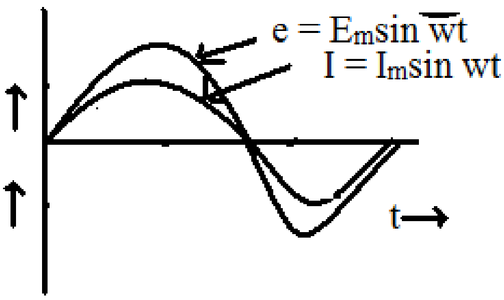
$= ei$

$=E_m\sin wt.I_m\sin wt$

$=E_mI_m\sin^2 wt$

$$=\frac{E_mI_m}{2}.2\sin^2 wt$$

$$=\frac{E_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}.(1-\cos 2wt)$$



$$P=\frac{E_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}-\frac{E_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}.\cos 2wt$$

i.e. $P=\frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}-\frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}.\cos 2wt$

Where $\frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}$ is called constant part of power.

$\frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos 2wt$ is called fluctuating part of power.

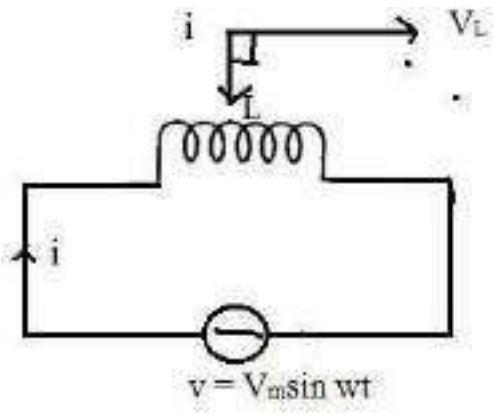
The fluctuating part $\frac{V_m I_m}{2}\cos 2wt$ of frequency double that of voltage and current waves.

Hence power for the whole cycle is $P=\frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}=V_{rms}.I_{rms}$

$\Rightarrow P=VI$ watts

A.C through Pure Inductance:→

Let inductance of 'L' henry is connected across the A.C. supply



$v=V_m \sin wt$ ----------(1)

According to Faraday's laws of electromagnetic inductance the emf induced across the inductance

$V=L\frac{di}{dt}$

$\frac{di}{dt}$ is the rate of change of current

$V_m \sin wt=L\frac{di}{dt}$

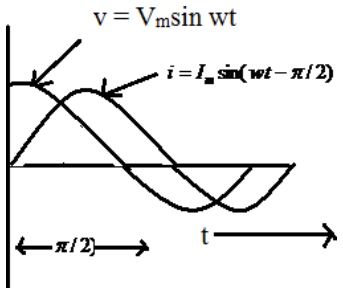
$\frac{di}{dt}=\frac{V_m \sin wt}{L}$

$\Rightarrow di=\frac{V_m}{L}\sin wt.dt$

Integrating both sides,

$\int di=\int \frac{V_m}{L}\sin wt.dt$

$i=\frac{V_m}{L}\left(\frac{-\cos wt}{w}\right)$



$i = -\frac{V_m \cos wt}{wL}$

$i = -\frac{V_m}{wL}\cos wt$

$i = -\frac{V_m}{wL}\sin\left(wt - \frac{\pi}{2}\right)$

$= -\frac{V_m}{X_L}\sin\left(wt - \frac{\pi}{2}\right)$ [Q $X = 2\pi fL = wL$]

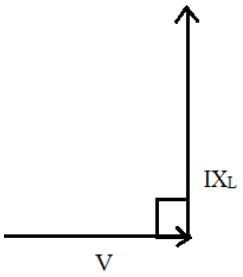
Maximum value of i is $I_m = \frac{V_m}{X_L}$ when $\sin\left(wt - \frac{\pi}{2}\right)$ is unity.

Hence the equation of current becomes $i = I_m \sin(wt - \pi/2)$

So we find that if applied voltage is rep[resented $v = V_m \sin wt$, then current by flowing in a purely inductive circuit is given by

$i = I_m \sin(wt - \pi/2)$

Here current lags voltage by an angle $\pi/2$ Radian.



Power factor $= \cos\phi$

$= \cos 90°$

$= 0$

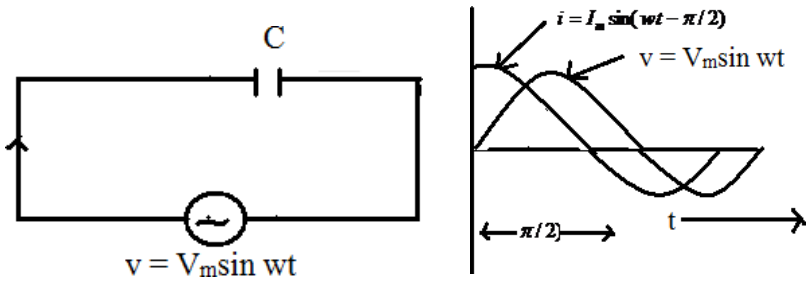
Power Consumed $= VI\cos\phi$

$= VI \times 0$

$= 0$

Hence, the power consumed by a purely Inductive circuit is zero.

A.C. Through Pure Capacitance:→



Let a capacitance of "C" farad is connected across the A.C. supply of applied voltage

$v = V_m \sin wt$ ----------(1)

Let 'q' = change on plates when p.d. between two plates of capacitor is 'v'

$q = cv$

$q = cV_m \sin wt$

$$\frac{dq}{dt}=c\frac{d}{dt}(V_m\sin wt)$$

$$i=cV_m sinwt$$

$$=wcV_m coswt$$

$$=\frac{V_m}{1/wc}=\cos wt$$

$$=\frac{V_m}{Xc}=\cos wt \qquad [Q X_c = \frac{1}{wc} = \frac{1}{2\pi fc}$$ isknown as capacitivereactance inohm.]

$$=I_m\cos wt$$

$$=I_m\sin(wt+\pi/2)$$

Herecurrentleadsthesupplyvoltagebyanangle$\pi/2$radian.

Powerfactor $=\cos\phi$

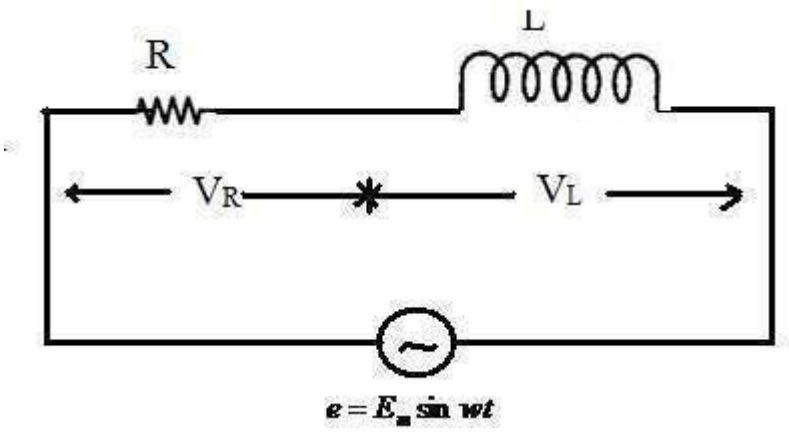
$= \cos 90° = 0$

Power Consumed= VI cos ϕ

$=VI\times 0 = 0$

Thepowerconsumedbyapurecapacitivecircuitiszero.

A.C.ThroughR-LSeriesCircuit:→



TheresistanceofR-ohmandinductanceofL-henryareconnectedinseries across the A.C. supply of applied voltage

$e=E_m\sin wt$---------- (1)

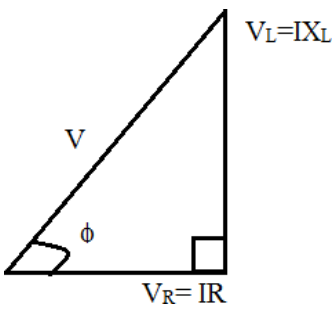
$$V=V_R+jV_L$$

$$=\sqrt{V_R^2+V_L^2}\angle\phi=\tan^{-1}\left(\frac{X_L}{R}\right)$$

$$=\sqrt{(IR)^2+(IX)_L^2}\angle\phi=\tan^{-1}\left(\frac{X_L}{R}\right)$$

$$=I\sqrt{R^2+X_L^2}\angle\phi=\tan^{-1}\left(\frac{X_L}{R}\right)$$

$$V=IZ\angle\phi=\tan^{-1}\left(\frac{X_L}{R}\right)$$



Where $Z = \sqrt{R^2+X^2_{\ L}}$

$= R + jX_L$ is known as impedance of R-L series Circuit.

$$I = \frac{V}{Z\angle\phi} = \frac{E_m \sin wt}{Z\angle\phi}$$

$I = I_m \sin(wt - \phi)$

Here current lags the supply voltage by an angle ϕ.

Power Factor: → It is the cosine of the angle between the voltage and current.

OR

It is the ratio of active power to apparent power.

OR

It is the ratio of resistance to inpedence.

Power: →

$= v.i$

$= V_m \sin wt . I_m \sin(wt - \phi)$

$= V_m I_m \sin wt . \sin(wt - \phi)$

$= \frac{1}{2^{mm}} VI \quad 2\sin wt . \sin(wt - \phi)$

$= \frac{1}{2^{mm}} VI \quad [\cos\phi - \cos 2(wt - \phi)]$

Obviously the power consists of two parts.

(i) a constant part $\frac{1}{2^{mm}} VI\cos\phi$ which contributes to real power.

(ii) a pulsating component $\frac{1}{2^{mm}} VI\cos(2wt - \phi)$ which has a frequency twice that of the voltage and current. It does not contribute to actual power since its average value over a complete cycle is zero.

Hence average power consumed

$$= \frac{1}{2^{mm}} VI\cos\phi$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

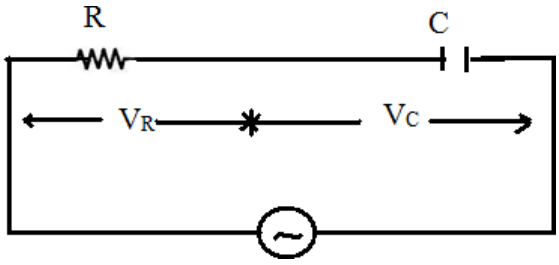
$$= VI\cos\phi$$

Where V & I represents the r.m.s value.

A.C. Through R-C Series Circuit: →

The resistance of 'R'-ohm and capacitance of 'C' farad is connected across the A.C. supply of applied voltage

$e=E_m \sin wt$ ----------(1)



$V=V_R+(-jV_C)$

$=IR+(-jIX_C)$

$=I(R-jX_C)$

$V=IZ$

Where $Z=R-jX_C=\sqrt{R^2+X^2{}_C}$ isknownasimpedanceofR-CseriesCircuit.

$Z=R-jX_C$

$=\sqrt{R^2+X^2{}_C}$

$\angle-\phi=\tan^{-1}\left(\frac{X_C}{R}\right)$

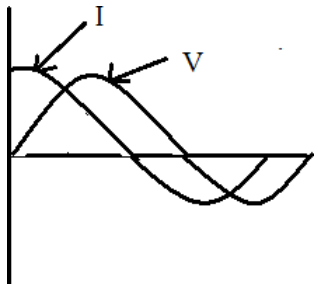
$V=IZ\angle-\phi$

$\Rightarrow I=\frac{V}{Z\angle-\phi}$

$=\frac{E_m \sin wt}{Z\angle-\phi}$

$=\frac{E_m}{Z\angle}\sin(wt+\phi)$

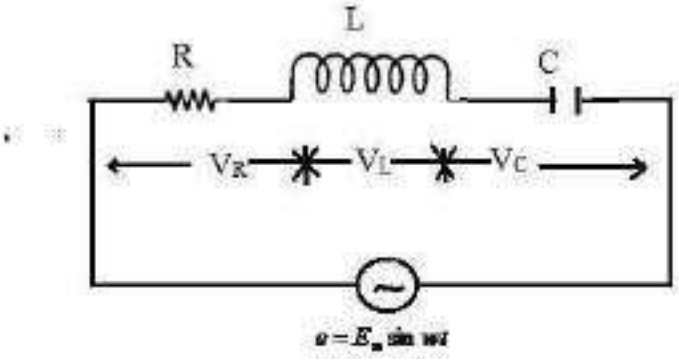
$\Rightarrow I=I_m \sin(wt+\phi)$



Herecurrentleadsthesupplyvoltagebyanangle‘ϕ’.

A.C.ThroughR-L-CSeriesCircuit:→

Letaresistanceof‘R’-ohminductanceof‘L’henryandacapacitanceof‘C’ farad are connectedacross the A.C. supply in series of applied voltage



$e=E_m \sin wt$---------- (1)

$$\vec{e}=\vec{V_R}+\vec{V_L}+\vec{V_C}$$

$$=V_R+jV_L-jV_C$$

$$=V_R+j(V_L-V_C)$$

$$=I_R+j(IX_L-IX_C)$$

$$=I[R+j(X_L-X_C)]$$

$$=I\sqrt{R^2+(X-X_{L\ \ C})^2} \qquad \angle\pm\phi=\tan^{-1}\left(\frac{X_L-X_C}{R}\right)$$

$$=IZ\angle\pm\phi$$

Where $Z=I\sqrt{R^2+(X-X_{L\ \ C})^2}$ isknownastheimpedanceofR-L-CSeries Circuit.

If$X_L>X_C$,thentheangleis+ve.

If$X_L<X_C$, then the angle is -ve.

Impedanceisdefinedasthephasorsumofresistanceandnetreactance

$$e=IZ\angle\pm\phi$$

$$\Rightarrow I=\frac{e}{Z\angle\pm\phi}IZ\angle\pm\phi \qquad =\frac{E_m\sin wt}{Z\angle\pm\phi} \quad =I_m\ \sin(wt\pm\phi)$$

(1) If $X_L>X_C$,thenP.fwillbelagging.

(2) If $X_L<X_C$,then,P.fwillbeleading.

(3) If $X_L=X_C$,then,thecircuitwillberesistiveone.Thep.f.becomesunity andtheresonanceoccurs.

REASONANCE

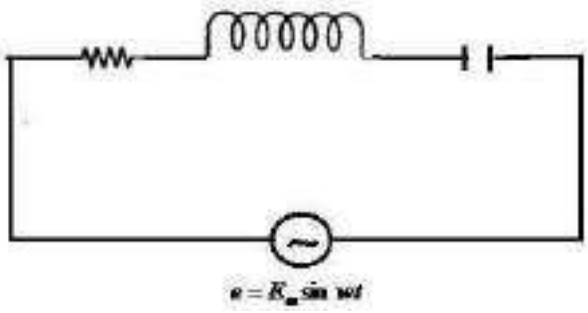
It is defined as the resonance in electrical circuit having passive or active elements represents a particular state when the current and the voltage in the circuitismaximumandminimumwithrespecttothemagnitudeofexcitationat a particular frequency and the impedances being either minimum or maximum at unity power factor

Resonanceareclassifiedintotwotypes.

(1) SeriesResonance

(2) ParallelResonance

(1) Series Resonance :- Letaresistanceof‘R’ohm,inductanceof‘L’ henry and capacitance of ‘C’ farad are connected in series across A.C. supply



$e=E_m \sin wt$

Theimpedanceofthecircuit

$Z=R+j(X_L-X_C)]$

$Z=\sqrt{R^2+(X-X_{L\quad C})^2}$

Theconditionofseriesresonance:

Theresonancewilloccurwhenthereactivepartofthelinecurrentiszero The p.f. becomes unity.

The net reactance will be zero.

The current becomes maximum.

Atresonancenetreactanceiszero

$X_L-X_C=0$

$\Rightarrow X_L=X_C$

$\Rightarrow W_oL=\frac{1}{W_oC}$

$\Rightarrow W_o^2LC=1$

$\Rightarrow W_o^2=\frac{1}{LC}$

$\Rightarrow W_o=\frac{1}{\sqrt{LC}}$

$\Rightarrow 2\pi f_o=\frac{1}{\sqrt{LC}}$

$\Rightarrow f_o=\frac{1}{2\pi\sqrt{LC}}$

Resonantfrequency$(f_o)=\frac{1}{2\pi}\cdot\frac{1}{\sqrt{LC}}$

ImpedanceatResonance

Z_0= R

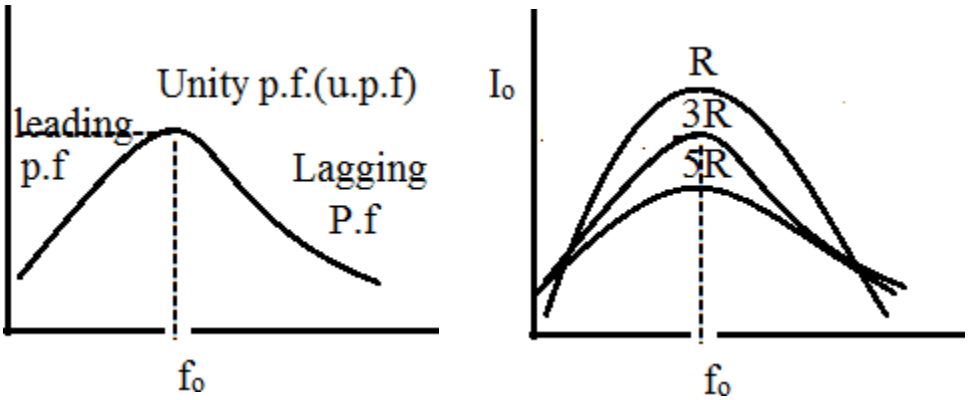
CurrentatResonance

$I_o=\frac{V}{R}$

Powerfactoratresonance

$p.f.=\frac{R}{Z_o}=\frac{R}{R}=1 \qquad [QZ_o=R]$

ResonanceCurve:-



At low frequency the $X_c$is greater and the circuit behaves leading and at high frequency the $X_L$becomes high and the circuit behaveslagging circuit.

Iftheresistancewillbelowthecurvewillbestiff(peak).

- If the resistance will go oh increasing the current goes on decreasing and the curve become flat.

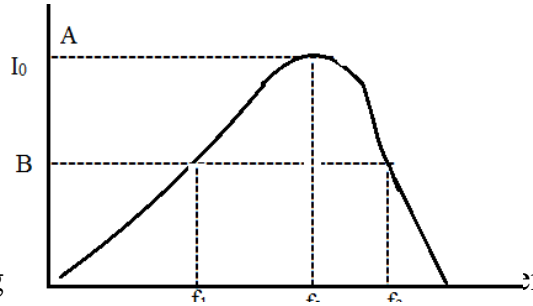
BandWidth:→

At point 'A' the power loss is I_0^2R.

Thefrequencyis$f_0$whichisatresonance.

Atpoint'B'thepowerlossis $\frac{I^2R}{2}$.

Thepowerlossis50%ofthepowerlossatpoint 'A"/



Hencethefrequencies corresponding er frequencies f_1& f_2.

f_1=Lowerhalfpowerfrequency

$$f_1 = f_0 - \frac{R}{4\pi L}$$

F_2=Upperhalfpowerfrequency

$$f_2 = f_0 + \frac{R}{4\pi L}$$

Bandwidth(B.W.)isdefinedasthedifferencebetweenupperhalfpower frequencyadlowerhalfpowerfrequency.

$$\text{B.W.} = f_2 - f_1 = \frac{R}{2\pi L}$$

Selectivity:→

Selectivityisdefinedastheratioof Bandwidthtoresonantfrequency

Selectivity= $\frac{B.W.}{f_0} = \frac{R}{2\pi L}$ $Selectivity = \frac{R}{2\pi f_o L}$

QualityFactor(Q-factor):→

Itisdefinedastheratioof $2\pi \times$ Maximumenergystoredtoenergydissipated per cycle

$$\text{Q-factor} = \frac{2\pi \times \frac{1}{2} LI^2}{I^2 RT}$$

$$= \frac{\pi L\left(\sqrt{2}I\right)^2}{I^2 RT}$$

$$= \frac{\pi L.2I^2}{I^2 RT}$$

$$= \frac{\pi L.2I^2}{I^2 RT}$$

$$= \frac{2\pi L.}{RT}$$

$$\text{Qualityfactor} = = \frac{2\pi f_0 L.}{R}$$

$$\left[Q = \frac{1.}{I} = f_0\right]$$

Qualityfactorisdefinedasthereciprocalofpowerfactor.

$$\text{Qfactor} = = \frac{1.}{\cos\phi}$$

Itisthereciprocalofselectivity.

