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MODULE#1

CHAPTER#1

1. Basic Concept of Control System

Control Engineering is concerned with techniques that are used to solve the following six problems in the most efficient manner possible.

(a)The identification problem :to measure the variables and convert data for analysis.

(b)The representation problem: to describe a system by an analytical form or mathematical model

(c)The solution problem: to determine the above system model response.

(d)The stability problem: general qualitative analysis of the system

(e)The design problem: modification of an existing system or develop a new one

(f)The optimization problem: from a variety of design to choose the best.

The two basic approaches to solve these six problems are conventional and modern approach. The electrical oriented conventional approach is based on complex function theory. The modern approach has mechanical orientation and based on the state variable theory.

Therefore, control engineering is not limited to any engineering discipline but is equally applicable to aeronautical, chemical, mechanical, environmental, civil and electrical engineering. For example, a control system often includes electrical, mechanical and chemical components. Furthermore, as the understanding of the dynamics of business, social and political systems increases; the ability to control these systems will also increase.

1.1. Basic terminologies in control system

System: A combination or arrangement of a number of different physical components to form a whole unit such that that combining unit performs to achieve a certain goal.

Control: The action to command, direct or regulate a system.

Plant or process: The part or component of a system that is required to be controlled.

Input: It is the signal or excitation supplied to a control system.

Output: It is the actual response obtained from the control system.

Controller: The part or component of a system that controls the plant.

Disturbances: The signal that has adverse effect on the performance of a control system.

Control system: A system that can command, direct or regulate itself or another system to achieve a certain goal.

Automation: The control of a process by automatic means

Control System: An interconnection of components forming a system configuration that will provide a desired response.

Actuator: It is the device that causes the process to provide the output. It is the device that provides the motive power to the process.

Design: The process of conceiving or inventing the forms, parts, and details of system to achieve a specified purpose.

Simulation: A model of a system that is used to investigate the behavior of a system by utilizing actual input signals.

Optimization: The adjustment of the parameters to achieve the most favorable or advantageous design.

Feedback Signal: A measure of the output of the system used for feedback to control the system.

Negative feedback: The output signal is feedback so that it subtracts from the input signal.

Block diagrams: Unidirectional, operational blocks that represent the transfer functions of the elements of the system.

Signal Flow Graph (SFG): A diagram that consists of nodes connected by several directed branches and that is a graphical representation of a set of linear relations.

Specifications: Statements that explicitly state what the device or product is to be and to do. It is also defined as a set of prescribed performance criteria.

Open-loop control system: A system that utilizes a device to control the process without using feedback. Thus the output has no effect upon the signal to the process.

Closed-loop feedback control system: A system that uses a measurement of the output and compares it with the desired output.

Regulator: The control system where the desired values of the controlled outputs are more or less fixed and the main problem is to reject disturbance effects.

Servo system: The control system where the outputs are mechanical quantities like acceleration, velocity or position.

Stability: It is a notion that describes whether the system will be able to follow the input command. In a non-rigorous sense, a system is said to be unstable if its output is out of control or increases without bound.

Multivariable Control System: A system with more than one input variable or more than one output variable.

Trade-off: The result of making a judgment about how much compromise must be made between conflicting criteria.

1.2. Classification

1.2.1. Natural control system and Man-made control system:

Natural control system: It is a control system that is created by nature, i.e. solar system, digestive system of any animal, etc.

Man-made control system: It is a control system that is created by humans, i.e. automobile, power plants etc.

1.2.2. Automatic control system and Combinational control system:

Automatic control system: It is a control system that is made by using basic theories from mathematics and engineering. This system mainly has sensors, actuators and responders.

Combinational control system: It is a control system that is a combination of natural and man-made control systems, i.e. driving a car etc.

1.2.3. Time-variant control system and Time-invariant control system:

Time-variant control system: It is a control system where any one or more parameters of the control system vary with time i.e. driving a vehicle.

Time-invariant control system: It is a control system where none of its parameters vary with time i.e. control system made up of inductors, capacitors and resistors only.

1.2.4. Linear control system and Non-linear control system:

Linear control system: It is a control system that satisfies properties of homogeneity and additive.

- Homogeneous property: $f(x+y)=f(x)+f(y)$
- Additive property: $f(\alpha x)=\alpha f(x)$

Non-linear control system: It is a control system that does not satisfy properties of homogeneity and additive, i.e. $f(x)=x^3$

1.2.5. Continuous-Time control system and Discrete-Time control system:

Continuous-Time control system: It is a control system where performances of all of its parameters are function of time, i.e. armature type speed control of motor.

Discrete -Time control system: It is a control system where performances of all of its parameters are function of discrete time i.e. microprocessor type speed control of motor.

1.2.6. Deterministic control system and Stochastic control system:

Deterministic control system: It is a control system where its output is predictable or repetitive for certain input signal or disturbance signal.

Stochastic control system: It is a control system where its output is unpredictable or non-repetitive for certain input signal or disturbance signal.

1.2.7. Lumped-parameter control system and Distributed-parameter control system:

Lumped-parameter control system: It is a control system where its mathematical model is represented by ordinary differential equations.

Distributed-parameter control system: It is a control system where its mathematical model is represented by an electrical network that is a combination of resistors, inductors and capacitors.

1.2.8. Single-input-single-output (SISO) control system and Multi-input-multi-output (MIMO) control system:

SISO control system: It is a control system that has only one input and one output.

MIMO control system: It is a control system that has only more than one input and more than one output.

1.2.9. Open-loop control system and Closed-loop control system:

Open-loop control system: It is a control system where its control action only depends on input signal and does not depend on its output response.

Closed-loop control system: It is a control system where its control action depends on both of its input signal and output response.

1.3. Open-loop control system and Closed-loop control system

1.3.1. Open-loop control system:

It is a control system where its control action only depends on input signal and does not depend on its output response as shown in Fig. 1.1.

Fig.1.1. An open-loop system

Examples: traffic signal, washing machine, bread toaster, etc.

Advantages:

- Simple design and easy to construct
- Economical
- Easy for maintenance
- Highly stable operation

Dis-advantages:

- Not accurate and reliable when input or system parameters are variable in nature
- Recalibration of the parameters are required time to time

1.3.2. Closed-loop control system:

It is a control system where its control action depends on both of its input signal and output response as shown in Fig.1.2.

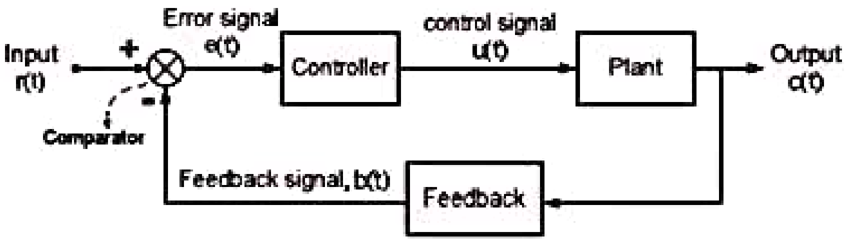


Fig.1.2. A closed-loop system

Examples: automatic electric iron, missile launcher, speed control of DC motor, etc.

Advantages:

- More accurate operation than that of open-loop control system
- Can operate efficiently when input or system parameters are variable in nature
- Less nonlinearity effect of these systems on output response
- High bandwidth of operation
- There is facility of automation
- Time to time recalibration of the parameters are not required

Dis-advantages:

- Complex design and difficult to construct

- Expensive than that of open-loop control system
- Complicate for maintenance
- Less stable operation than that of open-loop control system

1.3.3. Comparison between Open-loop and Closed-loop control systems:

It is a control system where its control action depends on both of its input signal and output response.

Sl. No.	Open-loop control systems	Closed-loop control systems
1	No feedback is given to the control system	A feedback is given to the control system
2	Cannot be intelligent	Intelligent controlling action
3	There is no possibility of undesirable system oscillation(hunting)	Closed loop control introduces the possibility of undesirable system oscillation(hunting)
4	The output will not very for a constant input, provided the system parameters remain unaltered	In the system the output may vary for a constant input, depending upon the feedback
5	System output variation due to variation in parameters of the system is greater and the output very in an uncontrolled way	System output variation due to variation in parameters of the system is less.
6	Error detection is not present	Error detection is present
7	Small bandwidth	Large bandwidth
8	More stable	Less stable or prone to instability
9	Affected by non-linearities	Not affected by non-linearities
10	Very sensitive in nature	Less sensitive to disturbances
11	Simple design	Complex design
12	Cheap	Costly

1.4. Servomechanism

It is the feedback unit used in a control system. In this system, the control variable is a mechanical signal such as position, velocity or acceleration. Here, the output signal is directly fed to the comparator as the feedback signal, b(t) of the closed-loop control system. This type of system is used where both the command and output signals are mechanical in nature. A position control system as shown in Fig.1.3 is a simple example of this type mechanism. The block diagram of the servomechanism of an automatic steering system is shown in Fig.1.4.

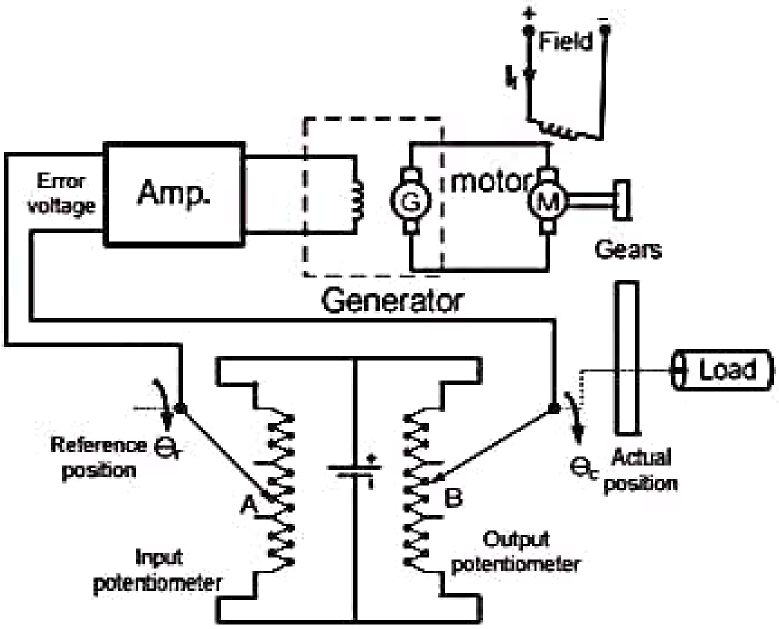


Fig.1.3. Schematic diagram of a servomechanism

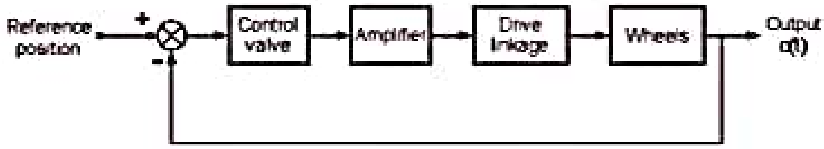


Fig.1.4. Block diagram of a servomechanism

Examples:

- Missile launcher
- Machine tool position control
- Power steering for an automobile
- Roll stabilization in ships, etc.

1.5. Regulators

It is also a feedback unit used in a control system like servomechanism. But, the output is kept constant at its desired value. The schematic diagram of a regulating system is shown in Fig.1.5. Its corresponding simplified block diagram model is shown in Fig.1.6.