Q-factorOrMagnificationfactor $= \frac{\text{Voltage across Inductor.}}{\text{Voltage across resistor}}$

$$= \frac{I_0 X_L}{I_0 R}$$

$$= \frac{X_L}{R}$$

$$= \frac{2\pi f_0 L}{R} = \frac{W_0 L}{R}$$

$$\text{Q-factor} = = \frac{W_0 L}{R}$$

Q-factorfactor $= \frac{\text{Voltage across Capacotor.}}{\text{Voltage across resistor}}$

$$= \frac{I_0 X_c}{I_0 R}$$

$$= \frac{X_C}{R}$$

$$= \frac{1}{2\pi f_0 C} = \frac{1}{2\pi f_0 CR}$$

Q-factor= $\frac{1}{W_0 CR}$

$Q^2 = \frac{W_0 L}{R} \times \frac{1}{W_0 CR}$

$Q^2 = \frac{1}{R^2 C}$

$Q = \sqrt{\frac{1}{R^2 C}}$

$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

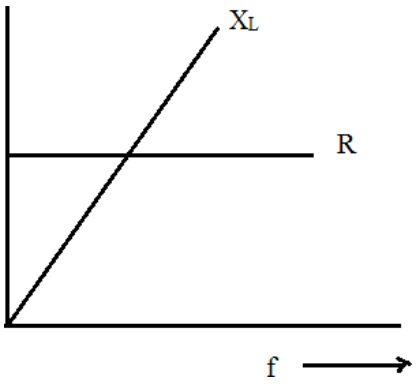
GraphicalMethod:→

(1) Resistanceisindependentoffrequency Itrepresentsastraightline.

(2) InductiveReactance $X_L = 2\pi fL$

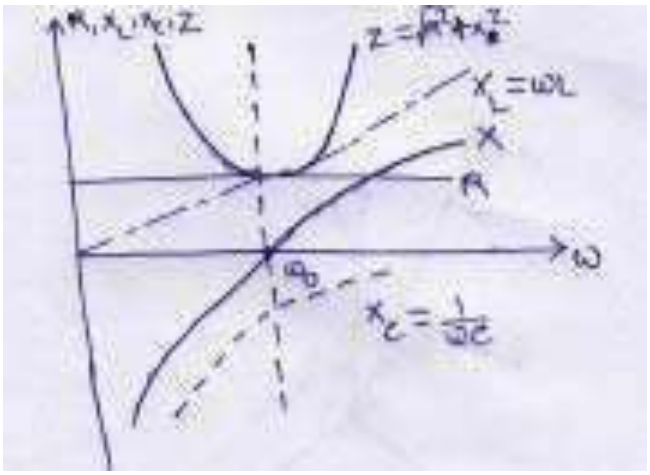
Itisdirectlyproportionaltofrequency. Asthefrequencyincreases, X_L increases

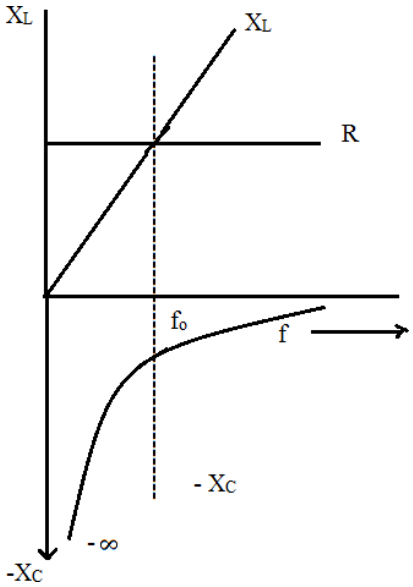
(3) CapacitiveReactance $X_C = = \frac{1}{2\pi fC}$



It isinversely proportional to frequency. As the frequency increases, X_C decreases.

When frequency increases, X_L increases and X_C decreases from the highervalue.



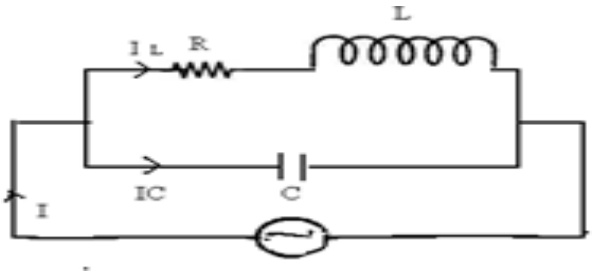


Atacertainfrequency.$X_L=X_C$

ThatparticularfrequencyisknownasResonantfrequency.

Variationofcircuitparameterinseriesresonance:

(2) Parallel Resonance :- Resonance willoccur when the reactive part of theline current is zero.



At resonance,

$I_C-I_L\sin\phi=0$

$I_C=I_L\sin\phi$

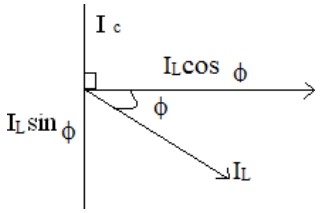
$$\Rightarrow \frac{V}{X_C} = \frac{V}{\sqrt{R^2+X_L^2}}\sin\phi$$

$$\Rightarrow \frac{V}{X_C} = \frac{V}{\sqrt{R^2+X_L^2}} \times \frac{X_L}{\sqrt{R^2+X_L^2}}$$

$$\Rightarrow \frac{1}{X_C} = \frac{X_L}{R^2+X_L^2}$$

$$\Rightarrow R^2+X^2=X_L.X_C$$

$$\Rightarrow Z^2=X_L.X_C=W_0L\times\frac{1}{W_0C}$$



$$Z^2=\frac{L}{C}$$

$$\Rightarrow R^2+X_L^2=\frac{L}{C}$$

$$\Rightarrow R^2+(2\pi f_0 L)^2=\frac{L}{C}$$

$$\Rightarrow R^2+4\pi^2 f_0^2 L^2=\frac{L}{C}$$

$$\Rightarrow 4\pi^2 f_0^2 L^2=\frac{L}{C}-R^2$$

$$\Rightarrow f_0^2=\frac{1}{4\pi^2 f_0^2 L^2}=\left(\frac{L}{C}-R^2\right)$$

$$\Rightarrow f_0=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$$

f_0=Resonantfrequencyinparallelcircuit.

CurrentatResonance=$I_L\cos\phi$

$$=\frac{V}{\sqrt{R^2+X_L^2}}\cdot\frac{R}{\sqrt{R^2+X_L^2}}$$

$$=\frac{VR}{R^2+X_L^2}$$

$$=\frac{VR}{Z^2}$$

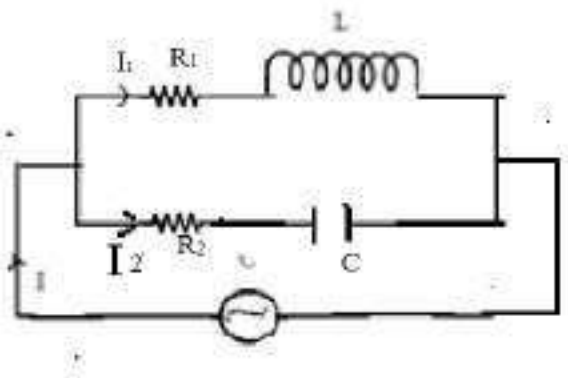
$$=\frac{VR}{L/C}=\frac{V}{L/RC}$$

$$=\frac{\text{V}}{\text{Dynamic Impedence}}$$

$L/RC\rightarrow$DynamicImpedanceofthecircuit.

or, dynamic impedances is defined as the impedance at resonance frequency in parallel circuit.

ParallelCircuit:→



Theparallelresonancecondition:

Whenthereactivepartofthelinecurrentiszero. The net reactance is zero.

Thelinecurrentwillbeminimum. The power factor will be unity

Impedance $Z_1=R_1+jX_L$

$Z_2=R_2-jX_C$

Admittance $Y_1=\frac{1}{Z_1}=\frac{1}{R+jX_L}$

$=\frac{(R_1+jX_L)}{(R_1+jX_L)(R_1-jX_L)}$

$=\frac{R_1+jX_L}{R_1^2+X_L^2}$

$Y_1=\frac{R_1}{R^2+X_L^2}-j\frac{X_L}{R_1^2+X_L^2}$

Admittance $Y_2=\frac{1}{Z_2}=\frac{1}{R_1+jX_C}$

$=\frac{(R_2+jX_C)}{(R2_1-jX_C)(R_2+jX_C)}$

$=\frac{R_2+jX_L}{R_2^2+X_C^2}$

$Y_2=\frac{R_2}{R^2+X_C^2}+j\frac{X_C}{R_2^2+X_C^2}$

TotalAdmittanceAdmittance $\left(\frac{1}{Z}\right)=\frac{1}{Z_1}+\frac{1}{Z_2}$

$\Rightarrow Y=Y_1+Y_2$

$\Rightarrow Y=\frac{R_1}{R^2+X_L^2}-j\frac{X_L}{R_1^2+X_L^2}+\frac{R2}{R_2^2+X_C^2}+j\frac{X_C}{R_2^2+X_C^2}$

$\Rightarrow Y=\frac{R_1}{R^2+X_L^2}+\frac{R2}{R^2+X_C^2}-j\left(\frac{X_L}{R^2+X_L^2}-\frac{X_C}{R^2+X_C^2}\right)$

AtResonance,

$\frac{X_L}{R_1^2+X_L^2}-\frac{X_C}{R_2^2+X_C^2}=0$

$\Rightarrow\frac{X_L}{R_1^2+X_L^2}=\frac{X_C}{R_2^2+X_C^2}$

$\Rightarrow X_L\left(R_2^2+X_C^2\right)=X_C\left(R_1^2+X_L^2\right)$

$\Rightarrow 2\pi fL\left(R^2+\frac{1}{4\pi^2f^2C^2}\right)=\frac{1}{2\pi fC}\left(R_1^2+4\pi^2f^2L^2\right)$

$\Rightarrow 2\pi fLR_2^2+\frac{L}{2\pi fC^2}=R_1^2+\frac{2\pi fL^2}{C}$

$$\Rightarrow \frac{L}{2\pi fC^2} - \frac{R_1^2}{2\pi fC} = \frac{2\pi fL^2}{C} - 2\pi fLR_2^2$$

$$\Rightarrow \frac{1}{2\pi fC}\left(\frac{L}{C} - R_1^2\right) = 2\pi fL\left(\frac{L}{C} - R_2^2\right)$$

$$\Rightarrow 4\pi^2 f^2 LC = C\,\frac{\frac{L}{C} - R_1^2}{\frac{L}{C} - R_2^2} = \frac{L - CR_1^2}{L - CR_2^2}$$

$$\Rightarrow 4\pi^2 f^2 = \frac{1}{LC}\left(\frac{L - CR_1^2}{L - CR_2^2}\right)$$

$$\Rightarrow f^2 = \frac{1}{4\pi^2 LC}\left(\frac{L - CR_1^2}{L - CR_2^2}\right)$$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(\frac{L - CR_1^2}{L - CR_2^2}\right)}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\left(\frac{L - CR_1^2}{LC - LCR_2^2}\right)}$$

f iscalledResonantfrequency.

If $R^2 = 0$

Then $f = \frac{1}{2\pi}\sqrt{\frac{L - CR_1^2}{L^2C}}$

$$= \frac{1}{2\pi L}\sqrt{\frac{L - CR_1^2}{C}}$$

$$= \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R_1^2}$$

$$= \frac{1}{2\pi}\sqrt{\frac{L}{L^2C} - \frac{R^2}{L^2}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{L}{LC} - \frac{R^2}{L^2}}}$$

If R_1 and $R_2 = 0$, then

$$f = \frac{1}{2\pi}\sqrt{\frac{L}{L^2C}}$$

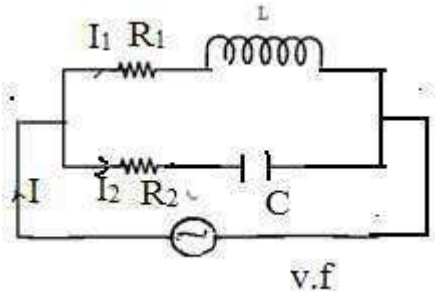
$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi\sqrt{LC}}}$$

ComparisonofSeriesandParallelResonantCircuit:→

Item	**Seriesckt(R-L-C)**	**Parallelckt(R–Land C)**

❖ ImpedanceatResonance	Minimum	Maximum
❖ CurrentatResonance	Maximum=$\frac{V}{R}$	Minimum=$\frac{V}{(L/CR)}$
❖ EffectiveImpedance	R	$\frac{L}{CR}$
❖ P.f.atResonance	Unity	Unity
❖ ResonantFrequency	$\frac{1}{2\pi\sqrt{LC}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$
❖ ItMagnifies	Voltage	Current
❖ Magnificationfactor	$\frac{WL}{R}$	$\frac{WL}{R}$

Parallelcircuit:→



$Z_1=R_1 + jX_L= \sqrt{R_1^2+X_L^2}\angle\phi_1$

$Z_2=R_1 -jX_C= \sqrt{R2_1^2+X_C^2}\angle-\phi_2$

$I_1=\frac{V}{Z\angle\phi_1}=\frac{V}{Z_1}\angle-\phi_1=I_1\angle-\phi_1$

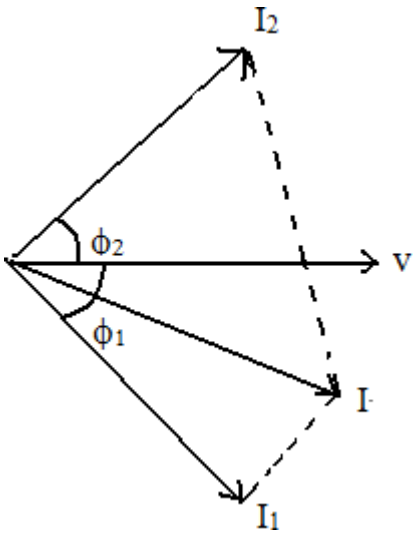
Where $\frac{V}{Z_1}=VY_1$

HereY_1→Admittanceofthecircuit

Admittanceisdefinedasthereciprocalofimpedence.

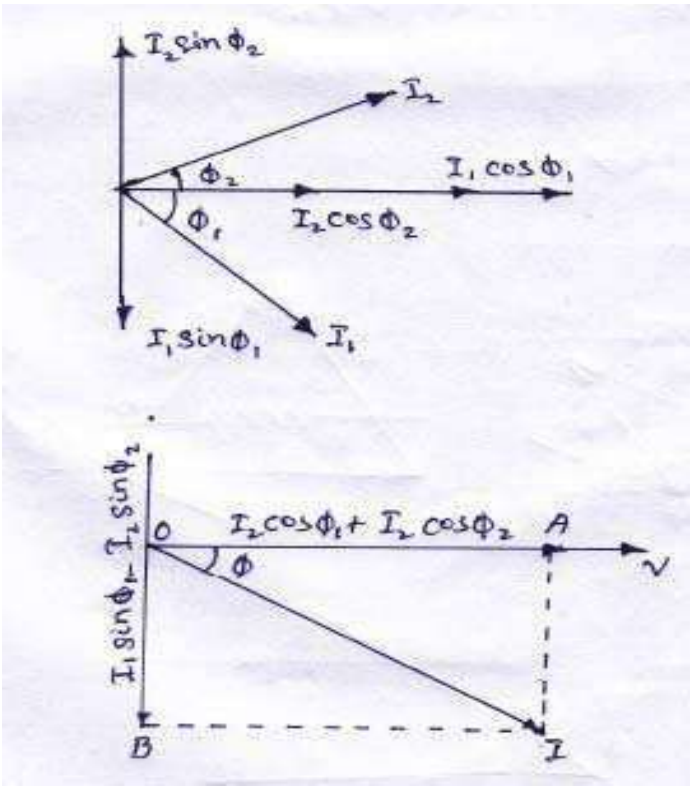
$$I_1 = VY_1 = \frac{v}{R_1 + jX_L}$$

$$I_2 = \frac{V}{Z_2 \angle -\phi 2_1} = \frac{V}{Z_2} \angle \phi_1 = VY_2 \angle \phi = I_{\ 2} \angle \phi_2$$



$$I = \sqrt{I_1^2 + I^2_{\ 2}} + 2I_1 I_2 \cos(\phi_1 + \phi_2)$$

$$I = I_1 \angle -\phi_1 + I_2 \angle \phi_2$$



The resultant current “I” is the vector sum of the branch currents I_1& I_2canbefoundbyusingparallelogramlowofvectorsorresolvingI_2intotheirX

–andY-components(oractiveandreactivecomponentsrespectively)andthen by combining these components.

SumofactivecomponentsofI$_1$andI$_2$=I$_1$cosϕ_1+I$_2$cosϕ_2
SumofthereactivecomponentsofI$_1$andI$_2$=I$_2$sinϕ_2-I$_1$sinϕ_1

EXP–01:

A60Hzvoltageof230Veffectivevalueisimpressedonaninductanceof 0.265H

(i) Writethetimeequationforthevoltageandtheresultingcurrent.Letthe zero axis of the voltage wave be att= 0.

(ii) Showthevoltageandcurrentonaphasordiagram.

(iii) Findthemaximumenergystoredintheinductance.

Solution:-

$V_{max}= \sqrt{2}V= \sqrt{2}\times 230V$

f=60Hz, $W=2\pi f$ $=2\pi\times 60=377 rad/s.$

$x_l=wl=377\times 0.265=100\Omega$.

(i) Thetime equationfor voltageis $V(t)=2302\sin 37\sqrt{t}.$

$I_{max}=V_{max}/x_l=230$ $\sqrt{2}$ /100.=2.3 $\sqrt{3}$

$\phi=90^o(lag)$.

QCurrentequationis.

$i(t)=2.32\sin(377t-\pi/2)$

or=2.32cos377t

(ii) Iti

(iii) or$E_{max}=\frac{1}{2}LI^2_{max}=\frac{1}{2}\times 0.265\times(2.32)^2=1.4J$

Example-02:

The potential difference measured across a coil is 4.5 v, when it carries a direct current of 9 A. The same coil when carries an alternating current of 9A at 25 Hz, the potential difference is 24 v. Find the power and the power factor when it is supplied by 50 v, 50 Hz supply.

Solution:

LetRbethed.c.resistanceandLbeinductanceofthecoil.

$R=V/I=4.5/9=0.5\Omega$

Witha.c.currentof25Hz,z=V/1.

$$\frac{24}{9}=2.66\Omega$$

$$x_l=\sqrt{Z^2-R^2}=\sqrt{2.66^2-0.5^2}$$

$$=2.62\Omega$$

$$x_l=2\pi\times25\times L$$

$$x_l=0.0167\Omega$$

At50Hz

$$x_l=2.62\times2=5.24\Omega$$

$$Z=\sqrt{0.5^2+5.24^2}$$

$$=5.06\ \Omega$$

$I=50/5.26=9.5A$

$P=I^2/R=9.5^2\times0.5=45$watt.

Example–03:

A50-µfcapacitorisconnectedacrossa230-v,50–Hzsupply.Calculate

(a) Thereactanceofferedbythecapacitor.

(b) Themaximumcurrentand

(c) Ther.m.svalueofthecurrentdrawnbythecapacitor.

Solution:

(a) $x_l=\frac{1}{wc}=\frac{1}{2\pi fe}=\frac{1}{2\pi\times50\times50\times10^{-6}}=63.6\Omega$

(c) Since230vrepresentsther.m.svalue

Q$I_{rms}=230/x_l=230/63.6=3.62A$

(b) $I_m=I_{r.m.s}\times\sqrt{2}=3.62\times\sqrt{2}=5.11A$

Example–04:

InaparticularR–Lseriescircuitavoltageof10vat50Hzproducesa current of 700 mA. What are the values of R and L in the circuit ?

Solution:

(i) $Z=\sqrt{R^2+(2\pi\times50L)^2}$

$=\sqrt{R^2+98696L^2}$

$V=1z$

$10=700\times10^{-3}\ \sqrt{(R^2+98696L^2)}$

$\sqrt{(R^2+98696L^2)}=10/700\times10^{-3}=100/7$

$R^2+98696L^2=10000/49$---------- (I)

(ii) Inthesecondcase$Z=\sqrt{R^2+(2\pi\times75L)^2}$

Q$10=500\times10^{-3}\ \sqrt{R^2+222066L^2)}=20$

$\sqrt{R^2+222066L^2)}=20$

CHAPTER-07TRANSIENTS

Whenever a network containing energy storage elements such as inductor or capacitor is switched from one condition to another,either by change in applied source or change in network elements,the response current and voltage change from one state to the other state.Thetimetakentochangefromaninitialsteadystatetothefinalsteadystateisknown as the *transient period*.This response is known as *transient response* or *transients.*The response of the network after it attains a final steady value is independent of time and is calledthesteady-stateresponse.Thecompleteresponseofthenetwork isdeterminedwith thehelpofadifferentialequation.