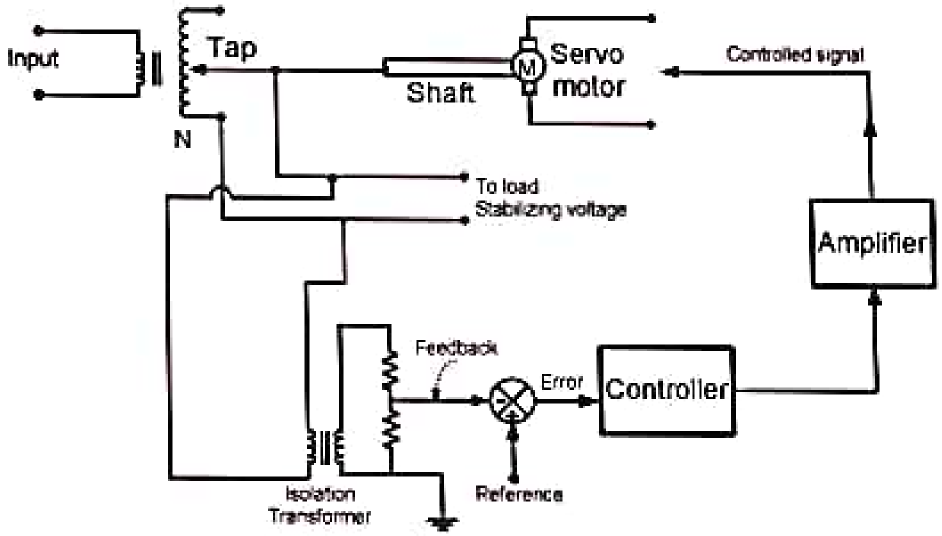


Fig.1.5. Schematic diagram of a regulating system

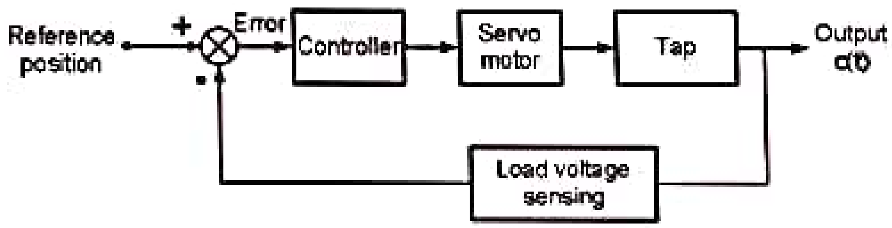


Fig.1.6. Block diagram of a regulating system

Examples:

- Temperature regulator
- Speed governor
- Frequency regulators, etc.

CHAPTER#2

2. Control System Dynamics

2.1. Definition: It is the study of characteristics behaviour of dynamic system, i.e.

(a) Differential equation

i. First-order systems
ii. Second-order systems

(b) System transfer function: Laplace transform

2.2. Laplace Transform: Laplace transforms convert differential equations into algebraic equations. They are related to frequency response.

$$L\{x(t)\} = X(s) = \int_0^{\infty} x(t)e^{-st}\,dt \quad (2.1)$$

$$L\{x(t)\} = X(s) = \int_0^{\infty} x(t)e^{-st}\,dt \quad (2.2)$$

No.	Function	Time-domain $x(t) = \mathcal{L}^{-1}\{X(s)\}$	Laplace domain $X(s) = \mathcal{L}\{x(t)\}$
1	Delay	$\delta(t-\tau)$	$e^{-s\tau}$
2	Unit impulse	$\delta(t)$	1
3	Unit step	$u(t)$	$\frac{1}{s}$
4	Ramp	t	$\frac{1}{s^2}$
5	Exponential decay	$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
6	Exponential approach	$\left(1-e^{-\alpha t}\right)$	$\frac{\alpha}{s(s+\alpha)}$
7	Sine	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
8	Cosine	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
9	Hyperbolic sine	$\sinh \alpha t$	$\frac{\alpha}{s^2-\alpha^2}$
10	Hyperbolic cosine	$\cosh \alpha t$	$\frac{s}{s^2-\alpha^2}$
11	Exponentially decaying sine wave	$e^{-\alpha t}\sin \omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
12	Exponentially decaying cosine wave	$e^{-\alpha t}\cos \omega t$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

2.3. Solution of system dynamics in Laplace form: Laplace transforms can be solved using partial fraction method.

A system is usually represented by following dynamic equation.

$$N(s) = \frac{A(s)}{B(s)} \quad (2.3)$$

The factor of denominator, B(s) is represented by following forms,

i. Unrepeated factors

ii. Repeated factors
iii. Unrepeated complex factors

(i) Unrepeated factors

$$\frac{N(s)}{(s+a)(s+b)}=\frac{A}{s+a}+\frac{B}{s+b}=\frac{A(s+b)+B(s+a)}{(s+a)(s+b)} \tag{2.4}$$

By equating both sides, determine A and B.

Example 2.1:
Expand the following equation of Laplacetransform in terms of its partial fractionsand obtain its time-domain response.

$$Y(s)=\frac{2s}{(s+1)(s+2)}$$

Solution:
The following equation in Laplacetransform is expandedwith its partial fractions as follows.

$$\frac{2s}{(s+1)(s+2)}=\frac{A}{(s+1)}+\frac{B}{(s+2)}$$

$$\Rightarrow \frac{2s}{(s+1)(s+2)}=\frac{A(s+2)+B(s+1)}{(s+1)(s+2)}$$

By equating both sides, A and B are determined as $A=-2, B=4$. Therefore,

$$Y(s)=-\frac{2}{(s+1)}+\frac{4}{(s+2)}$$

Taking Laplace inverse of above equation,

$$y(t)=-2e^{-t}+4e^{-2t}$$

(ii) Unrepeated factors

$$\frac{N(s)}{(s+a)^2}=\frac{A}{(s+a)^2}+\frac{B}{(s+a)}=\frac{A+B(s+a)}{(s+a)^2} \tag{2.5}$$

By equating both sides, determine A and B.

Example 2.2:
Expand the following equation of Laplacetransform in terms of its partial fractionsand obtain its time-domain response.

$$Y(s)=\frac{2s}{(s+1)^2(s+2)}$$

Solution:
The following equation in Laplacetransform is expandedwith its partial fractions as follows.

$$\frac{2s}{(s+1)^2(s+2)}=\frac{A}{(s+1)^2}+\frac{B}{(s+1)}+\frac{C}{(s+2)}$$

By equating both sides, A and B are determined as $A=-2, B=4$. Therefore,

$$Y(s)=-\frac{2}{(s+1)^2}+\frac{4}{(s+1)}-\frac{4}{(s+2)}$$

Taking Laplace inverse of above equation,

$$y(t)=-2te^{-t}+4e^{-t}-4e^{-2t}$$

(iii) Complex factors: They contain conjugate pairs in the denominator.

$$\frac{N(s)}{(s+a)(s+\bar{a})}=\frac{As+B}{(s+\alpha)^2+\beta^2} \tag{2.6}$$

By equating both sides, determine A and B.