STEADYSTATEANDTRANSIENTRESPONSE

In a network containing energy storage elements, with change in excitation, the currents and voltages in the circuit change from one state to other state. The behaviour of the voltageorcurrentwhen it is changedfrom onestatetoanother is calledthetransientstate. The time taken for the circuit to change from one steady state to another steady state is called the transient time. The application of KVL and KCL to circuits containing energy storageelementsresultsindifferential,ratherthanalgebraicequations.whenweconsidera circuit containing storage elements which are independent of the sources, the response depends upon the nature of the circuit and is called natural response. Storage elements deliver their energy to the resistances. Hence, the response changes, gets saturated after some time,and is referred to asthe transient response. When we consider a source acting on a circuit, the response depends on the nature of the source or sources.This response is called forced response. In other words,the complete response of a circuit consists of two parts; the forced response and the transient response. When we consider a differential equation, the complete solution consists of two parts: the complementary function and the particularsolution.Thecomplementaryfunctiondiesoutaftershort interval,andisreferred to as the transient response or source free response. The particular solution is the steady state response, or the forced response. The first step in finding the complete solution of a circuit is to form a differential equation for the circuit. By obtaining the differentialequation, several methods can be used to find out the complete solution.

DCRESPONSEOFANR-LCIRCUIT

Consideracircuitconsistingofaresistanceandinductanceasshowninfigure.Theinductor in the circuit is initially uncharged and is in series with the resistor.When the switch S is closed ,we can find the complete solution for the current.Application of kirchoff's voltage law to the circuit results in the following differential equation.

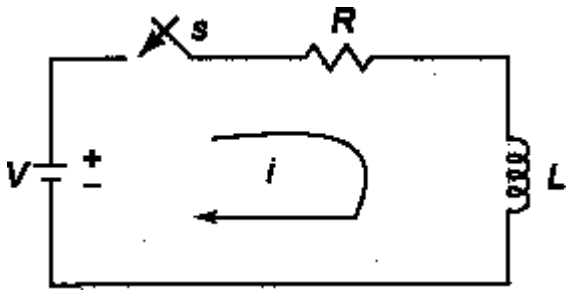


Figure1.1

$V = Ri + L\frac{di}{dt}$ 1.1

Or $\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$ 1.2

Intheaboveequation,thecurrentIisthesolutiontobefoundandVistheappliedconstant voltage.ThevoltageVisappliedtothecircuitonlywhentheswitchSisclosed. Theaboveequation is a linear differential equation of first order.comparing it with anon-homogenious differential equation

$\frac{dx}{dt} + P x = K$.....1.3

whosesolutionis

$X = e^{-Pt} \int K e^{+Pt} dt + c e^{-Pt}$.....1.4

Wherecisanarbitraryconstant.Inasimilarway,wecanwritethecurrentequationas

$$i = c e^{-\left(\frac{R}{L}\right)t} + e^{-\left(\frac{R}{L}\right)t} \int \frac{V}{L} e^{\left(\frac{R}{L}\right)t} dt$$

Hence, $i = c e^{-\left(\frac{R}{L}\right)t} + \frac{V}{R}$.....1.5

To determine the value of c in equation c , we use the initial conditions .In the circuit shown in Fig.1.1,theswitchsisclosedatt=0.att=0-,i.e.justbeforeclosingtheswitchs,thecurrentinthe inductor is zero. Since the inductordoesnotallowsuddenchanges in currents, at t=o+ just after the switch is closed,the current remains zero.

Thusatt=0,i=0

Substitutingtheaboveconditioninequationc,wehave $0 = c + \frac{V}{R}$

Substitutingthevalueofcinequationc,weget

$$i = \frac{V}{R} - \frac{V}{R} e^{\frac{-Rt}{L}}$$

$$i = \frac{V}{R}\left(1 - e^{\frac{-Rt}{L}}\right)$$

$i = I_o(1 - e^{\frac{-Rt}{L}})$ (where $I_o = \frac{V}{R}$)

$i = I_o(1 - e^{\frac{-t}{\tau}})$(where $\tau = Time\ constant = \frac{L}{R}$).....1.6

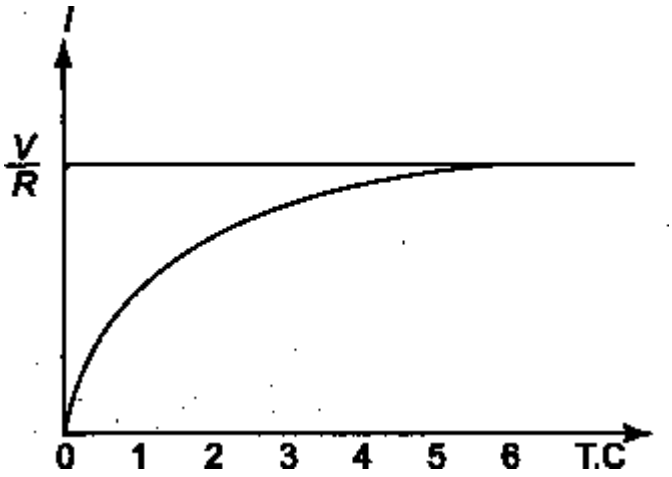


Figure1.2

Equationdconsistsoftwoparts,thesteadystatepart(I_o =V/R)andthetransientpart$I_o e^{\frac{-Rt}{L}}$.

WhenswitchSisclosed,theresponsereachesasteadystatevalueafteratimeintervalas shown in figure 1.2.

Here the transition period is defined as the time taken for the current to reach its final or stedy state value from its initial value.In the transient part of the solution, the quantityL/Risimportantindescribingthecurvesince L/Risthetimeperiodrequired for the current to reach its initial value of zero to the final value I_o=V/R. The time constant of a function $I_o e^{\frac{-Rt}{L}}$ isthetimeatwhichtheexponentofeisunity,wheree is the base of the natural logarithms.The term L/R is called the time constant and is denoted by τ .

So,$\tau = \frac{L}{R}$ sec

Hence,thetransientpartofthesolutionis

$$i = -\frac{V}{R} e^{\frac{-Rt}{L}} = \frac{V}{R} e^{\frac{-t}{\tau}}$$

AtoneTimeconstant,thetransienttermreaches36.8percentofitsinitialvalue.

$i(\tau) = -\frac{V}{R} e^{\frac{-\tau}{\tau}} = -\frac{V}{R} e^{-1} = -0.368 \frac{V}{R}$

Similarly,

$i(2\tau) = -\frac{V}{R} e^{-2} = -0.135 \frac{V}{R}$

$i(3\tau) = -\frac{V}{R} e^{-3} = -0.0498 \frac{V}{R}$

$i(5\tau) = -\frac{V}{R} e^{-5} = -0.0067 \frac{V}{R}$

After5TCthetransientpartreachesmorethan99percentofitsfinalvalue.

InfigureAwecanfindoutthevoltagesandpowersacrosseachelementbyusingthecurrent.

Voltageacrosstheresistoris

$$v_R = Ri = R \times \frac{V}{R}(1 - e^{\frac{-Rt}{L}})$$

Hence, $v_R = V\,(1 - e^{\frac{-Rt}{L}})$

Similarly,thevoltageacrosstheinductanceis v_L=

$$L\frac{di}{dt} = L\,\frac{V}{R} \times \frac{R}{L}\, e^{\frac{-Rt}{L}} = V\, e^{\frac{-Rt}{L}}$$

TheresponsesareshowninFigure1.3.

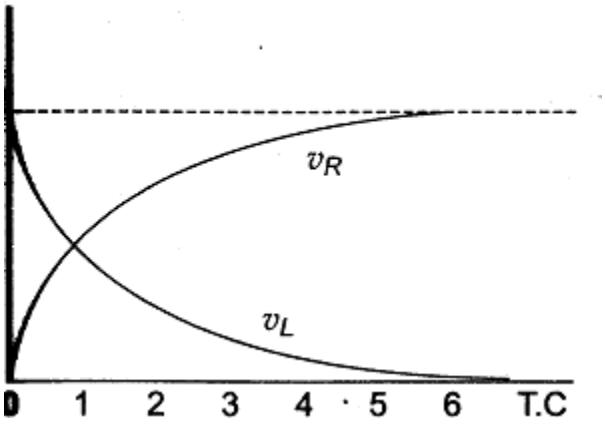


Figure1.3

Powerintheresistoris

$$P_R = v_R i = V\,(1 - e^{\frac{-Rt}{L}})\,(1 - e^{\frac{-Rt}{L}}) \times \frac{V}{R}$$

$$= \frac{V^2}{R}(1 - 2e^{\frac{-Rt}{L}}) + e^{\frac{-2Rt}{L}}$$

Powerintheinductoris

$$P_L = v_L i = V \quad e^{\frac{-Rt}{L}} \times \frac{V}{R}\,(1 - e^{\frac{-Rt}{L}})$$

$$= \frac{V^2}{R}(e^{\frac{-Rt}{L}} - e^{\frac{-2Rt}{L}})$$

Theresponsesareshowninfigure1.4.

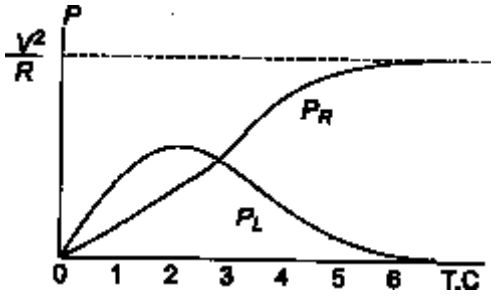


Figure1.4

Problem:1.1

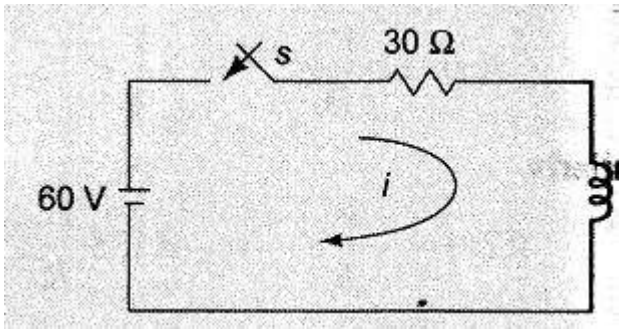


Figure1.5

AseriesR-L circuitwithR=30ΩandL=15Hhasaconstantvoltage V=50Vappliedatt=0as shown in Fig. 1.5 . determine the current i, the voltage across resistor and across inductor.

Solution:

ByapplyingKirchoff'svoltageLaw,weget

$15\frac{di}{dt}$ +30i=60

$=>\frac{di}{dt}$+2i=4

Thegeneralsolutionforalineardifferentialequationis i=c

$e^{-pt}+ e^{-pt}\int Ke^{pt}dt$

where P=2,K=4

puttingthe valuesi=c

$e^{-2t}+e^{-2t}\int 4e^{2t}dt$

=>$i=ce^{-2t}+ 2$

Att=0,theswitchsisclosed.

Sincetheinductorneverallowssuddenchangeincurrents.Att=0^+thecurrentinthecircuitis zero. Therefore at t=0^+, i =0

=>0=c + 2

=>c =- 2

Substitutingthevalueofcinthecurrentequation,wehave i=2(1-

e^{-2t}) A

voltageacrossresistor(V_R)=iR=2(1-e^{-2t})x30=60(1-e^{-2t})v

voltageacrossinductor(V_L)=L$\frac{di}{dt}$=15$\times$ $\frac{d}{dt}$2(1-e^{-2t})=30$\times$ $2e^{-2t}$v= $60e^{-2t}$

DCRESPONSEOFANR-CCIRCUIT

Consideracircuitconsistingofaresistanceandcapacitanceasshowninfigure.Thecapacitorinthe circuitis initiallyunchargedand isin series with theresistor.When the switch Sisclosedat t=0,we can find the complete solution for the current.Application of kirchoff's voltage law to the circuit results in the following differential equation.

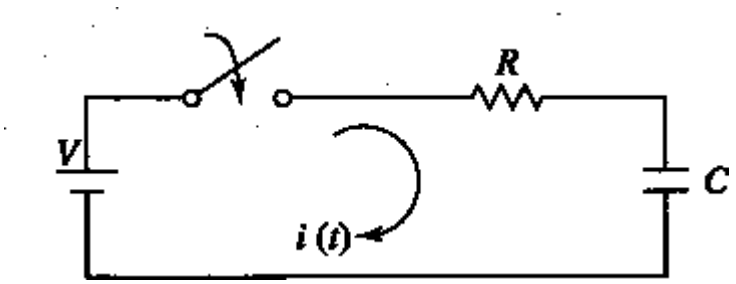


Figure1.6

V=Ri+$\frac{1}{C}\int i\,dt$.....1.7 By

differentiating the above equation, we get

0=R $\frac{di}{dt}$ + $\frac{i}{C}$ i 1.8

Or

$\frac{di}{dt}$ + $\frac{1}{RC}$ i=0 .....1.9

Equationcisalineardifferentialequationwithonlythecomplementaryfunction.Theparticular solution for the above equationis zero. The solution for this type of differential equationis

$i = c\, e^{-\left(\frac{t}{RC}\right)}$ 1.10

To determine the value of c in equation c , we use the initial conditions .In the circuit shown in Fig.theswitchsisclosedatt=0.Sincethecapacitordoesnotallowsuddenchangesinvoltage,it will act as a short circuitat t=o+ just after the switch is closed.

Sothecurrentinthecircuitatt=0+is $\frac{V}{R}$

Thusatt=0,thecurrenti= $\frac{V}{R}$

Substitutingtheaboveconditioninequationc,wehave $\frac{V}{R} = c$

Substitutingthevalueofcinequationc,weget

$i = \frac{V}{R} e^{-\left(\frac{t}{RC}\right)}$ 1.11

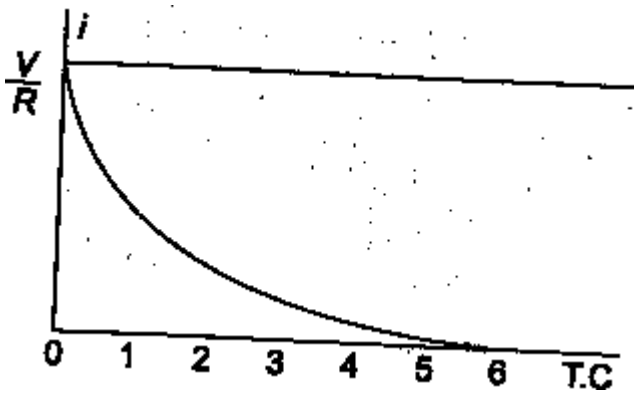


Figure1.7

WhenswitchSisclosed,theresponsedecaysasshowninfigurre. The term

RCis called the time constant and is denoted by τ .

So,τ=RCsec

After5TCthecurvereaches99percentofitsfinalvalue.

InfigureAwecanfindoutthevoltageacrosseachelementbyusingthecurrentequation. Voltage across the resistor is

$$v_R = R\,i = R \times \frac{V}{R} e^{\frac{-t}{RC}}$$

Hence, $v_R = V\, e^{\frac{-t}{RC}}$

Similarly,voltageacrossthecapacitoris v_C

$$= \frac{1}{C}\int i\, dt$$

$$= \frac{1}{C}\int \frac{V}{R} e^{\frac{-t}{RC}}\, dt$$

$$= -\left(\frac{V}{RC} \times RC\, e^{\frac{-t}{RC}}\right) + c$$

$$= -V e^{\frac{-t}{RC}} + c$$

Att=0,voltageacrosscapacitoriszero So, c

= V

And

$$V_C = V\left(1 - e^{\frac{-t}{RC}}\right)$$

TheresponsesareshowninFigure1.8.

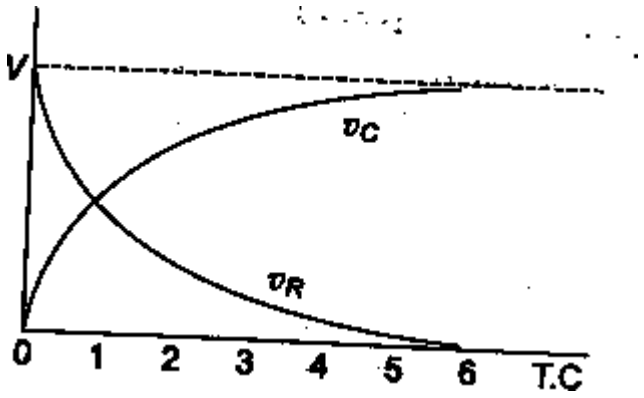


Figure1.8

Power in the resistor is

$$P_R = v_R i = V\, e^{\frac{-t}{RC}} \times \frac{V}{R} e^{\frac{-t}{RC}}$$

$$= \frac{V^2}{R} e^{\frac{-2t}{RC}}$$

Powerinthecapacitoris P_C

$$= v_C i = V\left(1 - e^{\frac{-t}{RC}}\right) \frac{V}{R} e^{\frac{-t}{RC}}$$

$$=\frac{V^2}{R}\left(e^{\frac{-t}{RC}}-e^{\frac{-2t}{RC}}\right)$$

Theresponsesareshowninfigure1.9.

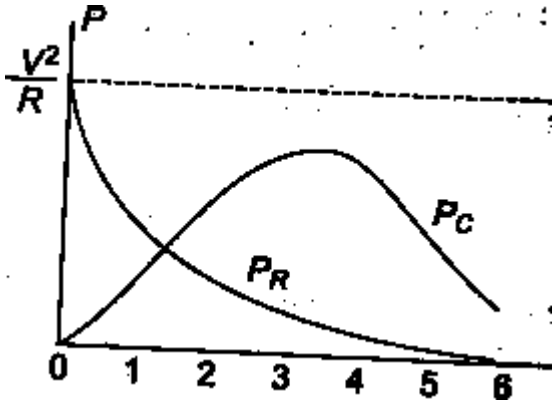


Figure1.9

Problem:1.2

AseriesR-CcircuitwithR=10Ωand C=0.1Fhasaconstantvoltage V=20Vappliedatt=0as shown in Fig. determine the current i, the voltage across resistor and acrosscapacitor.

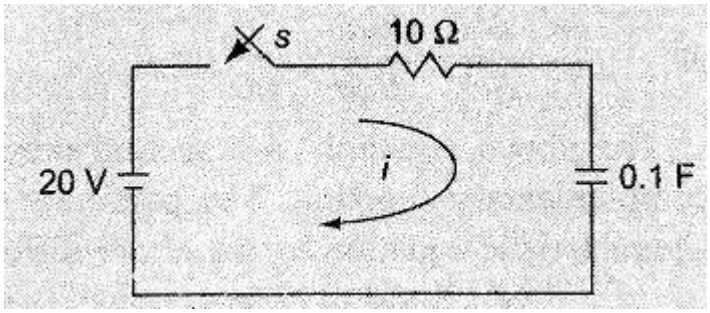


Figure1.10

Solution:

ByapplyingKirchoff'svoltageLaw,weget

$$10i+\frac{1}{0.1}\int i\,dt=20$$

Differentiatingw.r.t.tweget

$$10\frac{di}{dt}+\frac{i}{0.1}=0$$

$$=>\frac{di}{dt}+i=0$$

Thesolutionforaboveequationis

$i=ce^{-t}$

Att=0,theswitchsisclosed.

Sincethecapacitornever allowssuddenchangeinvoltages.Att=0^{+}thecurrentinthecircuitis i = V/R=20/10 =2 A

.Thereforeatt=0,i=2A

=>thecurrentequationisi=$2e^{-t}$

voltageacrossresistor(V_R)=iR=2 e^{-t}x10=$20e^{-t}$v

voltageacrosscapacitor(V_C)=V$(1-e^{\frac{-t}{RC}})$=20(1-e^{-t})V

DCRESPONSEOFANR-L-CCIRCUIT

Consider a circuit consisting of a resistance, inductanceandcapacitance as shown in figure.The capacitorandinductorinthecircuitisinitiallyunchargedandareinserieswiththeresistor.When the switch S is closedat t=0 , we can find the complete solution for the current.Application of kirchoff's voltage law to the circuit results in the following differential equation.

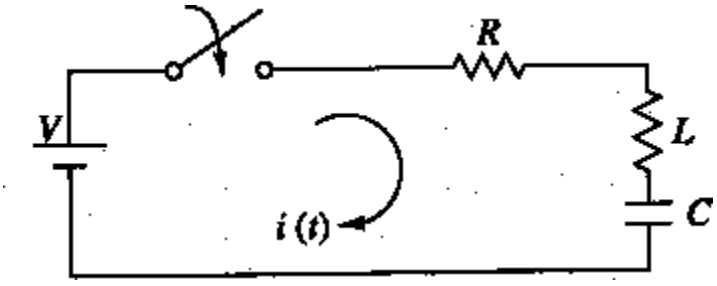


Figure1.11

V=Ri+L $\frac{di}{dt}+\frac{1}{C}\int i\,dt$.....1.12 By

differentiating the above equation, we get

0= R$\frac{di}{dt}$+ $L\,d^2i/dt^2$ +$\frac{i}{C}$i=.....1.13

Or

$d^2i/dt^2+\frac{R}{L}\frac{di}{dt}+\frac{1}{LC}$ i=0..... 1.14

The above equation c is a second orderlinear differential equation with only the complementary function.Theparticularsolutionfortheaboveequationiszero.Thecharacteristicsequationforthis type of differential equationis

$D^2+\frac{R}{L}D+\frac{1}{LC}=0$ 1.15

Therootsofequation1.15are

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

ByassumingK_1=-$\frac{R}{2L}$andK_2=$\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$D_1 = K_1 + K_2$and$D_2 = K_1 - K_2$

HereK_2maybepositive,negativeorzero.