Example 2.3:

Expand the following equation of Laplacetransform in terms of its partial fractionsand obtain its time-domain response.

$$Y(s)=\frac{2s+1}{(s+1+j)(s+1-j)}$$

Solution:

The following equation in Laplacetransform is expandedwith its partial fractions as follows.

$$Y(s)=\frac{2s}{(s+1)^2+1}+\frac{1}{(s+1)^2+1}$$

Taking Laplace inverse of above equation,

$$y(t)=2e^{-t}\cos t+e^{-t}\sin t$$

2.4. Initial value theorem:

$$\lim_{t\to 0}[y(t)]=\lim_{s\to\infty}\left[sY(s)\right] \tag{2.7}$$

Example 2.4:

Determine the initial value of the time-domain response of the following equation using the initial-value theorem.

$$Y(s)=\frac{2s+1}{(s+1+j)(s+1-j)}$$

Solution:

Solution of above equation,

$$y(t)=2e^{-t}\cos t+e^{-t}\sin t$$

Applying initial value theorem,

$$\lim_{s\to\infty}\frac{s(2s+1)}{(s+1+j)(s+1-j)}=2$$

2.5. Final value theorem:

$$\lim_{t\to\infty}(y(t))=\lim_{s\to 0}\left[sY(s)\right] \tag{2.8}$$

Example 2.5:

Determine the initial value of the time-domain response of the following equation using the initial-value theorem.

$$Y(s)=\frac{2s}{(s+1)^2(s+2)}$$

Solution:

Solution of above equation,

$$y(t) = -2te^{-t} + 4e^{-t} - 4e^{-2t}$$

Applying final value theorem,

$$\lim_{s \to \infty} \frac{s(2s+1)}{(s+1+j)(s+1-j)} = 2$$

CHAPTER#3

3. Transfer Function

3.1. Definition: It is the ratio of Laplace transform of output signal to Laplace transform of input signal assuming all the initial conditions to be zero. i.e.

Let, there is a given system with input r(t) and output c(t) as shown in Fig.3.1 (a), then its Laplace domain is shown in Fig.3.1 (b). Here, input and output are R(s) and C(s) respectively.

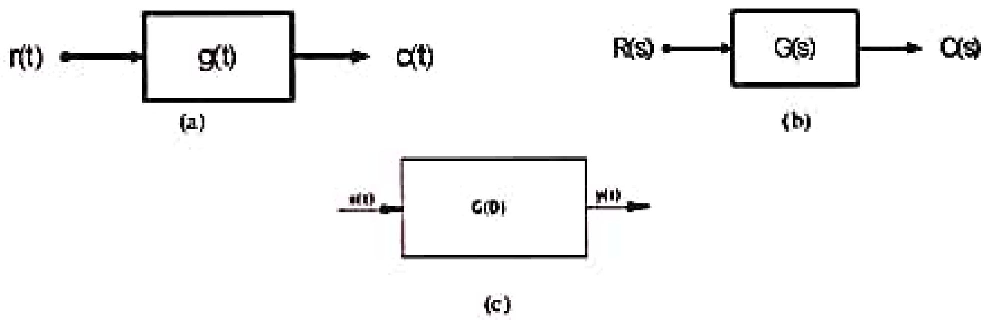


Fig.3.1.(a) A system in time domain, (b) a system in frequency domainand (c) transfer function with differential operator

G(s) is the transfer function of the system. It can be mathematically represented as follows.

$$G(s)=\left.\frac{C(s)}{R(s)}\right|_{zero\ \ initial\ \ condition}$$ Equation Section (Next)(3.1)

Example 3.1: Determine the transfer function of the system shown inFig.3.2.

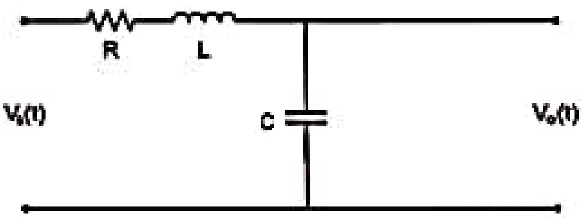


Fig.3.2 a system in time domain

Solution:

Fig.3.1 is redrawn in frequency domain as shown in Fig.3.2.

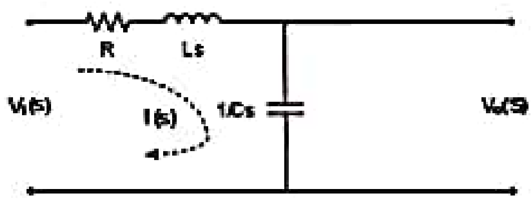


Fig.3.2 a system in frequency domain

Applying KVL to loop-1 of the Fig.3.2

$$V_i(s)=\left(R+Ls+\frac{1}{Cs}\right)I(s) \tag{3.2}$$

Applying KVL to loop-2 of the Fig.3.2

$$V_o(s)=\left(\frac{1}{Cs}\right)I(s) \tag{3.3}$$

From eq (2.12),

$$I(s)=V_o(s)/\left(\frac{1}{Cs}\right)=CsV_o(s) \tag{3.4}$$

Now, using eq (2.13) in eq (2.10),

$$V_i(s)=\left(R+Ls+\frac{1}{Cs}\right)CsV_o(s)$$
$$\Rightarrow \frac{V_o(s)}{V_i(s)}=\frac{1}{\left(R+Ls+\frac{1}{Cs}\right)Cs}=\frac{1}{LCs^2+RCs+1} \tag{3.5}$$

Then transfer function of the given system is

$$G(s)=\frac{1}{LCs^2+RCs+1} \tag{3.6}$$

3.2. General Form of Transfer Function

$$G(s)=\frac{K(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}=K\frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}^{n}(s-z_j)} \tag{3.7}$$

Where, z_1, $z_2 \ldots z_m$ are called zeros and p_1, $p_2 \ldots p_n$ are called poles.

Number of poles n will always be greater than the number of zeros m

Example 3.2:

Obtain the pole-zero map of the following transfer function.

$$G(s)=\frac{(s-2)(s+2+j4)(s+2-j4)}{(s-3)(s-4)(s-5)(s+1+j5)(s+1-j5)}$$

Solution:

The following equation in Laplacetransform is expandedwith its partial fractions as follows.