Case I : K_2 *is Positive* $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

Then,therootsareReal andUnequalandgiveanoverdampedResponseasshowninfigure 1.12.

Thesolutionfortheaboveequationis:i=$C_1\ e^{(K_1+K_2)t}+C_2\ e^{(K_1-K_2)t}$

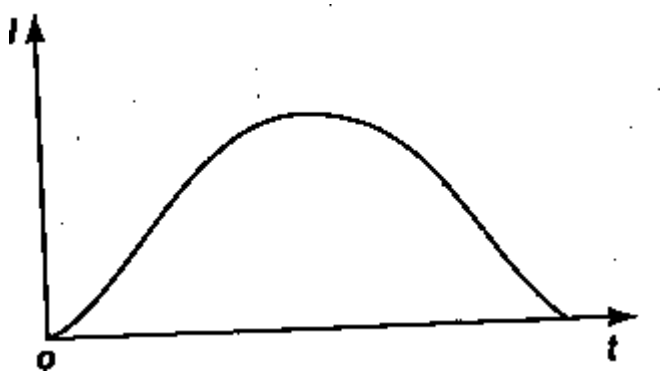


Figure1.12

Case II : K_2 *is Negative* $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

Then,therootsareComplexConjugate,andgiveanunder-dampedResponseasshownin figure 1.13.

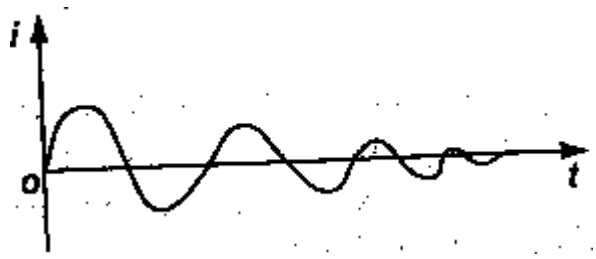


Figure1.13

Thesolutionfortheaboveequationis:i=$e^{K_1t}(C_1\cos K_2t + C_2\sin K_2t)$Case III :

Then,therootsareEqualandgivean K_2 is Zero $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ Critically-dampedResponseasshowninfigure1.14.

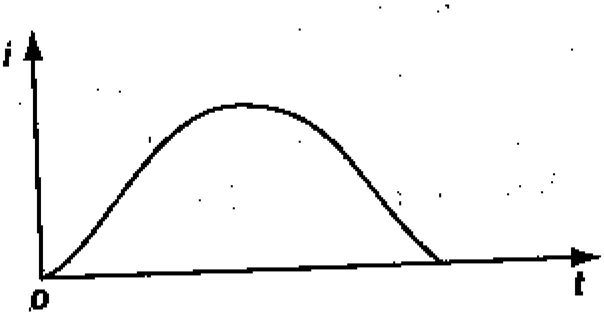


Figure1.14

Thesolutionfortheaboveequationis:i=$e^{K_1t}(C_1 + C_2t)$Problem :

1.3

AseriesR-L-CcircuitwithR=20Ω,L=0.05Hand C=20μFhasaconstantvoltageV=100V applied at t=0 as shown in Fig. determine the transient current i .

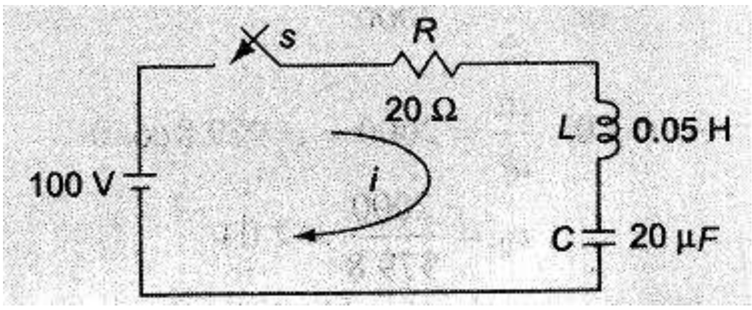


Figure1.15

Solution:

ByapplyingKirchoff'svoltageLaw,weget

100=30i+0.05 $\frac{di}{dt} + \frac{1}{20\times10^{-6}}\int i\,dt$

Differentiatingw.r.t.tweget

$0.05d^2i/dt^2$ +20 $\frac{di}{dt} + \frac{1}{20\times10^{-6}}$ i=0

$\Rightarrow d^2t/dt^2 + 400\frac{di}{dt} + 10^6 i = 0$

$\Rightarrow (D^2 + 400D + 10^6 i = 0$

Therootsofequationare

$D_1, D_2 = -\frac{400}{2} \pm \sqrt{\left(\frac{400}{2}\right)^2 - 10^6}$

$= -200 \pm \sqrt{(200)^2 - 10^6}$

$D_1 = -200 + j979.8$

$D_2 = -200 - j979.8$

Thereforethecurrent

$i = e^{+K_2 t}[C_1 \cos K_2 t + C_2 \cos K_2 t]$

$i = e^{-200t}[C_1 \cos 979.8t + C_2 \sin 979.8t]A$

Att=0,theswitchsisclosed.

Sincetheinductorneverallowssuddenchangeincurrents.Att=$0^+$the currentinthecircuitis zero. Therefore at t=0^+, i =0

$\Rightarrow i = 0 = (1)\ [C_1 \cos 0 + C_2 \sin 0]$

$\Rightarrow C_1 = 0 \text{ and } i = e^{-200t}[C_2 \sin 979.8t]A$

Differentiatingw.r.t.tweget

$\frac{di}{dt} = C_2[e^{-200t}979.8\cos 979.8t + e^{-200t}(-200)\sin 979.8t]$

Att=0,thevoltageacrosstheinductoris100V $\Rightarrow L\frac{di}{dt}$

=100 or $\frac{di}{dt} = 2000$

Att=0,$\frac{di}{dt} = 2000 = C_2 979.8 \cos 0$

$\Rightarrow C_2 = \frac{2000}{979.8} = 2.04$

Thecurrentequationis

$i = e^{-200t}(2.04 \sin 979.8t)\ A$

ANALYSISOFCIRCUITSUSINGLAPLACETRANSFORMTECHNIQUE

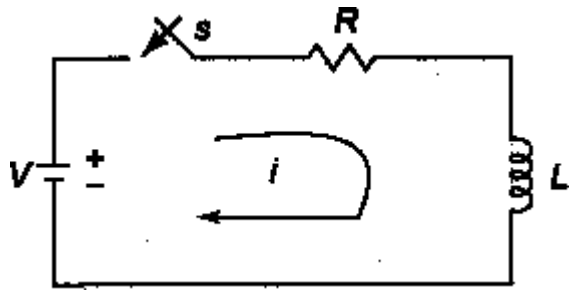
TheLaplacetransformisapowerfulAnalyticalTechniquethat iswidelyusedtostudythe behaviorofLinear,Lumpedparametercircuits.LaplaceTransformconvertsatimedomain functionf(t) toafrequencydomainfunctionF(s) andalsoInverseLaplacetransformation converts the frequency domain function F(s) back to a time domain function f(t).

$L\{f(t)\} = F(s) = \int_{-\infty}^{\infty} e^{-st} f(t)\,dt$ LT1

$\mathcal{L}^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi j}\int_{-j}^{+j} F(s)\, e^{st} ds$ LT2

DCRESPONSEOFANR-LCIRCUIT(LTMethod)

LetusdeterminethesolutionioftheFirstorderdifferentialequationgivenbyequationAwhich isfortheDC responseofaR-LCircuitunderthezeroinitialconditioni.e.currentiszero,i=0at t=$0^-$and hence i=0at t=0^+in the circuit in figure A by the property of Inductance not allowing thecurrenttochangeasswitchisclosedatt=0.



FigureLT1.1

$V = Ri + L\frac{di}{dt}$.....LT1.1

TakingtheLaplaceTransformofbothesidesweget,

$\frac{V}{s} = R\,I(s) + L\,[\,s\,I(s) - I(0)\,]$ LT1.2

$\Rightarrow \frac{V}{s} = R\,I(s) + L\,[\,s\,I(s)\,]$ (I(0)=0:zeroinitialcurrent)

$\Rightarrow \frac{V}{s} = I(s)[R + L\,s]$

$\Rightarrow I(s) = \frac{V}{s[R+Ls]}$ LT1.3

TakingtheLaplaceInverseTransformofbothsidesweget,

$\Rightarrow L^{-1}\{I(s)\} = i(t) = L^{-1}\{\frac{V}{s[R+Ls]}\}$

i(t)= $L^{-1}\{\frac{V/L}{s[R/L+s]}\}$(Dividingthenumeratoranddenominatorby L) putting

$\propto = R/L$we get

i(t)= $L^{-1}\{\frac{V/L}{s[s+\propto]}\} = L^{-1}\{\frac{V}{L}(\frac{1}{s} - \frac{1}{(s+\propto)})\frac{1}{\propto}\}$

i(t)= $L^{-1}\{\frac{V}{L}(\frac{1}{s} - \frac{1}{(s+R/L)})\frac{L}{R}\}$(againputtingbackthevalueof $\propto$)

i(t)= $L^{-1}\{\frac{V}{R}(\frac{1}{s} - \frac{1}{(s+R/L)})\} = \frac{V}{R}(1 - e^{\frac{-Rt}{L}}) = I_o(1 - e^{\frac{-Rt}{L}})$ (where $I_o = \frac{V}{R}$)

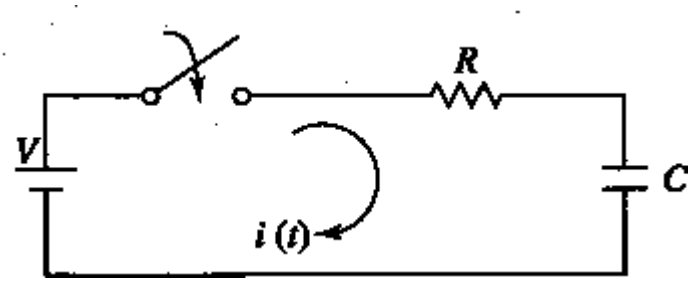
i(t)= $I_o(1 - e^{\frac{-t}{\tau}})$ (where $\tau = Time\ constant = \frac{L}{R}$) LT1.4

Itcanbeobservedthatsolutionfori(t)asobtainedbyLaplaceTransformtechniqueissameas that obtained by standard differential method .

DCRESPONSEOFANR-CCIRCUIT(L.T.Method)

Similarly,

Letusdeterminethesolutionofthefirstorderdifferentialequationgivenbyequation Awhich is for the DC response of a R-C Circuit under the zero initial conditioni.e.voltage across capacitor is zero, V_c=0at t=$0^-$and hence V_c=0at t=0^+in the circuit in figure A by the property ofcapacitancenotallowingthevoltageacrossittochangeasswitchisclosedatt=0.



FigureLT1.2

V=Ri+$\frac{1}{C}\int i\,dt$.....LT1.5

TakingtheLaplaceTransformofbothsidesweget,

$\frac{V}{S}$=R I(s) + $\frac{1}{C}[\frac{I(s)}{s}$+I (0)].....LT1.6

$\Rightarrow \frac{V}{S}$=R I(s) +$\frac{1}{C}[\frac{I(s)}{s}]$ (I(0)=0:zeroinitialcharge)

$\Rightarrow \frac{V}{S}$ =I(s)[R+$\frac{1}{Cs}$]=I(s)[$\frac{RCs+1}{Cs}$]

$\Rightarrow I(s) = \frac{V}{s}\left[\frac{Cs}{(RCs+1)}\right] = \frac{VC}{(RCs+1)}$ .....LT 1.7

TakingtheLaplaceInverseTransformofbothsidesweget,

$\Rightarrow L^{-1}\{I(s)\} = i(t) = L^{-1}\left\{\frac{VC}{(RCs+1)}\right\}$

$i(t) = L^{-1}\left\{\frac{\frac{VC}{RC}}{\left[s+\frac{1}{RC}\right]}\right\}$(DividingthenumeratoranddenominatorbyRC) putting

$\propto = \frac{1}{RC}$we get

$i(t) = L^{-1}\left\{\frac{V/R}{[s+\propto]}\right\} = \frac{V}{R}e^{-\propto t}$

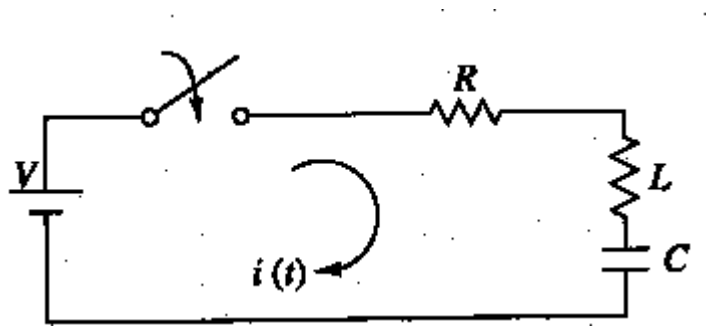
$i(t) = \frac{V}{R}e^{\frac{-t}{RC}}$(puttingbackthevalueof $\propto$)

$i(t) = I_o e^{\frac{-t}{RC}}$(where $I_o = \frac{V}{R}$) LT1.8

$i(t) = I_o e^{\frac{-t}{\tau}}$) (where $\tau = Time\ constant$ =RC)

Itcanbeobservedthatsolutionfori(t)asobtainedbyLaplaceTransformtechniqueinqis same as that obtained by standard differential method in d.

DCRESPONSEOFANR-L-CCIRCUIT(L.T.Method)



FigureLT1.3

Similarly,

Let us determine the solution i of the first order differential equation given by equation A which isfortheDC responseofaR-L-CCircuitunderthezeroinitialconditioni.e.theswitchsisclosed at t=0.at t=0-,i.e. just before closing the switch s , the current in the inductor is zero. Since the inductor does not allow sudden changes in currents, at t=o+ just after the switch is closed,the current remains zero. also thevoltage across capacitor is zero i.e. V_c=0at t=$0^-$and hence V_c=0 att=0^+inthecircuitinfigurebythepropertyofcapacitancenotallowingthevoltageacrossit $V_c$to suddenly change as switch is closed at t=0.

$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\ dt$.....LT1.9

TakingtheLaplaceTransformofbothsidesweget,

$\frac{V}{s}$=R I(s) ++ L [s I(s) –I(0)]+$\frac{1}{C}$[$\frac{I(s)}{s}$+I (0)]…… LT1.10

$\Rightarrow \frac{V}{s}$=R I(s) + L [s I(s)] + $\frac{1}{C}$[$\frac{I(s)}{s}$] ($I(0) = 0$: zero initial current &I(0)=0:zeroinitial charge)

$\Rightarrow \frac{V}{s}$= I(s)[R +Ls + $\frac{1}{Cs}$] = I(s)[$\frac{LCs^2+Rcs+1}{Cs}$]

$\Rightarrow$I(s) =$\frac{V}{s}$[$\frac{Cs}{(LCs^2+RCs+1)}$] =$\frac{VC}{(LCs^2+RCs+1)}$ ……LT1.11

Taking the Laplace Inverse Transform of both sides we get,

$\Rightarrow L^{-1}${I(s)}= $i(t) = L^{-1}\{\frac{VC}{(LCs^2+RCs+1)}\}$

i(t)= $L^{-1}\{\frac{\frac{VC}{LC}}{\left[s^2+s\frac{RC}{LC}+\frac{1}{LC}\right]}\}$(Dividingthenumeratoranddenominator byLC) i(t) =

$L^{-1}\{\frac{\frac{V}{L}}{\left[s^2+s\frac{R}{L}+\frac{1}{LC}\right]}\}$

putting$\propto = \frac{R}{2L}$ and $\omega=\sqrt{\frac{1}{LC}}$weget i(t) =

$L^{-1}\{\frac{\frac{V}{L}}{[s^2+2\propto s+\omega^2]}\}$

Thedenominatorpolynomialbecomes=$[s^2 + 2 \propto s + \omega^2]$

where,s_1 ,s_2 = $\frac{-2\propto\pm\sqrt{4\propto^2-4\omega^2}}{2}$=$-\propto\pm\sqrt{\propto^2-\omega^2}=-\propto\pm\beta$

where,$\propto=\frac{R}{2L}$;$\omega=\sqrt{\frac{1}{LC}}and\beta=\sqrt{\propto^2-\omega^2}$

BypartialFractionexpansion,ofI(s),

$$I(s) = \frac{A}{S-S_1}+\frac{B}{S-S_1}$$

$$A=(S-S_1)\ I(s)]s=S_1$$

$$=\frac{\frac{V}{L}}{(S_1-S_2)}$$

$$B=(S-S_2)\ I(s)]s=S_2$$

$$=\frac{\frac{V}{L}}{(S_2-S_1)}=-\frac{\frac{V}{L}}{(S_1-S_2)}$$

$$I(s)=\frac{\frac{V}{L}}{(S_1-S_2)}\left(\frac{1}{(s-s_1)}-\frac{1}{(s-s_2)}\right)$$

TakingtheInverseLaplaceTransform

$$i(t)=A_1 e^{s_1 t}+A_2 e^{s_2 t}$$

Where A_1 and A_2 areconstantstobe determinedand s_1 and s_2 arentheroots ofthe equation.

Nowdependinguponthevaluesof s_1 and s_2, wehavethreecasesoftheresponse. CASE I :

When the roots are Real and Unequal, it gives an over-damped response.

$\frac{R}{2L} > \sqrt{\frac{1}{LC}}$ or $\propto > \omega$;Inthiscase,thesolutionisgivenby

$i(t)=e^{-\propto t}\left(A_1 e^{\beta t}+A_2 e^{-\beta t}\right)$......................................LT1.12

or $i(t)=A_1 e^{s_1 t}+A_2 e^{s_2 t}$ fort$>$0

CASEII:WhentherootsareRealandEqual,itgivesanCritically-dampedresponse.

$\frac{R}{2L}=\sqrt{\frac{1}{LC}}$ or $\propto = \omega$;Inthiscase,thesolutionisgivenby or

$i(t)=e^{-\propto t}(A_1+A_2 t)$ fort$>$0......................................LT1.13

CASEIII:WhentherootsareComplexConjugate,itgivesanunder-dampedresponse. or

$\frac{R}{2L} < \sqrt{\frac{1}{LC}}$ $\propto < \omega$; In this case, the solution is given by

$i(t)=A_1 e^{s_1 t}+A_2 e^{s_2 t}$ fort$>$0

where, $s_1, s_2 = \frac{-2\propto \pm \sqrt{4\propto^2-4\omega^2}}{2} = -\propto \pm \sqrt{\propto^2-\omega^2}$

Let $\sqrt{\propto^2-\omega^2}=\sqrt{-1}\sqrt{\omega^2-\propto^2}$ $=j\omega_d$ where $j=\sqrt{-1}$ and $\omega_d=\sqrt{\omega^2-\propto^2}$

Hence, $i(t)=e^{-\propto t}\left(A_1 e^{j\omega_d t}+A_2 e^{-j\omega_d t}\right)$

$$i(t)=e^{-\propto t}\left[(A_1+A_2)\left\{\frac{e^{j\omega_d t}+e^{-j\omega_d t}}{2}\right\}+j(A_1-A_2)\left\{\frac{e^{j\omega_d t}-e^{-j\omega_d t}}{2j}\right\}\right]$$

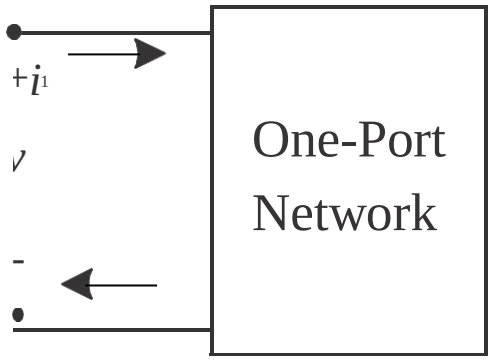
$$i(t)=e^{-\propto t}\left[(A_1+A_2)\cos\omega_d t+j(A_1-A_2)\sin\omega_d t\right]$$

$i(t)=e^{-\propto t}(B_1\cos\omega_d t+B_2\sin\omega_d t)$......................................LT1.14

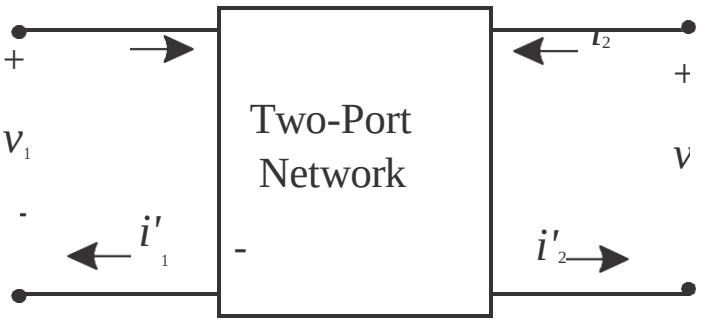
,,,XXXXXXXXXXXXXXXXX,,,

CHAPTER8

TWO-PORTNETWORKS



a) **One port network** is a two terminal electrical network in which, current entersthroughoneterminalandleavesthroughanotherterminal.Resistors, inductorsandcapacitorsaretheexamplesofoneportnetworkbecauseeach one has two terminals. One port network representation is shown in the following figure.
b) A pair of terminals at which asignal (voltageorcurrent) may enter or leaveis called a port.
c) Anetworkhavingonlyonesuchpairofterminalsiscalledaone-portnetwork.
d) Noconnectionsmaybemadetoanyother nodesinternaltothenetwork.
e) ByKCL,wethereforehavei_1=i_1



- **two port network** is a pair of two terminal electrical network in which, current entersthroughoneterminalandleavesthroughanotherterminalofeachport.Two port network representation is shown in the following figure.Type equation here.
- Two-portnetworksareusedto describetherelationship between apairof terminals
- The analysis methods we will discuss require the following conditions be met
    1. Linearity
    2. Noindependentsourcesinsidethenetwork
    3. Nostoredenergyinsidethenetwork(zeroinitialconditions)
    4. i_1=$i_1$andi_2=i

TwoPortNetworkParameters

Therearevariousparametersneededtoanalyzeatwoport network. For examples, Z parameters, Y parameters, h parameters, g parameters, ABCD parametersetc. Letusdiscussthesenetworkparametersonebyonetogaina better understanding of their application and uses.