Zeros	Poles
s=2	s=3
s=-2-j4	s=4
s=-2+j4	s=5

	s=-1-j5
	s=-1+j5

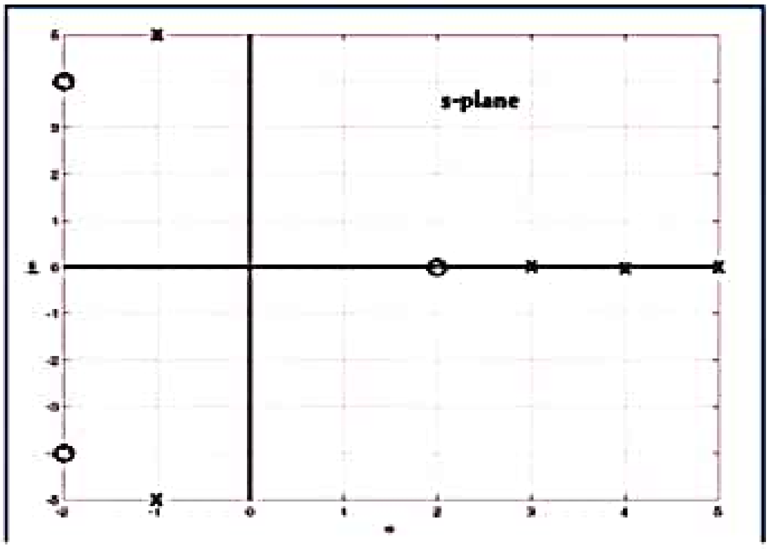


Fig 3.3. pole-zero map

3.3. Properties of Transfer function:

- Zero initial condition
- It is same as Laplace transform of its impulse response
- Replacing 's' by $\frac{d}{dt}$ in the transfer function, the differential equation can be obtained
- Poles and zeros can be obtained from the transfer function
- Stability can be known
- Can be applicable to linear system only

3.4. Advantages of Transfer function:

- It is a mathematical model and gain of the system
- Replacing 's' by $\frac{d}{dt}$ in the transfer function, the differential equation can be obtained
- Poles and zeros can be obtained from the transfer function
- Stability can be known
- Impulse response can be found

3.5. Disadvantages of Transfer function:

- Applicable only to linear system
- Not applicable if initial condition cannot be neglected
- It gives no information about the actual structure of a physical system

CHAPTER#4

4. Description of physical system

4.1. Components of a mechanical system:

Mechanical systems are of two types, i.e. (i) translational mechanical system and (ii) rotational mechanical system.

4.1.1. Translational mechanical system

There are three basic elements in a translational mechanical system, i.e. (a) mass, (b) spring and (c) damper.

(a) **Mass:** A mass is denoted by M. If a force f is applied on it and it displays distance x, then $f = M\frac{d^2x}{dt^2}$ as shown in Fig.4.1.

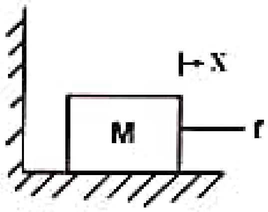


Fig.4.1. Force applied on a mass with displacement in one direction

If a force f is applied on a massM and it displays distance $x_1$in the direction of f and distance x_2 in the opposite direction, then $f = M\left(\frac{d^2x_1}{dt^2} - \frac{d^2x_2}{dt^2}\right)$ as shown in Fig.4.2.

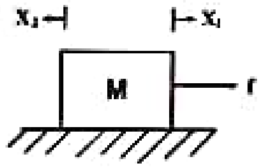


Fig.4.2. Force applied on a mass with displacement two directions

(b) **Spring:** A spring is denoted by K. If a force f is applied on it and it displays distance x, then $f = Kx$ as shown in Fig.4.3.

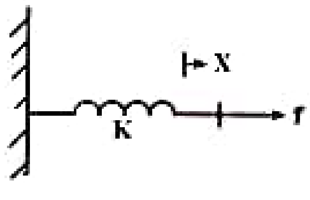


Fig.4.3. Force applied on a spring with displacement in one direction

If a force f is applied on a springK and it displays distance $x_1$in the direction of f and distance x_2 in the opposite direction, then $f = K(x_1 - x_2)$ as shown in Fig.4.4.

Fig.4.4. Force applied on a spring with displacement in two directions

(c) **Damper:** A damper is denoted by D. If a force f is applied on it and it displays distance x, then $f = D\frac{dx}{dt}$ as shown in Fig.4.5.

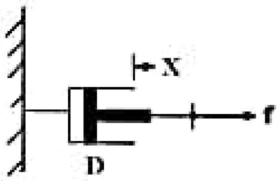


Fig.4.5. Force applied on a damper with displacement in one direction

If a force f is applied on a damper D and it displays distance x_1 in the direction of f and distance x_2 in the opposite direction, then $f = D\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right)$ as shown in Fig.4.6.

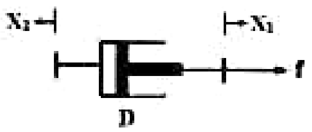


Fig.4.6. Force applied on a damper with displacement in two directions

4.1.2. Rotational mechanical system

There are three basic elements in a Rotational mechanical system, i.e. (a) inertia, (b) spring and (c) damper.

(a) **Inertia:** A body with an inertia is denoted by J. If a torque T is applied on it and it displays distance θ, then $T = J\frac{d^2\theta}{dt^2}$. If a torque T is applied on a body with inertia J and it displays distance θ_1 in the direction of T and distance θ_2 in the opposite direction, then $T = J\left(\frac{d^2\theta_1}{dt^2} - \frac{d^2\theta_2}{dt^2}\right)$.

(b) **Spring:** A spring is denoted by K. If a torque T is applied on it and it displays distance θ, then $T = K\theta$. If a torque T is applied on a body with inertia J and it displays distance θ_1 in the direction of T and distance θ_2 in the opposite direction, then $T = K(\theta_1 - \theta_2)$.

(c) **Damper:** A damper is denoted by D. If a torque T is applied on it and it displays distance θ, then $T = D\frac{d\theta}{dt}$. If a torque T is applied on a body with inertia J and it displays distance θ_1 in the direction of T and distance θ_2 in the opposite direction, then $T = D\left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right)$.

4.2. Components of an electrical system: There are three basic elements in an electrical system, i.e. (a) resistor (R), (b) inductor(L) and (c) capacitor (C). Electrical systems are of two types, i.e. (i) voltage source electrical system and (ii) current source electrical system.

4.2.1. Voltage source electrical system: If i is the current through a resistor(Fig.4.7) and v is the voltage drop in it, then $v = Ri$.

If i is the current through an inductor (Fig.4.7) and v is the voltage developed in it, then $v = L\frac{di}{dt}$.

If i is the current through a capacitor(Fig.4.7) and v is the voltage developed in it, then $v = \frac{1}{C}\int idt$.

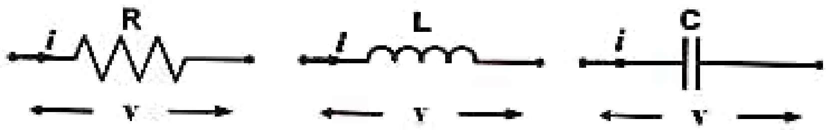


Fig.4.7. Current and voltage shown in resistor, inductor and capacitor

4.2.2. Current source electrical system:

If i is the current through a resistor and v is the voltage drop in it, then $i = \frac{v}{R}$.

If i is the current through an inductor and v is the voltage developed in it, then $i = \frac{1}{L}\int vdt$.

If i is the current through a capacitor and v is the voltage developed in it, then $i = C\frac{dv}{dt}$.

4.2.3. Work out problems:

Q.4.1. Find system transfer function betweenvoltage drop across the capacitanceand input voltage in the followingRC circuit as shown in Fig.4.8.

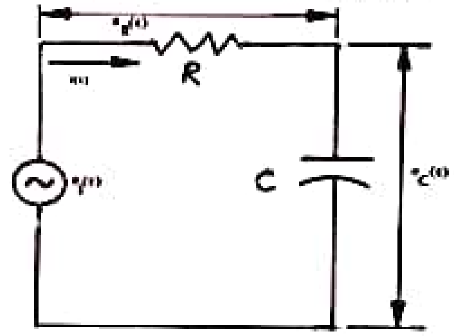


Fig.4.8.

Solution

Voltage across resistance, $e_R(t) = i(t)R$

Voltage across capacitance, $e_C(t) = \frac{1}{C}\int i(t)dt$

Total voltage drop, $e_i = e_R + e_C = i(t)R + \frac{1}{C}\int i(t)dt$

Laplace transform of above equation, $E_i(s) = I(s)\left(R + \frac{1}{Cs}\right)$

System transfer function betweenvoltage drop across the capacitanceand input voltage, $\frac{E_C(s)}{E_i(s)} = \frac{1}{RCs+1} = \frac{1}{\tau s+1}$

where, $RC = \tau$ is the time-constant

Q.4.2. Find system transfer function betweenfunction between the inductance currentto the source currentin the followingRL circuit as shown in Fig.4.9.

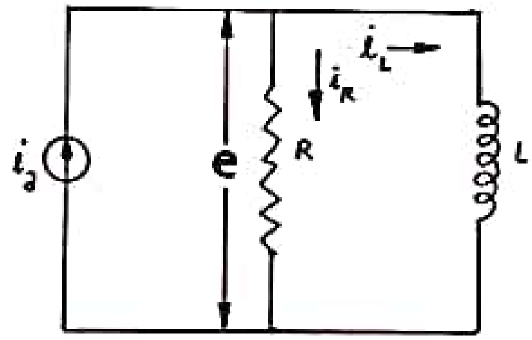


Fig.4.9.

Voltage across the Resistance, $e(t) = i_R R \Rightarrow i_R = \frac{e(t)}{R}$

Voltage across the Inductance, $e(t) = L\frac{di_L}{dt} \Rightarrow i_L = \frac{1}{L}\int e(t)dt$

Total current, $i_a = i_R + i_L = \frac{e(t)}{R} + \frac{1}{L}\int e(t)dt$

Laplace transform of the current source,

$I_a(s) = E(s)\left(\frac{1}{R} + \frac{1}{Ls}\right)$ and $I_L(s) = \frac{E}{Ls}$

Transfer function between the inductance current to the source current,

$$\frac{I_L(s)}{I_a(s)} = \frac{1}{\frac{L}{R}s+1} = \frac{1}{\tau s+1}$$

where $\tau = \frac{L}{R}$ is the time-constant

Q.4.3. Find system transfer function betweenfunction between the capacitance voltageto the source voltage in the followingRLC circuit as shown in Fig.4.10.

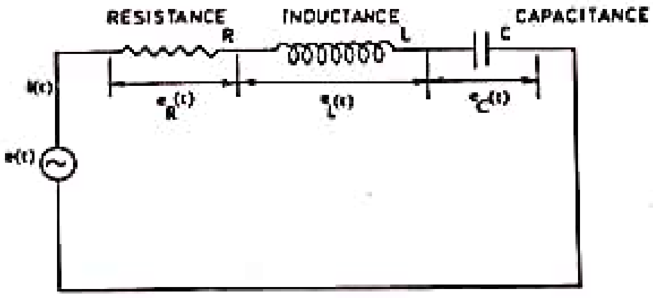


Fig.4.10.

Voltage across the Resistance, $e_R(t) = iR$

Voltage across the Inductance, $e_L(t) = L\frac{di}{dt}$

Voltage across thecapacitance, $e_C(t) = \frac{1}{C}\int idt$

Total voltage, $e(t) = iR + L\frac{di}{dt} + \frac{1}{C}\int idt$

Laplace transform of the voltage source, $E(s) = I(s)\left(R + Ls + \frac{1}{Cs}\right)$

Transfer function between capacitance voltage and source voltage

$$\frac{E_C(s)}{E(s)} = \frac{1}{Cs\left(R + Ls + \frac{1}{Cs}\right)} = \frac{\omega_n^2}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

where $\omega_n = \frac{1}{\sqrt{LC}}$ and $\zeta = \frac{R}{2\sqrt{\frac{L}{C}}}$

Q.4.4.Find the transfer function of the following Spring-mass-damperas shown in Fig.4.11.

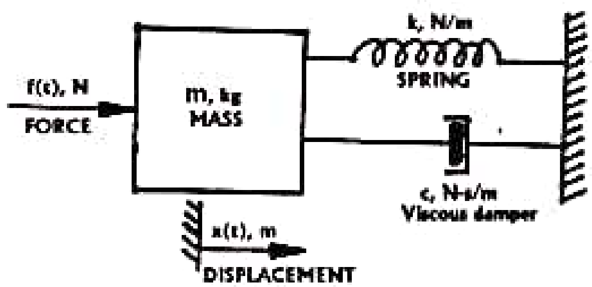


Fig.4.11.

Solution

$$\frac{X(s)}{F(s)}=\frac{1}{ms^2+cs+k}=\frac{1}{m\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$$

4.3. Analogous system: Fig.4.12 shows a translational mechanical system, a rotational control system and a voltage-source electrical system.