ImpedanceParameters

- Supposethecurrentsandvoltagescanbemeasured.
- Alternatively,ifthecircuitintheboxisknown,V_1andV_2canbecalculated based on circuit analysis.
- Relationshipcanbewrittenintermsoftheimpedanceparameters.
- Wecanalsocalculatetheimpedanceparametersaftermakingtwosetsof measurements.

$$V_1=z_{11}I_1+z_{12}I_2$$

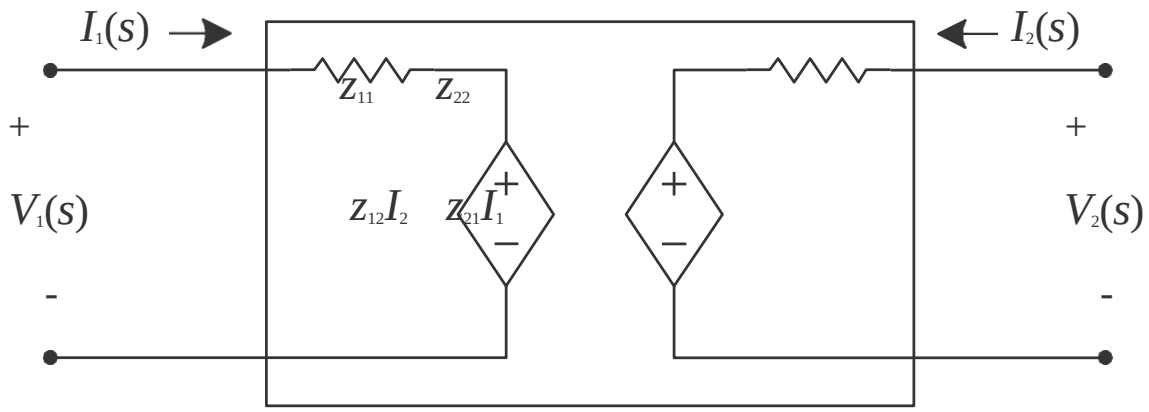
$$V_2=z_{21}I_1+z_{22}I_2$$

Iftherightportisanopencircuit (I_2=0),thenwecan easilysolvefor two of the impedance parameters: Similarly by open circuiting left hand port ($I_{1=0}$) we can solve for the other two parameters.

$$Z_{11} = inputimpedence = \frac{V_1}{I_1}\Big|I_2 = 0 \qquad Z_{21} = forwardtransferimpedence = \frac{V_2}{I_1}\Big|I_2 = 0$$

$$Z_{12} = reversetransferimpedence = \frac{V_1}{I_2}\Big|I_1 = 0 \quad Z_{22} = outputimpedence = \frac{V_2}{I_2}\Big|I_1 = 0$$

ImpedanceParameterEquivalent

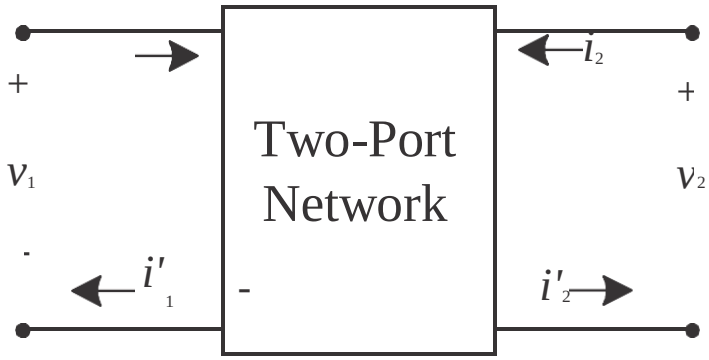


$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

- Onceweknowwhattheimpedanceparametersare,wecanmodel the behavior of the two-port with an equivalent circuit.
- NoticethesimilaritytoTh´eveninandNorton equivalents

AdmittanceParameters



$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

Y_{11}=inputadmittance= $\left.\frac{I_1}{V_1}\right|_{V_2=0}$

Y_{21}=forwardtransfer admittance= $\left.\frac{I_2}{V_1}\right|_{V_2=0}$

Y_{22} =outputadmittance= $\left.\frac{I_1}{V_2}\right|_{V_1=0}$

Y_{12}=reversetransferadmittance= $\left.\frac{I_1}{V_2}\right|_{V_1=0}$

HybridParameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

h_{11}=inputimpedance= $\left.\frac{V_1}{I_1}\right|_{V_2=0}$

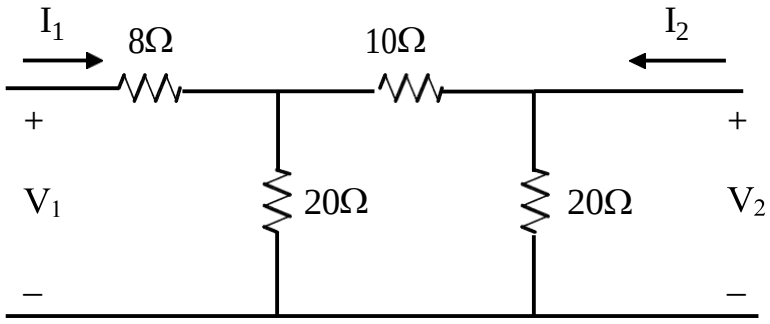
h_{21} = forwardcurrentratio = $\left.\frac{I_2}{I_1}\right|_{V_2=0}$

h_{12} =reversevoltageratio= $\left.\frac{V_1}{V_2}\right|_{I_1=0}$

h_{22} =output admittance= $\left.\frac{I_2}{V_2}\right|_{I_1=0}$

Example:

Giventhefollowingcircuit.DeterminetheZparameters.



$Z_{11}=8+20||30=20\Omega$

$Z_{22}=20||30=12\Omega$

$$Z_{12}=\frac{V_1}{I_2}\Big|_{I_1=0}$$

$$V_1=\frac{20xI_2x20}{20+30}=8xI_2 \qquad \text{Therefore} \quad z_{12}=\frac{8xI_2}{I_2}=8\Omega=z_{21}$$

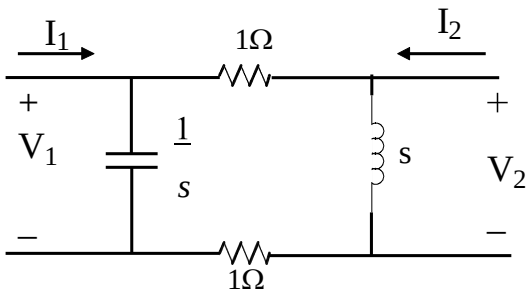
TheZparameterequationscanbeexpressedinmatrixformasfollows.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

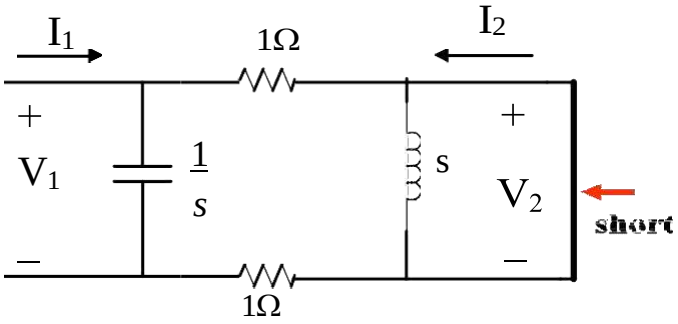
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 8 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Example:

Giventhefollowingcircuit.DeterminetheYparameters.



$I_1=y_{11}V_1+y_{12}V_2 I_2=$
$y_{21}V_1+y_{22}V_2$



To find y_{11}

$$V_1 = I_1\left(\frac{2/s}{2+1/s}\right) = I_1\left[\frac{2}{2s+1}\right]$$

$$\frac{V_1}{I_1} = \frac{2}{2s+1}$$

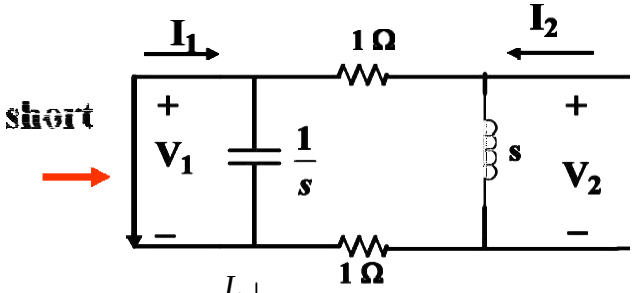
$$\text{So } y_{11} = \frac{I_1}{V_1} y_{11} = \frac{1}{V_1}\Big|_{V_2=0} = s+0.5$$

To find y_{12} and y_{21} we reverse the things and short V_1

$$y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

$$V_1 = -2I_2$$

$$y_{21} = \frac{I_2}{V_1} = -0.5S$$



$$y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

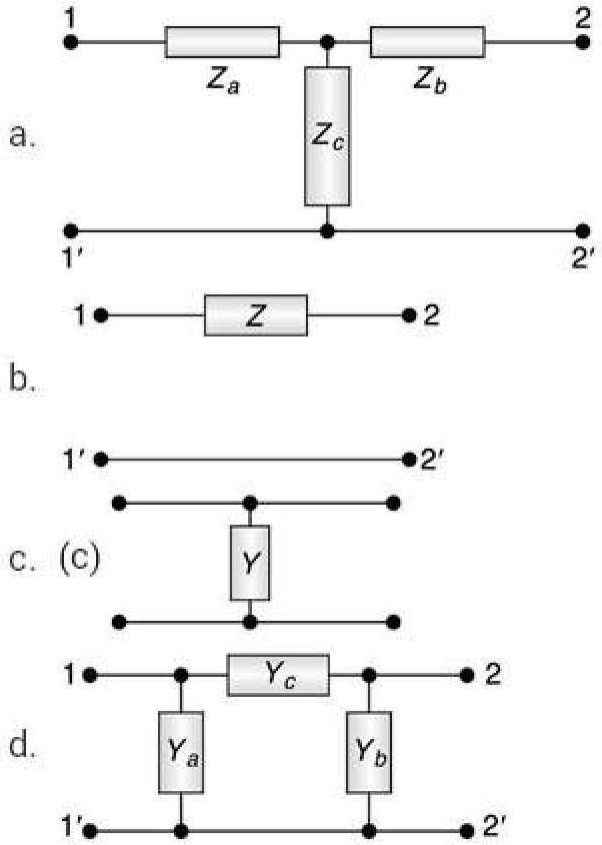
$$V_2 = -2I_1 \qquad y_{12} = \frac{I_1}{V_2} = -0.5s$$

$$y_{22} = 0.5 + \frac{1}{s}$$

$$y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} \qquad V_2 = I_2\,\frac{2s}{(s+2)} \qquad y_{22} = 0.5 + \frac{1}{s}$$

Problem1

Find the *Z* and *Y* parameter for the networks shown in figure.



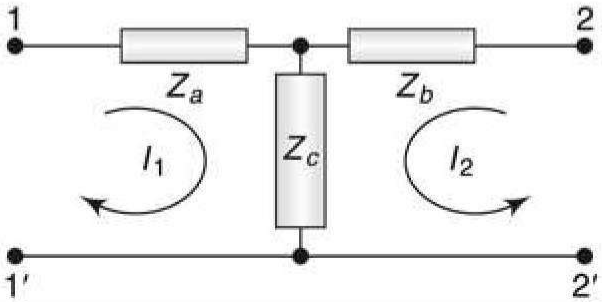
Solution

a. By KVL, $(Z_a + Z_c)I_1 + Z_c I_2 = V_1$

and $Z_c I_1 + (Z_b + Z_c)I_2 = V_2$

Thus, the Z-parameters are:

$$z_{11} = (Z_a + Z_c),\ z_{12} = z_{21} = Z_c,\quad z_{22} = (Z_b + Z_c)$$



b. By KCL,

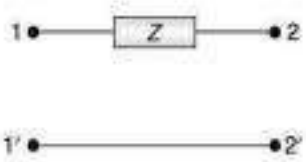
$$I_1 = \frac{V_1 - V_2}{Z} = \frac{1}{Z}V_1 - \frac{1}{Z}V_2$$

and $I_2 = \frac{V_2 - V_1}{Z} = -\frac{1}{Z}V_1 + \frac{1}{Z}V_2$

Thus, the y-parameters are,

$$y_{11} = \frac{1}{Z} = y_{22} \quad y_{12} = y_{21} = -\frac{1}{Z}$$

Since, $\Delta y = y_{11}y_{22} - y_{12}y_{21} = 0$, the z-parameters do not exist for this network.



c. By KVL,

$V_1 =$

$$\frac{I_1 + I_2}{Y} = V_2 \quad \text{or,} V_1 = \left(\frac{1}{Y}\right)I_1 + \left(\frac{1}{Y}\right)I_2 \quad \text{and} \quad V_2 = \left(\frac{1}{Y}\right)I_1 + \left(\frac{1}{Y}\right)I_2$$

Thus, the z-parameters are,

$$z_{11} = z_{22} = \frac{1}{Y} = z_{12} = z_{21}$$

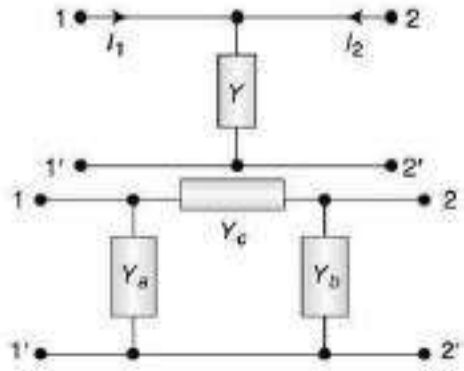
Since, $\Delta z = z_{11}z_{22} - z_{12}z_{21} = 0$, the y-parameters do not exist for this network.

d. By KCL,

$$I_1 = Y_a V_1 + (V_1 - V_2)Y_c = V_1(Y_a + Y_c) - V_2 Y_c$$
$$I_2 = Y_b V_2 + (V_2 - V_1)Y_c = -V_1 Y_c + V_2(Y_b + Y_c)$$

Thus, the y-parameters are:

$$y_{11} = Y_a + Y_c;\ y_{12} = y_{21} = -Y_c;\ y_{22} = Y_b + Y_c$$



Problem2

a. The following equations give the voltages V_1 and V_2 at the two ports of a two port network, $V_1 = 5I_1+2I_2$, $V_2 = 2I_1+I_2$;

   A load resistance of 3 Ω is connected across port-2. Calculate the input impedance.

b. The *z*-parameters of a two port network are z_{11} = 5 Ω, z_{22} = 2 Ω, z_{12} = z_{21} = 3 Ω. Load resistance of 4 Ω is connected across the output port. Calculate the input impedance.

Solution

a. From the given equations,

$V_1 = 5I_1 + 2I_2$ (i)

$V_2 = 2I_1 + I_2$ (ii)

At the output, $V_2 = -I_2R_L = -3I_2$

Putting this value in (ii),

$-3I_2 = 2I_1 + I_2$ fi $I_2 = -I_1/2$

Putting in (i), $V_1 = 5I_1 + \left(\frac{-I_1}{2}\right) = 4I_1$

∴ Input impedance, $Z_{in} = \frac{V_1}{I_1} = 4\Omega$

b. [Same as Prob. (a)] $Z_{in} = \frac{V_1}{I_1} = 3.5\Omega$

Problem3

Determine the h-parameter with the following data:

i. with the output terminals short circuited, $V_1 = 25$ V, $I_1 = 1$ A, $I_2 = 2$ A

ii. with the input terminals open circuited, $V_1 = 10$ V, $V_2 = 50$ V, $I_2 = 2$ A

Solution

The *h*-parameter equations are,

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

a. With output short-circuited, $V_2 = 0$, given: $V_1 = 25$ V, $I_1 = 1$ A and $I_2 = 2$ A.

$$\left.\begin{aligned} \therefore \quad 25 &= h_{11} \times 1 \\ \text{and} \quad 2 &= h_{21} \times 1 \end{aligned}\right\} \Rightarrow h_{11} = 25\ \Omega, \text{ and } h_{21} = 2$$

b. With input open-circuited, $I_1 = 0$, given: $V_1 = 10$ V, $V_2 = 50$ V and $I_2 = 2$ A.

$$\left.\begin{aligned} \therefore \quad 10 &= h_{12} \times 50 \\ \text{and} \quad 2 &= h_{22} \times 50 \end{aligned}\right\} \Rightarrow h_{12} = \frac{1}{5} = 0.2 \text{ and } h_{23} = \frac{1}{25}\ \mho = 0.04\ \mho$$

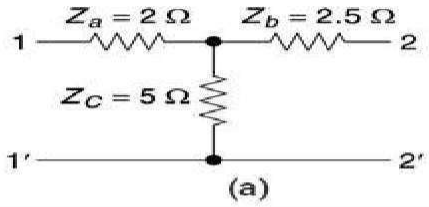
Thus, the *h*-parameters are:

$$[h] = \begin{bmatrix} 25\ \Omega & 0.2 \\ 2 & 0.04\ \Omega^{-1} \end{bmatrix}$$

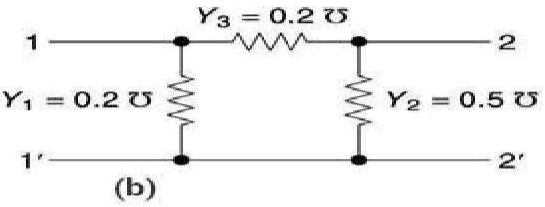
Problem4

a. Find the equivalent π-network for the *T*-network shown in the Fig. (a).

b. Find the equivalent *T* -network for the π-network shown in the Fig. (b).



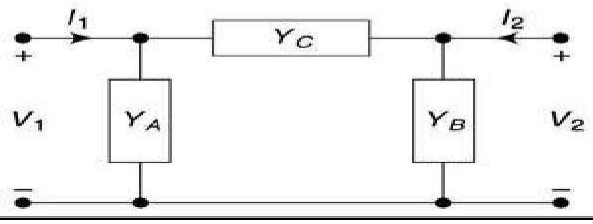
(a)



(b)

Solution

a. Let the equivalent π-network have Y_C as the series admittance and Y_A and Y_B as the shunt admittances at port-1 and port-2, respectively.



Now, the *z*-parameters are given as:

$$z_{11} = (Z_A + Z_C) = 7\ \Omega,\ z_{12} = z_{21} = Z_C = 5\ \Omega,\ z_{22} = (Z_B + Z_C) = 7.5\ \Omega$$

$$\therefore \Delta z = (7 \times 7.5 - 5 \times 5) = 27.5\ \Omega^2$$

$$\therefore y_{11} = \frac{z_{22}}{\Delta z} = \frac{7.5}{27.5}\ \mho$$

$$y_{12} = y_{21} = -\frac{z_C}{\Delta z} = -\frac{5}{27.5}\ \mho$$

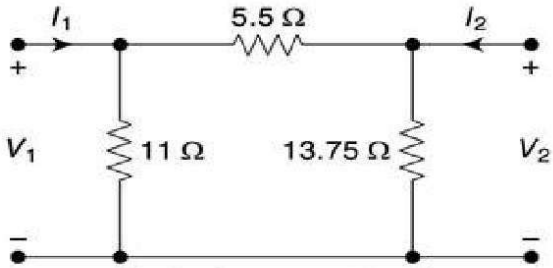
$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{7}{27.5}\ \mho$$

$$\therefore Y_A = (y_{11} + y_{12}) = \frac{2.5}{27.5} = \frac{1}{11}\ \mho$$

$$\therefore Y_B = (y_{22} + y_{12}) = \frac{2}{27.5}\ \mho$$

$$\text{and } Y_C = -y_{21} = \frac{5}{27.5} = \frac{2}{11}\ \mho$$

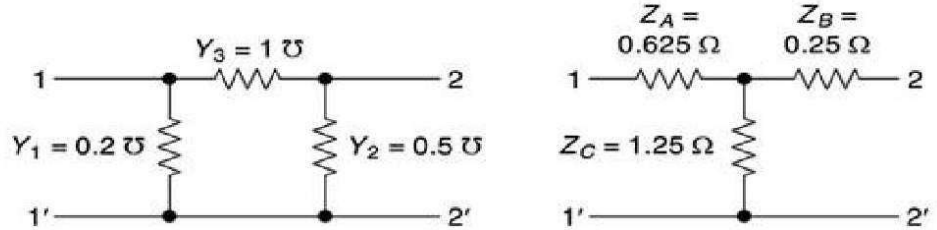
Thus, the impedances of the equivalent π-networks are:



Equivalent π-network

$$\left.\begin{aligned} Z_A &= \frac{1}{Y_A} = 11\ \Omega, \\ Z_B &= \frac{1}{Y_B} = 13.75\ \Omega, \\ Z_C &= \frac{1}{Y_C} = 5.5\ \Omega \end{aligned}\right\}$$

b.