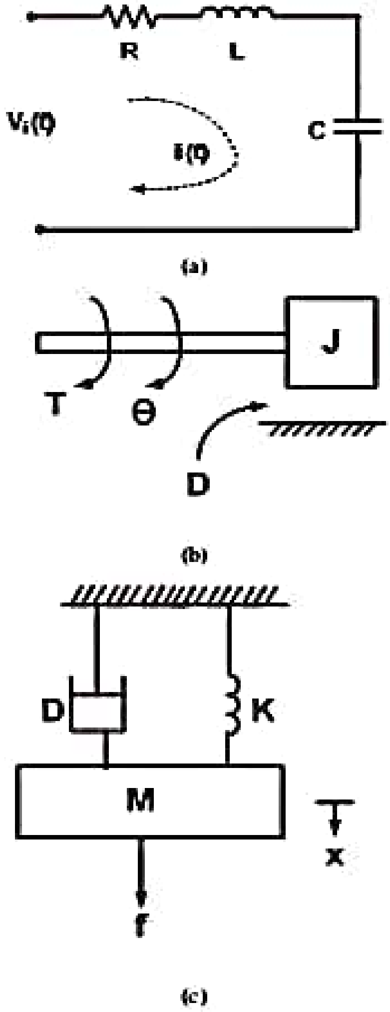


Fig.4.12. (a) a voltage-source electrical system,(b) a translational mechanical system and (c) a rotational control system

From Fig4. 12 (a), (b) and (c), we have

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=v(t)$$

$$J\frac{d^2\theta}{dt^2}+D\frac{d\theta}{dt}+K\theta=T \qquad \text{Equation Chapter 8 Section 0}(4.1)$$

$$M\frac{d^2x}{dt^2}+D\frac{dx}{dt}+Kx=f$$

Where,

$$q = \int i dt \tag{4.2}$$

The solutions for all the above three equations given by eq (4.2) are same. Therefore, the above shown three figures are analogous to each other. There are two important types of analogous systems, i.e. force-voltage (f-v) analogy and force-current analogy. From eq (4.2), f-v analogy can be drawn as follows.

Translational	Rotational	Electrical
Force (f)	Torque (T)	Voltage (v)
Mass (M)	Inertia (J)	Inductance (L)
Damper (D)	Damper (D)	Resistance (R)
Spring (K)	Spring (K)	Elastance (1/C)
Displacement (x)	Displacement (Θ)	Charge (q)
Velocity (u) = $\dot{x}$	Velocity (u) = $\dot{\theta}$	Current (i) = $\dot{q}$

Similarly, f-i analogy that can be obtainedfrom eq (4.1), can be drawn as follows.

Translational	Rotational	Electrical
Force (f)	Torque (T)	Current (i)
Mass (M)	Inertia (J)	Capacitance (C)
Damper (D)	Damper (D)	Conductance (1/R)
Spring (K)	Spring (K)	Reciprocal of Inductance (1/L)
Displacement (x)	Displacement (Θ)	Flux linkage (ψ)
Velocity (u) = $\dot{x}$	Velocity (u) = $\dot{\theta}$	Voltage (v) = $\dot{\psi}$

4.4. Mathematical model of armature controlled DC motor: The armature control type speed control system of a DC motor is shown in Fig.4.6. The following components are used in this system.

R_a=resistance of armature

L_a=inductance of armature winding

i_a=armature current

I_f=field current

E_a=applied armature voltage

E_b=back emf

T_m=torque developed by motor

Θ=angular displacement of motor shaft

J=equivalent moment of inertia and load referred to motor shaft

f=equivalent viscous friction coefficient of motor and load referred to motor shaft

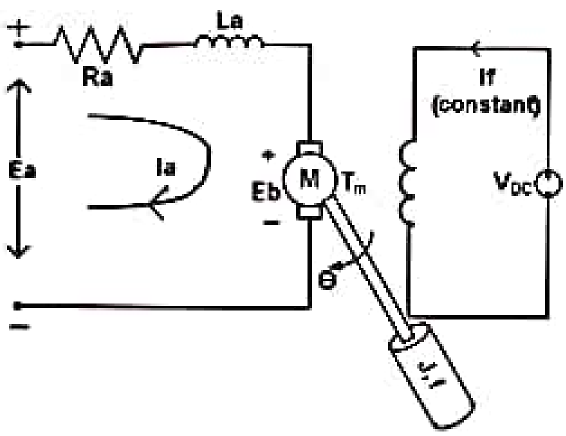


Fig.4.6. Schematic diagram of armature control type speed control system of a DC motor

The air-gap flux ϕ is proportional of the field current i.e.

$$\phi = K_f I_f \tag{4.3}$$

The torque T_m developed by the motor is proportional to the product of armature current and air gap flux i.e.

$$T_m = k_1 K_f I_f i_a \tag{4.4}$$

In armature-controlled D.C. motor,the field current is kept constant,so that eq(4.4) can be written as follows.

$$T_m = K_t i_a \tag{4.5}$$

The motor back emf being proportional to speed is given as follows.

$$E_b = K_b \left(\frac{d\theta}{dt}\right) \tag{4.6}$$

The differential equation of the armature circuit is

$$L_a\left(\frac{di_a}{dt}\right) + R_a i_a + E_b = E_a \tag{4.7}$$

The torque equation is

$$J\left(\frac{d^2\theta}{dt^2}\right) + f\left(\frac{d\theta}{dt}\right) = T_m = K_t I_a \tag{4.8}$$

Taking the Laplace transforms of equations (4.6), (4.7) and (4.8), assuming zero initial conditions, we get

$$E_b(s) = sK_b\theta(s) \tag{4.9}$$

$$(sL_a + R_a)I_a(s) = E_a(s) - E_b(s) \tag{4.10}$$

$$(s^2 J + sf)\theta(s) = T_m(s) = K_t I_a \tag{4.11}$$

From eq(4.9) to (4.11) the transfer function of the system is obtained as,

$$G(s)=\frac{\theta(s)}{E_a(s)}=\frac{K_t}{s\left[(R_a+sL_a)(sJ+f)+K_tK_b\right]} \qquad (4.12)$$

Eq(4.12) can be rewritten as

$$G(s)=\frac{\theta(s)}{E_a(s)}=\left[\frac{\dfrac{K_t}{(R_a+sL_a)(sJ+f)}}{1+\dfrac{K_tK_b}{(R_a+sL_a)(sJ+f)}}\right]\frac{1}{s} \qquad (4.13)$$

The block diagram that is constructed from eq (4.13) is shown in Fig.4.7.