π-network Equivalent T-network

The y-parameters,

$$y_{11} = 1.2\ \mho,\ y_{12} = y_{21} = -1\ \mho, \text{ and } y_{22} = 1.5\ \mho$$

$$\therefore \Delta y = (1.2 \times 1.5 - 1) = 0.8$$

$$\therefore z_{11} = \frac{y_{22}}{\Delta y} = \frac{1.5}{0.8}\ \Omega,\ z_{12} = z_{21} = -\frac{y_{12}}{\Delta y} = \frac{1}{0.8}\ \Omega,\ z_{22} = \frac{y_{11}}{\Delta y} = \frac{1.2}{0.8}\ \Omega$$

$$\left.\begin{aligned} Z_A &= (z_{11} - z_{12}) = \frac{0.5}{0.8} = 0.625\ \Omega \\ \therefore Z_B &= (z_{22} - z_{12}) = \frac{0.2}{0.8} = 0.25\ \Omega \\ Z_C &= z_{12} = \frac{1}{0.8} = 1.25\ \Omega \end{aligned}\right\}$$

CHAPTER9

LOWPASSFILTERINTRODUCTION

Basically, an electrical filter is a circuit that can be designed to modify, reshapeorreject allunwantedfrequenciesofanelectricalsignalandacceptorpassonly thosesignalswantedby thecircuit's designer.Inother wordsthey"filter-out" unwanted signals and an ideal filter will separate and pass sinusoidal input signals based upon their frequency.

In low frequency applications(up to 100kHz),passivefiltersaregenerally constructed using simple RC(Resistor-Capacitor) networks, while higher frequency filters (above 100kHz) are usually made from RLC (Resistor-Inductor-Capacitor) components.

Passive Filters are made up of passive components such as resistors, capacitorsand inductorsand haveno amplifying elements(transistors,op-amps,etc)so have no signal gain, therefore their output level is always less than the input.

Filters are so named according to the frequency range of signals that they allow to pass through them, while blocking or "attenuating" the rest. The most commonly used filter designs are the:

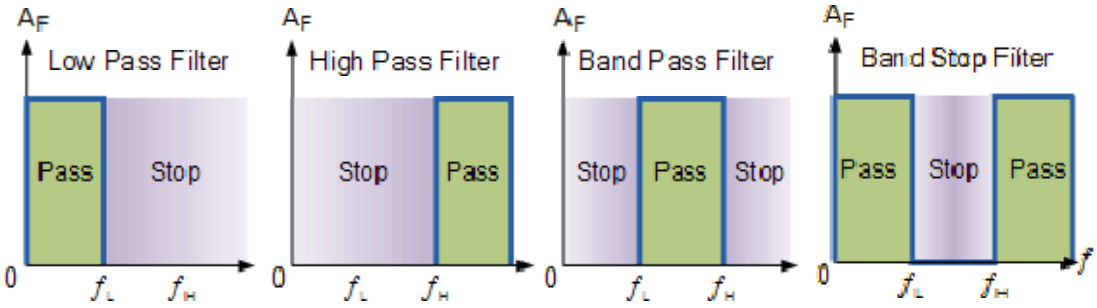
- 1. The Low Pass Filter – the low pass filter only allows low frequency signals from 0Hz to its cut-off frequency, ƒc point to pass while blocking those any higher.
- 2. The High Pass Filter – the high pass filter only allows high frequency signals from its cut-off frequency, ƒc point and higher to infinity to pass through while blocking those any lower.
- 3. The Band Pass Filter – the band pass filter allows signals falling within a certain frequencybandsetupbetweentwopointsto passthroughwhileblockingboth the lower and higher frequencies either side of this frequency band.
- 4 Band Stop Filter - It is so called *band-elimination*, *band-reject*, or *notch* filters; this kind of filter passes all frequencies above and below a particular range set by the component values.

SimpleFirst-orderpassivefilters(1storder)canbemadebyconnecting together a single resistor and a single capacitor in series across an input signal, (Vin) with the output of the filter, (Vout) taken from the junction of these two components. Depending onwhichway aroundweconnect theresistor and thecapacitorwithregards to the output signal determines the type of filter construction resulting in either a Low Pass Filter or a High Pass Filter.

As the function of any filter is to allow signals of a given band of frequencies topass unalteredwhile attenuatingorweakeningallothers those are not

wanted, we can define the amplitude response characteristics of an ideal filter by using an ideal frequency response curve of the four basic filter types as shown.

IDEALFILTERRESPONSECURVES

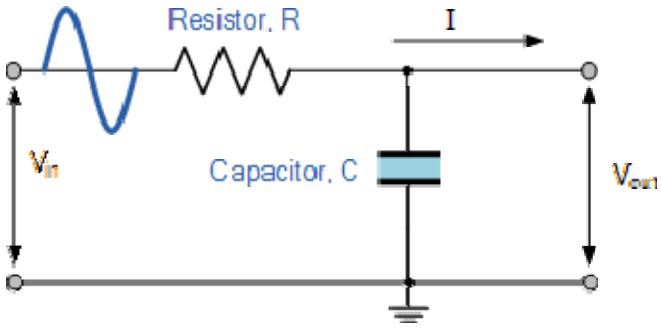


A Low Pass Filter can be a combination of capacitance, inductance or resistance intended to produce high attenuation above a specified frequency and little or no attenuation below that frequency. The frequency at which the transition occurs is called the "cutoff" frequency. The simplest low pass filters consist of a resistor and capacitor but more sophisticated low pass filters have a combination of series inductors and parallel capacitors. In this tutorial we will look at the simplest type, a passive two component RC low pass filter.

THELOWPASSFILTER

A simple passive RC Low Pass Filter or LPF, can be easily made by connecting together in series a single Resistor with a single Capacitor as shown below.In this type of filter arrangement the input signal (Vin) is applied to the series combination (both the Resistor and Capacitor together) but the output signal (Vout) is taken across the capacitor only. This type of filter is known generally as a "first-order filter" or "one-pole filter", why first-order or single-pole?, because it has only "one" reactive component, the capacitor, in the circuit.

RCLOWPASSFILTERCIRCUIT



As mentioned previously in theCapacitive Reactance tutorial, the reactance of a capacitor varies inversely with frequency, while the value of the resistor remains constant as the frequency changes. At lowfrequencies the capacitive reactance, (Xc)ofthecapacitorwillbevery large compared to the resistive value of the resistor, R and as a result the voltage across the capacitor, Vc will also be large while the voltage drop across the resistor, Vr will be much lower. At high frequencies the reverse is true with Vc being small and Vr being large.

While the circuit above is that of anRC Low Pass Filtercircuit, it can also beclassedasafrequencyvariablepotentialdividercircuitsimilartotheonewelooked

at in theResistorstutorial. In that tutorial we used the following equation to calculatethe output voltage for two single resistors connected in series.

$$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$$

where: $R_1 + R_2 = R_T$, the total resistance of the circuit

Wealsoknowthatthecapacitivereactanceofacapacitorinan ACcircuit isgiven as:

$$X_C = \frac{1}{2\pi f C} \text{ in Ohm's}$$

OppositiontocurrentflowinanACcircuitiscalledimpedance,symbol Z and for a series circuit consisting of a single resistor in series with a single capacitor, the circuit impedance is calculated as:

$$Z = \sqrt{R^2 + X_C^2}$$

Then by substituting our equation for impedance above into the resistive potential divider equation gives us:

RCPOTENTIALDIVIDEREQUATION

$$V_{out} = V_{in} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = V_{in} \frac{X_C}{Z}$$

So, by using the potential divider equation of two resistors in series and substituting for impedance we can calculate the output voltage of an RC Filter for any given frequency.

LOWPASSFILTEREXAMPLE

A Low Pass Filter circuit consisting of a resistor of 4k7Ω in series with a capacitor of 47nF is connected across a 10v sinusoidal supply. Calculate the output voltage (Vout) at a frequency of 100Hz and again at frequency of 10,000Hz or 10kHz.

VoltageOutputata Frequencyof100Hz.

$$Xc = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 100 \times 47 \times 10^{-9}} = 33{,}863\Omega$$

$$V_{OUT} = V_{IN} \times \frac{Xc}{\sqrt{R^2 + X_C^2}} = 10 \times \frac{33863}{\sqrt{4700^2 + 33863^2}} = 9.9v$$

Voltage Outputata Frequencyof10,000Hz(10kHz).

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10{,}000 \times 47 \times 10^{-9}} = 338.6\Omega$$

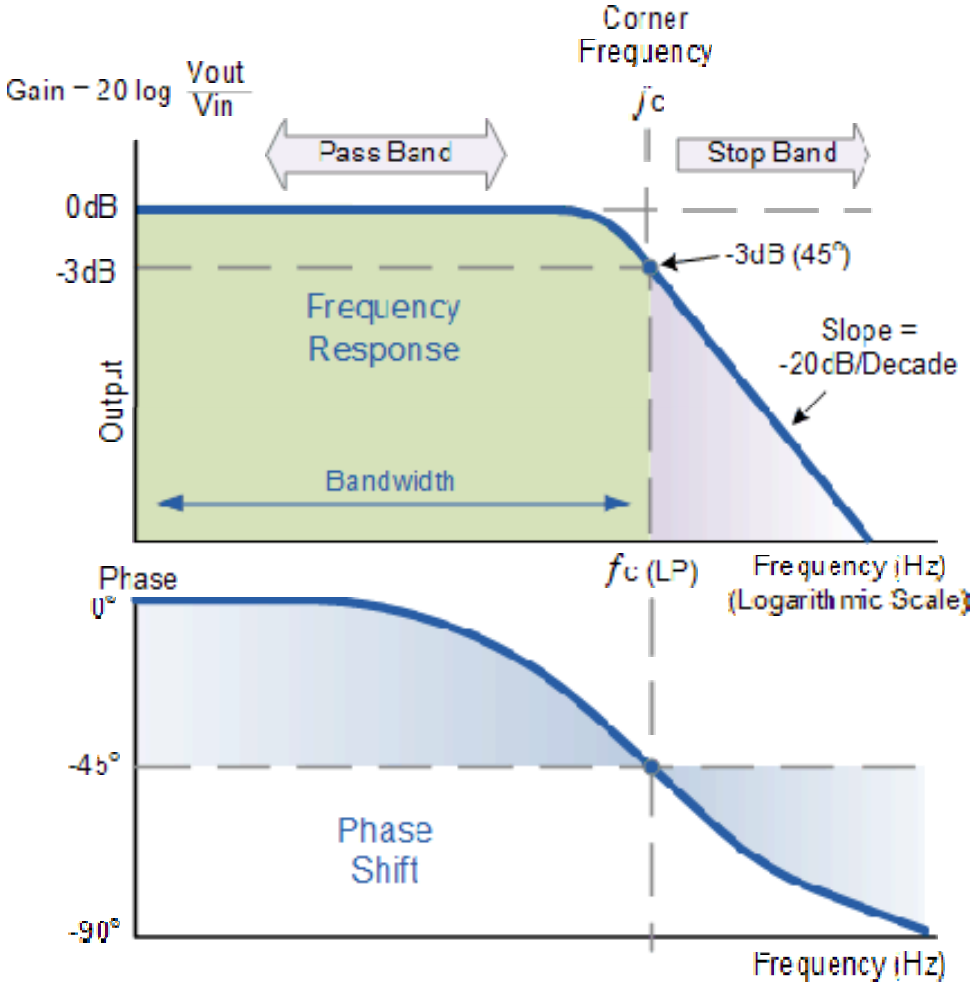
$$V_{OUT} = V_{IN} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = 10 \times \frac{338.6}{\sqrt{4700^2 + 338.6^2}} = 0.718v$$

FREQUENCYRESPONSE

We can see from the results above that as the frequency applied to the RC network increasesfrom 100Hz to 10 kHz,thevoltagedropped acrossthecapacitorand therefore the output voltage (Vout) from the circuit decreases from 9.9v to 0.718v.

Byplottingthenetworksoutputvoltageagainstdifferentvaluesofinputfrequency, the Frequency Response Curve or Bode Plot function of the low pass filter circuit can be found, as shown below.

Frequency Response of a 1st-order Low Pass Filter



The Bode Plotshows the Frequency Response of the filter to be nearly flat for low frequencies and the entire input signal is passed directly to the output, resulting in a gain of nearly 1, called unity, until it reaches its Cut-off Frequency point (ƒc). This is because the reactance of the capacitor is high at low frequencies and blocks any current flow through the capacitor.

After this cut-off frequency point the response of the circuit decreases to zero at a slope of -20dB/ Decade or (-6dB/Octave) "roll-off". Note that the angle of the slope, this -20dB/ Decade roll-off will always be the same for any RC combination.

Any high frequency signals applied to the low pass filter circuit above this cut-off frequency point will become greatly attenuated, that is they rapidly decrease. This happens because at very high frequencies the reactance of the capacitor becomesso low that it gives the effect of a short circuit condition on the output terminals resulting in zero output.

Then by carefully selecting the correct resistor-capacitor combination, we can create a RC circuit that allows a range of frequencies below a certain value to pass through the circuit unaffected while any frequencies applied to the circuit above thiscut-off point to be attenuated, creating what is commonly called a Low Pass Filter.

For this type of "Low Pass Filter" circuit, all the frequencies below thiscut-off, ƒc point that are unaltered with little or no attenuation and are said to be in the filters Pass band zone. This pass band zone also represents the Bandwidth of the filter. Any signal frequencies above this point cut-off point are generally said to be inthe filters Stop band zone and they will be greatly attenuated.

This "Cut-off", "Corner" or "Breakpoint" frequency is defined as being the frequency point where the capacitive reactance and resistance are equal, R = Xc = 4k7Ω. When this occurs the output signal is attenuated to 70.7% of the input signal value or -3dB (20 log (Vout/Vin)) of the input. Although R = Xc, the output is not half of the input signal. This is because it is equal to the vector sum of the two and is therefore 0.707 of the input.

As the filter contains a capacitor, the Phase Angle(Φ)oftheoutputsignal LAGS behindthatoftheinputandatthe-3dBcut-offfrequency(ƒc)andis- 45° outofphase.Thisis dueto thetimetaken to chargetheplates ofthecapacitor as the input voltage changes, resulting in the output voltage (the voltage across the capacitor) "lagging" behind that of the input signal. The higher the input frequency applied to the filter the more the capacitor lags and the circuit becomes more and more"out ofphase".

The cut-off frequency point and phase shift angle can be found by using the following equation:

CUT-OFFFREQUENCYANDPHASESHIFT

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 4700 \times 47 \times 10^{-9}} = 720\text{Hz}$$

$$\text{Phase Shift } \varphi = -\arctan(2\pi f RC)$$

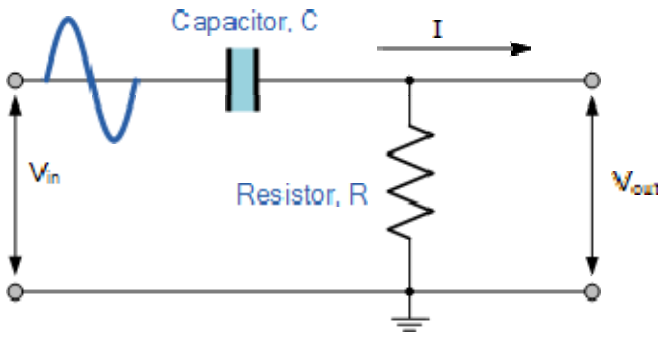
Then for our simple example of a "Low Pass Filter" circuit above, the cut-off frequency (ƒc) is given as720Hz with an output voltage of 70.7% of the input voltage value and a phase shift angle of -45°.

HIGHPASSFILTERS

A High Pass Filter or HPF, is the exact opposite to that of the previously seen Low Pass filter circuit, as now the two components have been interchanged with the output signal (Vout) being taken from across the resistor as shown.

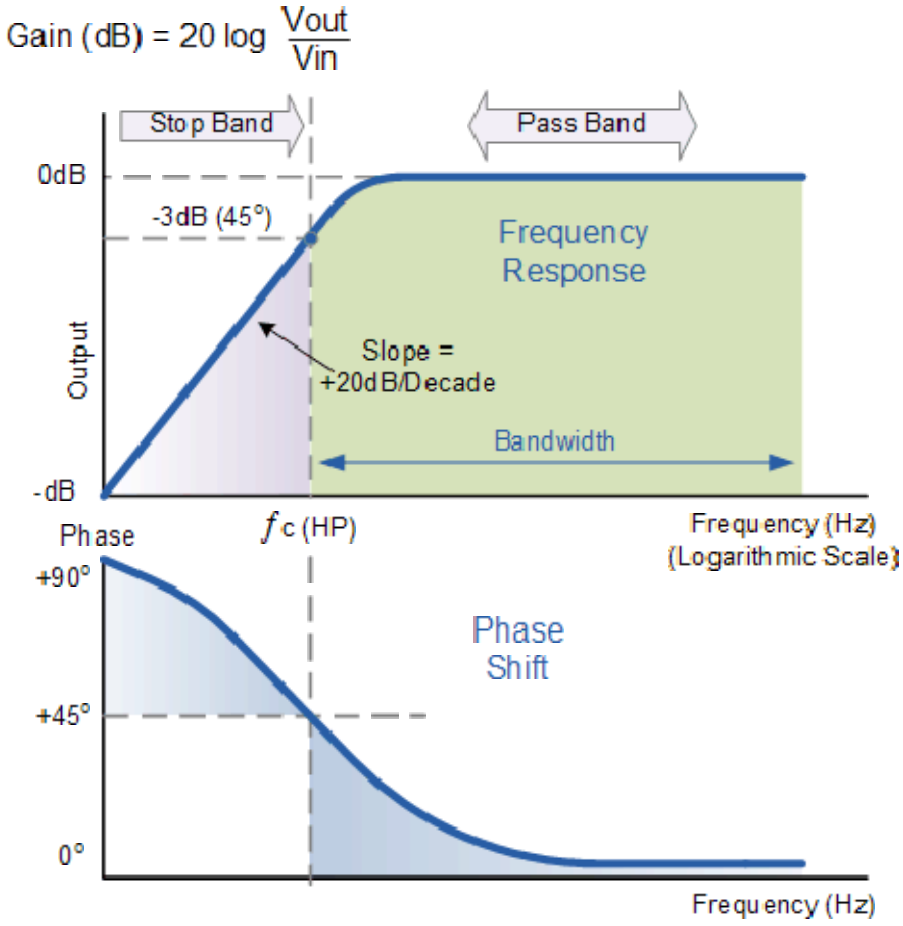
Where as the low pass filter only allowed signals to pass below its cut-off frequency point, ƒc, the passive high pass filter circuit as its name implies, only passes signals above the selected cut-off point, ƒc eliminating any low frequency signals from the waveform. Consider the circuit below.

THEHIGHPASSFILTERCIRCUIT



In this circuit arrangement, the reactance of the capacitor is very high atlowfrequenciessothecapacitoractslikeanopencircuitandblocksanyinputsignals at Vin until the cut-off frequency point (ƒc) is reached. Above thiscut-offfrequency point the reactance of the capacitor has reduced sufficiently as to now act more like a short circuit allowing the entire input signal to pass directly to the output as shown below in the High Pass Frequency Response Curve.

FREQUENCYRESPONSEOFA1STORDERHIGHPASSFILTER.



TheBodePlotor FrequencyResponse Curve abovefor a High Pass filter is the exact opposite to that of a low pass filter. Here the signal is attenuated or damped at low frequencies with the output increasing at +20dB/Decade (6dB/Octave) until the frequency reaches thecut-off point (ƒc) where again R = Xc. It has a response curve that extends down from infinity to the cut-off frequency, where the output voltage amplitude is $1/\sqrt{2}$ = 70.7% of the input signal value or -3dB (20 log (Vout/Vin)) of the input value.

Also we can see that the phase angle (Φ) of the output signal LEADS thatof the input and is equal to+45°at frequency ƒc. Thefrequency responsecurvefor a high pass filter implies that the filter can pass all signals out to infinity. However in practice, the high pass filter response does not extend to infinity but is limited by the electrical characteristics of the components used.

The cut-off frequency point for a first order high pass filter can be found using the same equation as that of the low pass filter, but the equation for the phaseshift is modified slightly to account for the positive phase angle as shown below.

CUT-OFFFREQUENCYANDPHASESHIFT

$$fc = \frac{1}{2\pi RC}$$

$$Phase\ Shift\ \phi = \arctan\frac{1}{2\pi fRC}$$

Thecircuitgain,Avwhichisgivenas Vout/Vin(magnitude)andiscalculatedas:

$$A_V = \frac{V_{OUT}}{V_{IN}} = \frac{R}{\sqrt{R^2 + XC^2}} = \frac{R}{Z}$$

$$\text{at low } f: \ Xc \to \infty, \ Vout = 0$$
$$\text{at high } f: \ Xc \to 0, \ Vout = Vin$$

HIGHPASSFILTEREXAMPLE.

Calculate the cut-off or “breakpoint” frequency (ƒc) for asimple highpass filter consisting of an82pF capacitor connected in series with a 240kΩ resistor.

$$fc = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 240{,}000 \times 82 \times 10^{-12}} = 8{,}087\ Hz\ or\ 8kHz$$

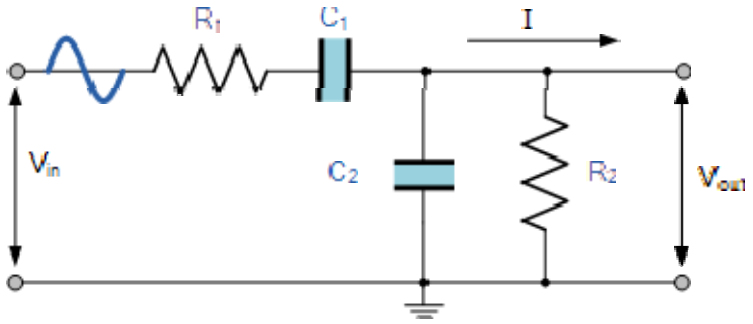
BANDPASSFILTERS

The cut-off frequency or ƒc point in a simple RC passive filter can be accurately controlled using just a single resistor in series with anon-polarized capacitor, and depending upon which way around they are connected either a low pass or a high pass filter is obtained.

One simple use for these types of Passive Filters is in audio amplifier applications or circuits such as in loudspeaker crossover filters or pre-amplifier tone controls. Sometimes it is necessary to only pass a certain range of frequencies that do not begin at 0Hz, (DC) or end at some high frequency point but are within a certain frequency band, either narrow or wide.

By connectingor "cascading" togetherasingleLow PassFilter circuitwith a High Pass Filter circuit, we can produce another type of passive RC filter that passes a selected range or "band" of frequencies that can be either narrow or wide while attenuating all those outside of this range. This new type of passive filter arrangement produces a frequency selective filter known commonly as a Band Pass Filter or BPF for short.

BANDPASSFILTERCIRCUIT



Unlike a low pass filterthat only passsignals of a low frequency range orahighpassfilterwhichpasssignalsofahigherfrequencyrange,aBandPass Filters passes signals within a certain "band" or "spread" of frequencies without distorting the input signal or introducing extra noise. This band of frequencies can be any width and is commonly known as the filters Bandwidth.

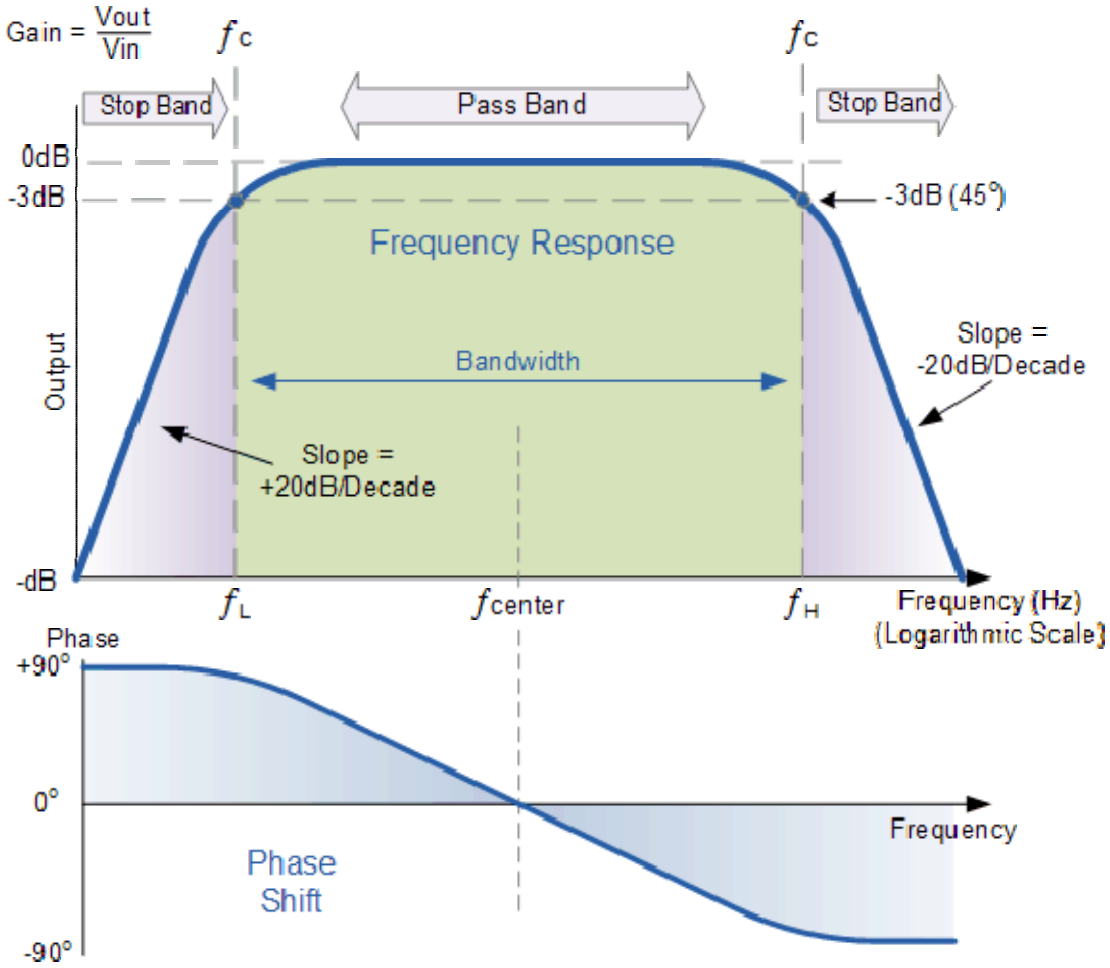
Bandwidth is commonly defined as the frequency range that exists between two specified frequency cut-off points (ƒc), that are 3dB below the maximum centre or resonant peak while attenuating or weakening the others outside of these two points.

Then for widely spread frequencies, we can simply define the term "bandwidth", BW as being the difference between the lower cut-off frequency ($ƒc_{LOWER}$) and the higher cut-off frequency ($ƒc_{HIGHER}$) points. In other words, $BW = ƒ_H - ƒ_L$. Clearly for a pass band filter to function correctly, the cut-off frequency of the low pass filter must be higher than the cut-off frequency for the high pass filter.

The"ideal"BandPassFiltercanalsobe usedto isolateorfilteroutcertain frequenciesthatliewithinaparticularbandoffrequencies,forexample,noise

cancellation. Band pass filters are known generally as second-order filters, (two-pole) because they have "two" reactive component, the capacitors, within their circuit design. One capacitor in the low pass circuit and another capacitor in the high pass circuit.

FrequencyResponseofa2ndOrderBandPassFilter.



The Bode Plot or frequency response curve above shows the characteristics of the band pass filter. Here the signal is attenuated at low frequencies with the output increasing at a slope of +20dB/Decade (6dB/Octave) untilthe frequency reaches the "lower cut-off" point f_L. At this frequency the output voltage is again $1/\sqrt{2}$ = 70.7% of the input signal value or -3dB (20 log (Vout/Vin)) of the input.

The output continues at maximum gainuntil itreaches the "upper cut-off" point $f_H$where the output decreases at a rate of -20dB/Decade (6dB/Octave)attenuating any high frequency signals. The point of maximum output gain is generally the geometric mean of the two -3dB value between the lower and upper cut-off points and is called the "Centre Frequency" or "Resonant Peak" value f_r. This geometric mean value is calculated as being $f_r^2 = f_{(UPPER)} x\, f_{(LOWER)}$.

A band pass filter is regarded as a second-order (two-pole) type filter because it has "two" reactive components within its circuit structure, then the phase anglewillbetwicethatofthepreviouslyseenfirst-orderfilters,i.e.,180°.Thephase

angle of the output signal LEADSthat of the input by+90° up to the centre or resonant frequency,frpoint were itecbmes "zero" degrees (0°) or "in-phase" and then changes to LAG the input by -90°as the output frequency increases.

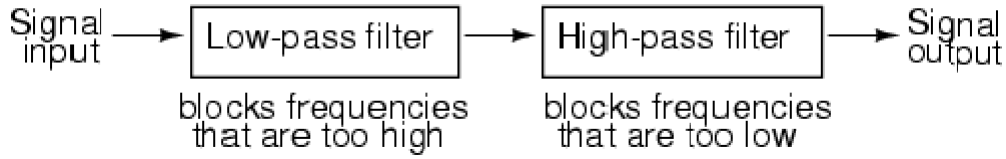
Theupperandlower cut-offfrequencypointsfor abandpassfilter can be foundusingthesameformulaasthatfor boththelowandhighpassfilters,For example.

$$f_C = \frac{1}{2\pi RC}\ Hz$$

Thenclearly,thewidthofthepassbandofthefiltercanbecontrolledbythe positioning of the two cut-off frequency points of the two filters.

BandPassFilterExample

Asecond-order bandpassfilter istobeconstructedusingRC components that will only allow a range of frequencies to pass above 1kHz (1,000Hz) andbelow30kHz(30,000Hz).Assumingthatboththe resistors havevalues of10kΩ´s, calculatethevaluesofthet wo capacitorsrequired.



TheHighPassFilterStage

The value of thecapacitor C1 required to give a cut-off frequency f_L of 1kHz with a resistor value of10kΩ is calculated as:

$$C = \frac{1}{2\pi fc.R} = \frac{1}{2\pi\, x1{,}000 x10{,}000} = 15.8\,nF$$

Then, the values ofR1 andC1required for the high pass stage to give a cut-off frequency of 1.0kHz are: R1 = 10kΩ´s and C1 = 15nF.

TheLowPassFilterStage

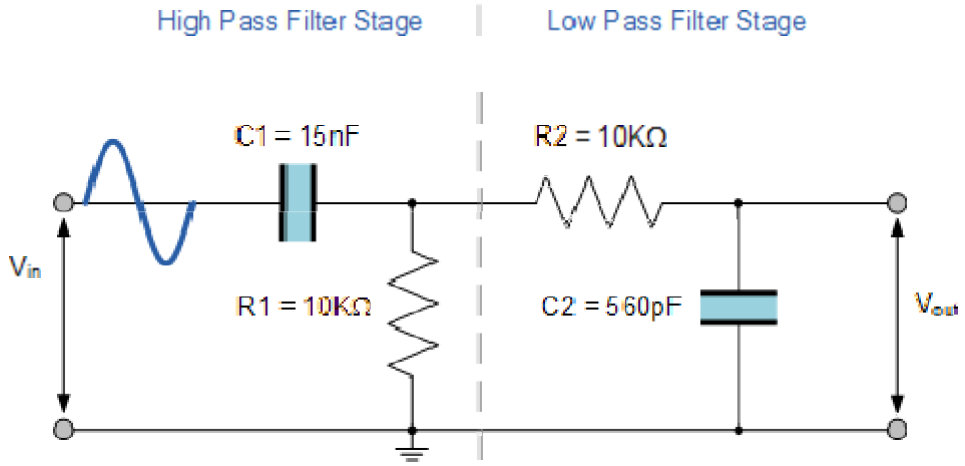
Thevalueofthecapacitor C2 requiredtogiveacut-offfrequency $f_H$of30kHzwitha resistor value of10kΩ is calculated as:

$$C = \frac{1}{2\pi fc.R} = \frac{1}{2\pi\, x30{,}000 x10{,}000} = 510\,pF$$

Then,thevaluesofR2and C2requiredforthelowpassstage to givea cut- off frequency of 30kHz are, R = 10kΩ´s and C = 510pF. However, the nearest preferred value of the calculated capacitor value of 510pF is 560pF so this is used instead.

With the values of both the resistances R1 and R2 given as 10kΩ, and the twovaluesofthecapacitorsC1 and C2 foundforthehighpassandlowpassfilters as 15nF and 560pF respectively, then the circuit for oursimplepassive BandPassFilter is given as.

CompletedBandPassFilterCircuit



BandPassFilterResonantFrequency

We can also calculate the “Resonant” or “Centre Frequency” (fr) point of the band pass filter were the output gain is at its maximum or peak value. This peak value is not the arithmetic averageof theupperand lower -3dBcut-off pointsasyoumight expect but is in factthe “geometric” or mean value. This geometricmeanvalueiscalculatedas being $fr^2 = fc_{(UPPER)} \times fc_{(LOWER)}$for example:

CentreFrequencyEquation

$$fr = \sqrt{f_L \; x \; f_H}$$

- Where,$f_r$istheresonantorcentrefrequency
- $f_L$isthelower-3dBcut-offfrequencypoint
- $f_H$istheupper-3dbcut-offfrequencypoint

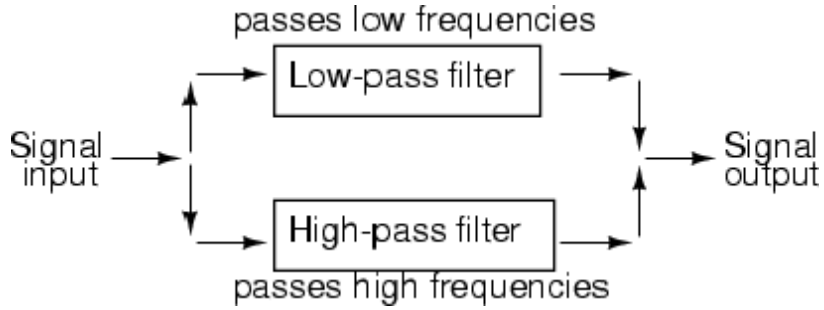
And in our simple example above, the calculated cut-off frequencies were found tobe f_L=1,060Hzandf_H=28,420Hzusingthe filtervalues.

Then by substituting these values into the above equation gives a central resonant frequency of:

$$fr = \sqrt{f_L \; x \; f_H} = \sqrt{1{,}060 \; x \; 28{,}420} = 5{,}48\,kHz$$

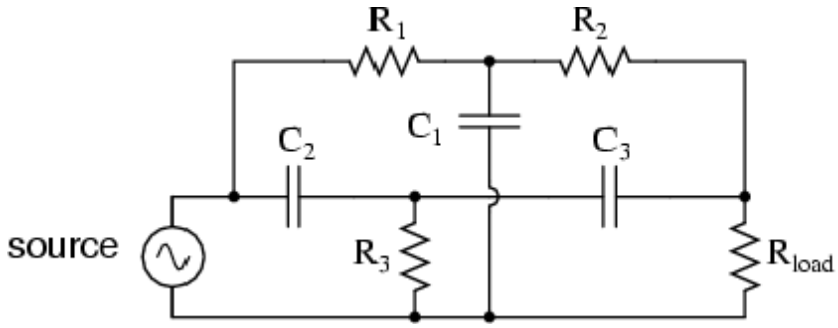
Band-stop filters

It is so called*band-elimination*,*band-reject*, or*notch* filters; this kind of filter passes all frequencies above and below a particular range sety the component values. Not surprisingly, it can be made out of a low-pass and a high-pass filter, just like the band-pass design, except that this time we connect the two filter sections in parallel with each other instead of in series. (Figure below)



Systemlevelblockdiagramofaband-stopfilter.

Constructed using two capacitive filter sections, it looks something like (Figure below).



ButterworthFilter

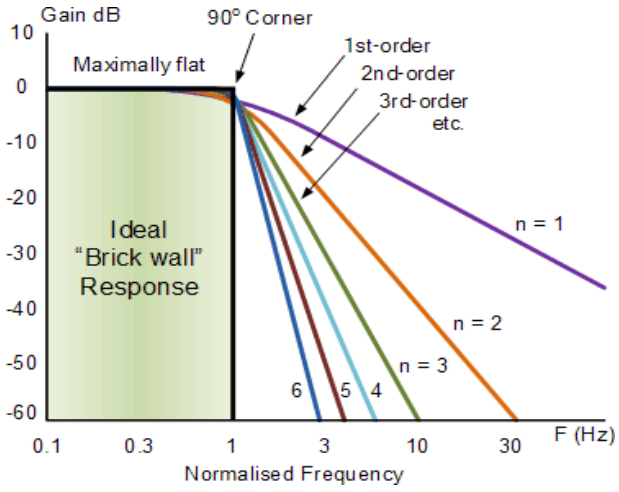
A Butterworth filter is a type of signal processing filter designed to have a frequency responseasflataspossibleinthepassband.HencetheButterworthfilterisalsoknownas "maximally flat magnitude filter". It was invented in 1930 by the British engineer and physicist Stephen Butterworth in his paper titled "*On the Theory of Filter Amplifiers*". The frequency response of the Butterworth filter is flat in the passband (i.e. a bandpassfilter)androll-offstowardszerointhestopband.Therateofroll-offresponsedependson the order of the filter. The number of reactive elements used in the filter circuit will decide the order of the filter.

The inductorand capacitorare reactive elements used in filters. But in the case of Butterworthfilteronlycapacitorsareused.So,thenumberofcapacitorswilldecidethe order of the filter.

Here,wewilldiscusstheButterworthfilterwithalowpassfilter.Similarly,the highpassfiltercan be designed by just changing the position of resistanceand capacitance.

ButterworthLowPassFilterDesign

While designingthe filter, the designer tries to achieve a response near to the ideal filter. It is very difficult to match results with the exact ideal characteristic. We need to use complexhigher-order If youincrease theorder of thefilter,thenumber of cascade stages with the filter is also increased. But in practice, we cannot achieve Butterworth's ideal frequency response. Because it produces excessive ripple in the passband.In Butterworth filter, mathematicallyitispossibletogetflatfrequencyresponsefrom0Hztothecut-off frequencyat -3dB with no ripple. If the frequency is more than the cut-off frequency, it will roll-off towards zero with the rate of -20 dB/decade for the first-order filter.If you increase the order of the filter, the rate of a roll-off period is also increased. And for second-order, it is -40 dB/decade. The quality factorfor the Butterworth filter is 0.707. ThebelowfigureshowsthefrequencyresponseoftheButterworthfilterforvarious orders of the filter



FrequencyResponseofButterworthFilterThegeneralizedformoffrequencyresponsefor nth-order Butterworth low-pass filter is;

$$H(j\omega) = \frac{1}{\sqrt{1 + \varepsilon^2 (\frac{\omega}{\omega_C})^{2n}}}$$

Where,

n=order of thefilter,
ω=operatingfrequency(passbandfrequency)ofcircuit
ω_C=Cut-offfrequency
ε=maximumpassbandgain=A_{max}

Thebelowequationisusedtofindthevalueofε.

$$H_1 = \frac{H_0}{\sqrt{1 + \varepsilon^2}}$$

Where,
H_1 =minimumpassband gain
H_0=maximumpassbandgain

First-orderLowpassButterworthFilter

The lowpass filter is a filter thatallows the signal with the frequency is lower than the cutoff frequency and attenuates the signals with the frequency is more than cutoff frequency.Inthefirst-orderfilter,thenumberofreactivecomponentsisonlyone.The below figure shows the circuit diagram of the first-order lowpass Butterworth filter.

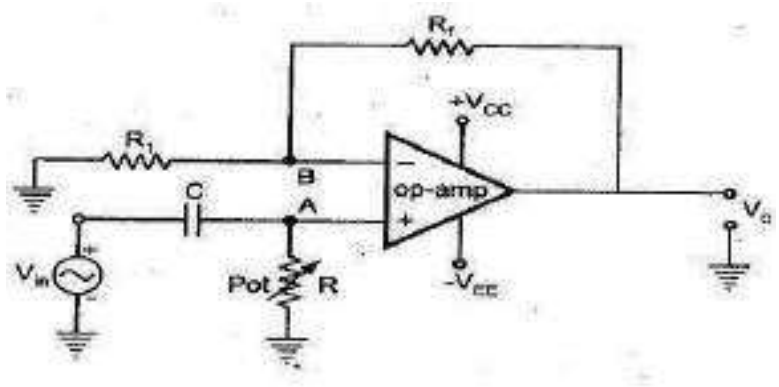


Fig. 2.79 First order high pass Butterworth filter

The low pass Butterworth filter is an active Low pass filteras it consists of the op-amp. Thisop-ampoperatesonnon-invertingmode.Hence,thegainofthefilterwilldecideby the resistor$R_1$and R_F. And the cutoff frequency decides by R and C. Now,ifyouapplythevoltagedividerruleatpointVaandfindthevoltageacrossa capacitor. It is given as;

$$V_a = \frac{-jX_C}{R - jX_C} V_{in}$$

$$V_a = \frac{-j(\frac{1}{2\pi fC})}{R - j(\frac{1}{2\pi fC})} V_{in}$$

$$V_a = \frac{-j}{2\pi fRC - j} V_{in}$$

$$V_a = \frac{V_{in}}{1 - \frac{2\pi fRC}{j}}$$

$$V_a = \frac{V_{in}}{1 + j2\pi fRC}$$

Becauseofthe non-invertingconfigurationofanop-amp,

$$V_0 = \left(1 + \frac{R_f}{R_1}\right) V_a$$

$$V_0 = \left(1 + \frac{R_f}{R_1}\right) \frac{V_{in}}{1 + j2\pi fRC}$$

$$\frac{V_0}{V_a} = \frac{A_f}{1 + j\frac{f}{f_c}}$$

WHERE

$$A_f = 1 + \frac{R_F}{R_1}$$

A_f = Gain of filter in Passband

$$f_c = \frac{1}{2\pi RC}$$

f_c = Cutoff Frequency

f = Operating Frequency

$$\frac{V_0}{V_a} = \left|\frac{V_0}{V_a}\right| \angle \phi$$

$$\left|\frac{V_0}{V_a}\right| = \frac{A_f}{\sqrt{1 + j\left(\frac{f}{f_c}\right)^2}}$$

$$\phi = -\tan^{-1}\left(\frac{f}{f_c}\right)$$

1. At very low frequency, $f \ll f_c$

$$\left|\frac{V_0}{V_a}\right| \approx A_f(Constant)$$

2. At cutoff frequency, $f = f_c$

$$\left|\frac{V_0}{V_a}\right| = \frac{A_f}{\sqrt{2}} = 0.707A_f$$

3. At high frequency, $f > f_c$

$$\left|\frac{V_0}{V_a}\right| < A_f$$

Thebelowfigureshowsthefrequencyresponseoffirst-orderlowpassButterworth filter.

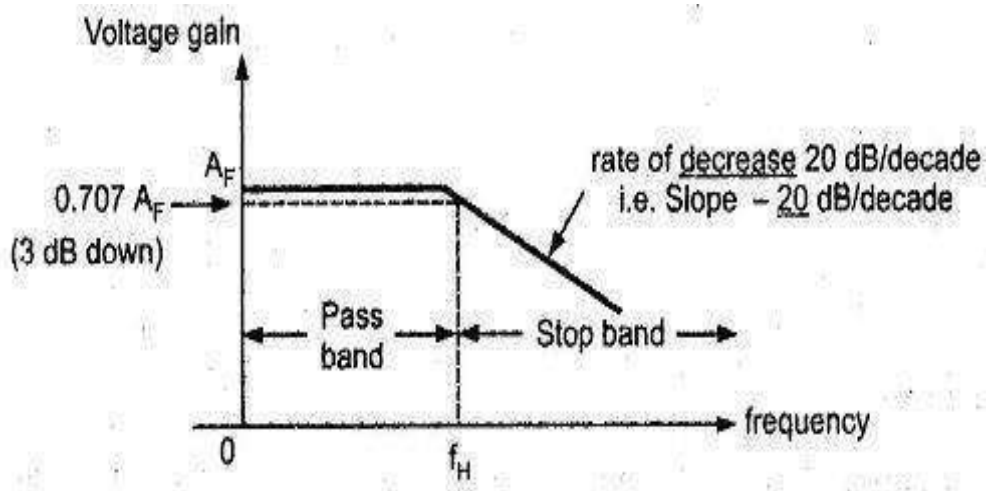


Fig. 2.75 Frequency response

Second-orderButterworthFilter

The second-order Butterworth filter consists of two reactive components. The circuitdiagramofasecond-orderlowpassButterworthfilterisasshowninthe below figure.

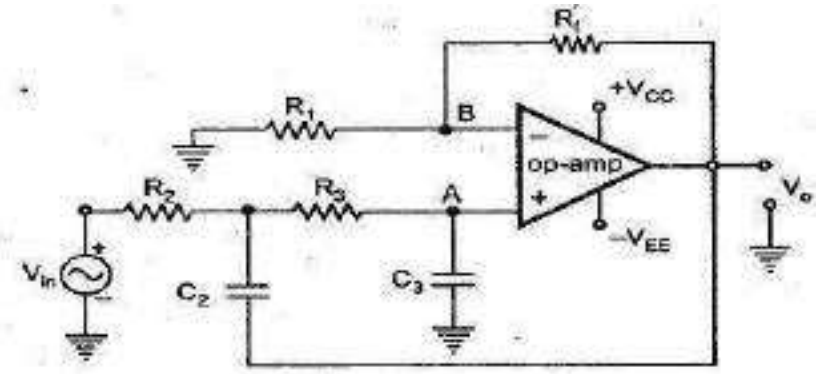


Fig. 2.76 Second order low pass butterworth filter

Inthis type of filter, resistor R and $R_F$are the negative feedback ofop-amp. And the cutoff frequency of the filter decides by R_2, R_3, C_2, and C_3.The second-order lowpass Butterworth filter consists oftwo back-to-back connected RC networks. And$R_L$istheloadresistance. First-orderandsecond-orderButterworthfiltersare very important. Because we can get higher-order Butterworth filter by just cascading of the first-order and second-order Butterworth filters.

Let'sanalysethecircuitofsecond-orderButterworthfilter,

ApplyKirchhoff'sCurrentLawatpointV_1.

$$I_1 = I_2 + I_3$$

$$\frac{V_{in} - V_1}{R_2} = \frac{V_1 - V_0}{\frac{1}{sC_2}} + \frac{V_1 - V_a}{R_3} \tag{1}$$

Using potential divider rule at point V_a

$$V_a = V_1 \left[\frac{\frac{1}{sC_3}}{R_3 + \frac{1}{sC_3}} \right]$$

$$V_a = V_1 \left[\frac{\frac{1}{sC_3}}{\frac{R_3 sC_3 + 1}{sC_3}} \right]$$

$$V_a = \frac{V_1}{1 + sR_3C_3}$$

Put the value of V_1 in equation-(1) $V_1 = V_a(1 + sR_3C_3)$

$$\frac{V_{in}-V_a(1+sR_3C_3)}{R_2}=\frac{V_a(1+sR_3C_3)-V_0}{\frac{1}{sC_2}}+\frac{V_a(1+sR_3C_3)-V_a}{\frac{1}{R_3}}$$

$$\frac{V_{in}}{R_2}-\frac{V_a(1+sR_3C_3)}{R_2}=\frac{V_a(1+sR_3C_3)}{\frac{1}{sC_2}}-\frac{V_0}{\frac{1}{sC_2}}+\frac{V_a(1+sR_3C_3)}{R_3}-\frac{V_a}{R_3}$$

$$\frac{V_{in}}{R_2}+\frac{V_0}{\frac{1}{sC_2}}=\frac{V_a(1+sR_3C_3)}{\frac{1}{sC_2}}+\frac{V_a(1+sR_3C_3)}{R_2}+\frac{V_a(1+sR_3C_3)}{R_3}-\frac{V_a}{R_3}$$

$$\frac{V_{in}}{R_2}+V_0sC_2=V_a\left[sC_2(1+sR_3C_3)+\frac{(1+sR_3C_3)}{R_2}+\frac{(1+sR_3C_3)}{R_3}-\frac{1}{R_3}\right]$$

$$\frac{V_{in}+V_0sC_2R_2}{R_2}=V_a\left[\frac{R_3R_2sC_2(1+sR_3C_3)+R_3(1+sR_3C_3)+R_2(1+sR_3C_3)-R_2}{R_2R_3}\right]$$

$$R_3(V_{in}+V_0sC_2R_2)=V_a\left[R_3R_2sC_2(1+sR_3C_3)+R_3(1+sR_3C_3)+R_2(1+sR_3C_3)-R_2\right]$$

$$R_3V_{in}+V_0sC_2R_2R_3=V_a\left[(1+sR_3C_3)\left(R_3R_2sC_2+R_3+R_2\right)-R_2\right]$$

$$V_a=\frac{R_3V_{in}+V_0sC_2R_2R_3}{(1+sR_3C_3)\left(R_3R_2sC_2+R_3+R_2\right)-R_2}$$

Because of the non-inverting configuration of an op-amp,

$$V_0=A_fV_a$$

Where,

$$A_f=1+\frac{R_f}{R_1}=Gain\,of\,filter\,in\,passband$$

$$V_0=A_f\left[\frac{R_3V_{in}+V_0sC_2R_2R_3}{(1+sR_3C_3)\left(R_3R_2sC_2+R_3+R_2\right)-R_2}\right]$$

$$V_0-\frac{A_fV_0sC_2R_2R_3}{(1+sR_3C_3)\left(R_3R_2sC_2+R_3+R_2\right)-R_2}=\frac{A_fR_3V_{in}}{(1+sR_3C_3)\left(R_3R_2sC_2+R_3+R_2\right)-R_2}$$

$$V_0\left[(1+sR_3C_3)(R_3R_2sC_2+R_3+R_2)-R_2-A_fsC_2R_2R_3\right]=A_fR_3V_{in}$$

$$\frac{V_0}{V_{in}}=\frac{A_fR_3}{\left[(1+sR_3C_3)(R_3R_2sC_2+R_3+R_2)-R_2-A_fsC_2R_2R_3\right]}$$

Rearrangethisequation,

$$\frac{V_0}{V_{in}} = \frac{A_f R_3}{[(1+sR_3C_3)(R_2+R_3+sR_2R_3C_2) - R_2 - sA_f R_2 R_3 C_2]}$$

$$\frac{V_0}{V_{in}} = \frac{A_f R_3}{[(R_2+R_3+sR_2R_3C_2+sR_2R_3C_3+sR_3^2C_3+s^2R_2R_3^2C_2C_3) - R_2 - sA_f R_2 R_3 C_2]}$$

$$\frac{V_0}{V_{in}} = \frac{A_f R_3}{s^2R_2R_3^2C_2C_3 + s(R_2R_3C_2 + R_2R_3C_3 + R_3^2C_3 - A_f R_2R_3C_2) + R_3}$$

$$\frac{V_0}{V_{in}} = \frac{A_f R_3}{R_2R_3^2C_2C_3\left(s^2 + s\frac{R_2R_3C_2+R_2R_3C_3+R_3^2C_3-A_fR_2R_3C_2}{R_2R_3^2C_2C_3} + \frac{R_3}{R_2R_3^2C_2C_3}\right)}$$

$$\frac{V_0}{V_{in}} = \frac{A_f}{R_2R_3C_2C_3\left(s^2 + s\frac{R_2C_2+R_2C_3+R_3C_3-A_fR_2C_2}{R_2R_3C_2C_3} + \frac{1}{R_2R_3C_2C_3}\right)}$$

$$\frac{V_0}{V_{in}} = \frac{\frac{A_f}{R_2R_3C_2C_3}}{\left(s^2 + s\frac{R_2C_2+R_2C_3+R_3C_3-A_fR_2C_2}{R_2R_3C_2C_3} + \frac{1}{R_2R_3C_2C_3}\right)}$$

Comparethisequationwiththestandard formtransfer functionforsecond-order Butterworth filter. And that is,

$$\frac{V_0}{V_{in}} = \frac{A}{s^2 + 2\zeta\omega_c s + \omega_c^2}$$

Bycomparingaboveequations,wecanfindtheequationofcutofffrequencyand overall gain for the second-order lowpass Butterworth filter.

Thegainoffilteris,

$$A_{max} = \frac{A_f}{R_2 R_3 C_2 C_3}$$

And the Cutoff frequency of filter is ,

$$\omega_c^2 = \frac{1}{R_2 R_3 C_2 C_3}$$

$$f_c = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}}$$

Now, if we consider the value of R_2 is same as R_3 and the value of C_2 is same as C_3.

$$R_2 = R_3 = R \quad and \quad C_2 = C_3 = C$$

$$f_c = \frac{1}{2\pi RC}$$

Now if we put above values in transfer function,

$$\frac{V_0}{V_{in}} = \frac{\frac{A_f}{R^2C^2}}{s^2 + s\frac{RC+RC+RC-A_f RC}{R^2C^2} + \frac{1}{R^2C^2}}$$

$$\omega_c = \frac{1}{RC}$$

$$\frac{V_0}{V_{in}} = \frac{A_f \omega^2}{s^2 + s(3 - A_f)\omega + \omega^2}$$

From above equation, the quality factor Q is equal to,

$$Q = \frac{1}{3 - A_f}$$

Wecansaythat,thequalityfactorisonlydependsonthe gainoffilter.Andthe value of gain should not more than 3. If the value of gain is more than 3, the system will be unstable.

ThevalueofqualityfactorisO.707fortheButterworthfilter.And ifweputthis value inequation ofquality factor, we can find the value of gain.

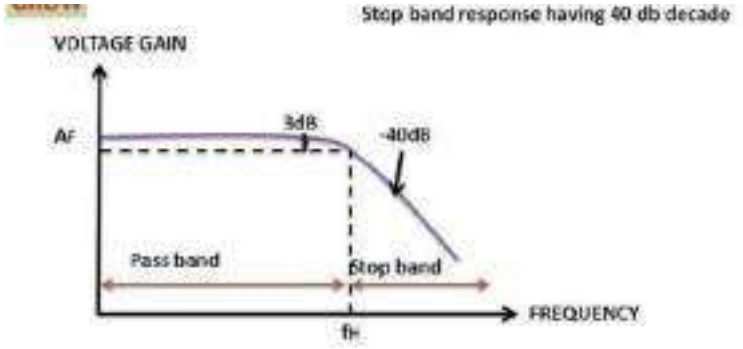
$$0.707 = \frac{1}{3 - A_f}$$

$$A_f = 1.586$$

$$1 + R_f R_1 = 1.586$$

$$R_f R_1 = 0.586$$

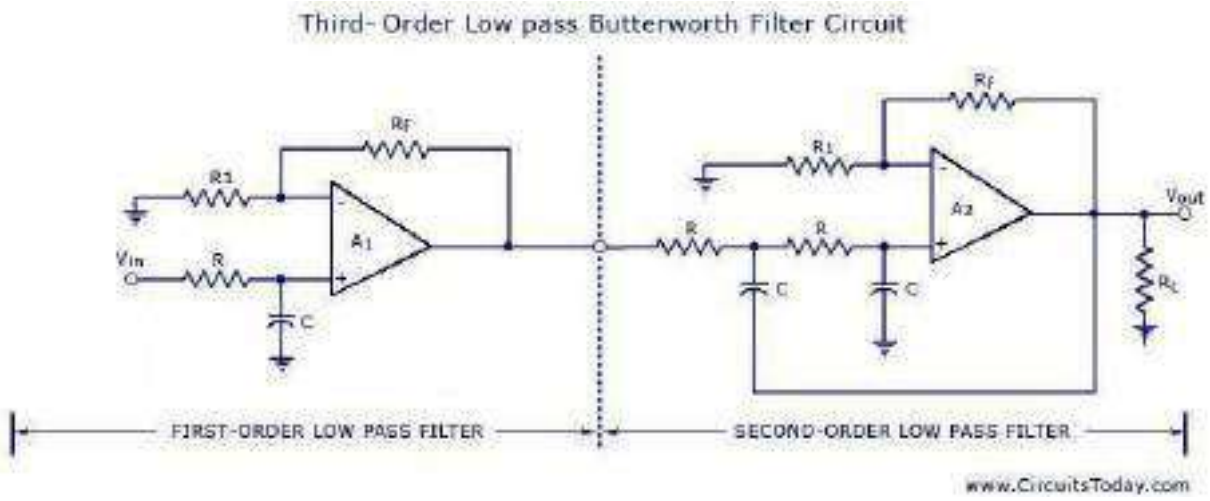
Whiledesigningthesecond-orderButterworthfilteraboverelationmustbesatisfy. Thefrequencyresponseofthisfilterisasshowninbelowfigure.



Third-orderLowpassButterworthFilter

Third-orderlowpassButterworthfiltercandesignbycascadingthefirst-orderand second-order Butterworth filter.

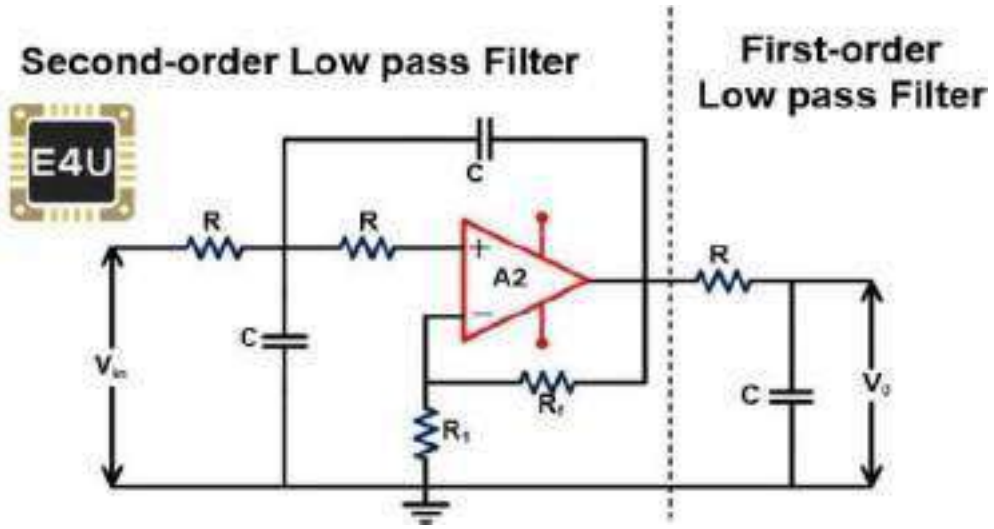
Thebelowfigureshowsthecircuitdiagramofthethird-orderlowpassButterworthfilter.



Third-orderLowPassButterworthFilter

Inthisfigure,thefirstpartshowsthe first-orderlowpassButterworthfilter,andthe second part shows the second-order lowpass Butterworth filter.

But in this condition, the voltage gain of the first part is optional and itcan be set atany value.Therefore,thefirstop-ampisnottakingpartinvoltage gain.Hence,thefigurefor the third-order low pass filter can be expressed as below figure also;

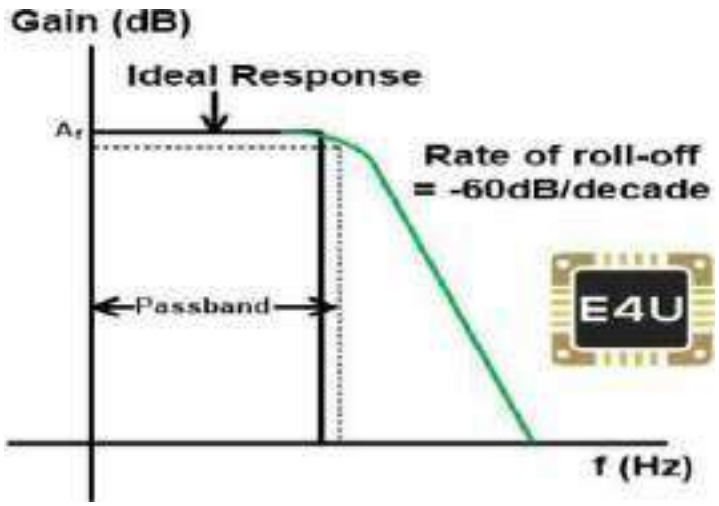


;

Thevoltage gainofasecond-orderfilteraffectsthe flatnessoffrequencyresponse. If the gain of the second-order filter is kept at 1.586, the gain will down 3db for each part. So, the overall gain will down 6dB at the cutoff frequency.

Byincreasingthe voltage gainofthesecond-orderfilter,wecanoffsetthe cumulative loss of voltage gain.

Inthethird-orderButterworthfilter,therateofaroll-offperiod is-60dB/decade. And the frequency response ofthis filter is nearer to the ideal Butterworth filter compared to the first and second-order filters. The frequ



(frequencyresponseofthisfilterisasshowninthebelowfigure.)

Fourth-orderLowpassButterworthFilter

Fourth-order Butterworth filter is established by the cascade connection of two second-orderlowpassButterworthfilters.Thecircuitdiagramofthe fourth-order lowpass Butterworth filter is as shown in the below figure.