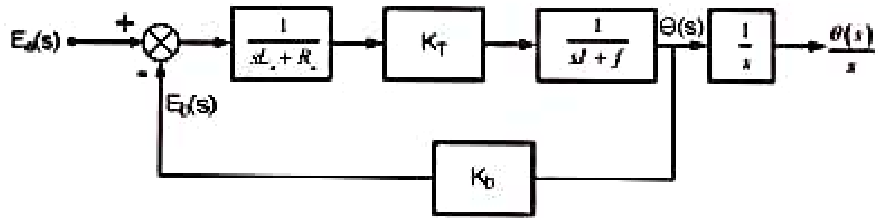


Fig.4.7. Block diagram of armature control type speed control system of a DC motor

The armature circuit inductance L_a is usually negligible. Therefore, eq(4.13) can be simplifiedas follows.

$$\frac{\theta(s)}{E_a(s)}=s^2\left(\frac{K_t}{R_a}\right)J+s\left(f+\frac{K_tK_b}{R_a}\right) \qquad (4.14)$$

The term $\left(f+\frac{K_tK_b}{R_a}\right)$ indicates that the back emf of the motor effectively increases the viscous friction of the system. Let,

$$f'=f+\frac{K_tK_b}{R_a} \qquad (4.15)$$

Where f' be the effective viscous friction coefficient. The transfer function given by eq(4.15) may be written in the following form.

$$\frac{\theta(s)}{E_a(s)}=\frac{K_m}{s(s\tau+1)} \qquad (4.16)$$

Here $K_m=\frac{K_t}{R_af}$ = motor gain constant, and $\tau=\frac{J}{f'}$ = motor time constant.Therefore, the motor torque and back emf constant K_t, K_b are interrelated.

4.5. Mathematical model of field controlled DC motor: The field control type speed control system of a DC motor is shown in Fig.4.8. The following components are used in this system.

R_f=Field winding resistance

L_f=inductance of field winding

I_f=field current

e_f=field control voltage

T_m=torque developed by motor

Θ=angular displacement of motor shaft

J=equivalent moment of inertia and load referred to motor shaft

f=equivalent viscous friction coefficient of motor and load referred to motor shaft

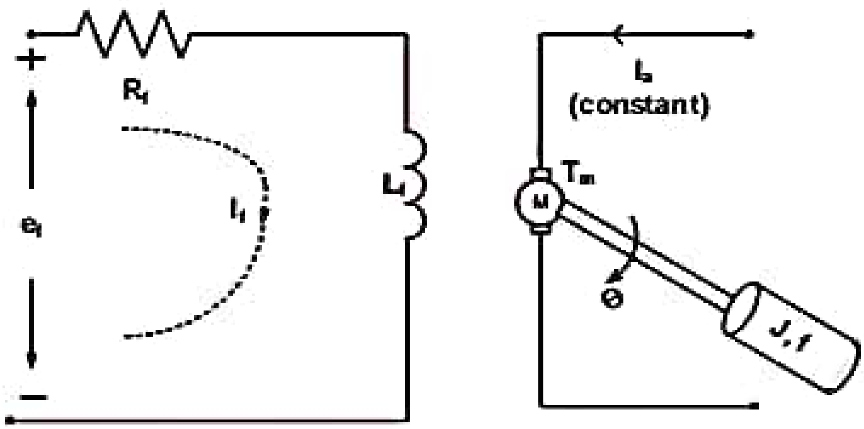


Fig.4.8. Block diagram of field control type speed control system of a DC motor

In field control motor the armature current is fed from a constant current source.The air-gap flux Φ is proportional of the field current i.e.

$$\phi = K_f I_f \tag{4.17}$$

The torque Tm developed by the motor is proportional to the product of armature current and air gap flux i.e.

$$T_m = k_1 K_f I_f I_a = K_t I_f \tag{4.18}$$

The equation for the field circuit is

$$L_f \frac{dI_f}{dt} + R_f I_f = E_f \tag{4.19}$$

The torque equation is

$$J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} = T_m = K_t I_f \tag{4.20}$$

Taking the Laplace transforms of equations (4.19) and (4.20) assuming zero initial conditions, we get the following equations

$$\left(L_f s + R_f\right) I_f(s) = E_f(s) \tag{4.21}$$

and

$$\left(Js^2 + fs\right)\theta(s) = T_m(s) = K_t I_f(s) \tag{4.22}$$

From eq(4.21) and (4.22) the transfer function of the system is obtained as

$$G(s) = \frac{\theta(s)}{E_f(s)} = \frac{K_t}{s\left(R_f + sL_f\right)(Js + f)} \tag{4.23}$$

The transfer function given by eq(4.23) may be written in the following form.

$$\frac{\theta(s)}{E_a(s)} = \frac{K_t}{s\left(L_f s + R_f\right)(Js + f)} = \frac{K_m}{s(s\tau + 1)(s\tau' + 1)} \tag{4.24}$$

Here $K_m = \frac{K_t}{R_f f}$ = motor gain constant, and $\tau = \frac{L_f}{R_f}$ = time constant of field circuit and $\tau' = \frac{J}{f}$ = mechanical time constant.For small size motors field control is advantageous.The block diagram that is constructed from eq (4.24) is shown in Fig.4.9.

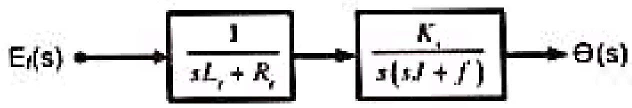


Fig.4.9. Block diagram of field control type speed control system of a DC motor

CHAPTER#5

5. Block Diagram Algebra

5.1. Basic Definition in Block Diagram model:

Block diagram: It is the pictorial representation of the cause-and-response relationship between input and output of a physical system.

Fig.5.1. (a) A Block diagram representation of a system and (b) A Block diagram representation with gain of a system

Output: The value of input multiplied by the gain of the system.

$$C(s)=G(s)R(s) \tag{5.1}$$

Summing point: It is the component of a block diagram model at which two or more signals can be added or subtracted. In Fig.15, inputs **R(s)** and **B(s)** have been given to a summing point and its output signal is **E(s)**. Here,

$$E(s)=R(s)-B(s) \tag{5.2}$$

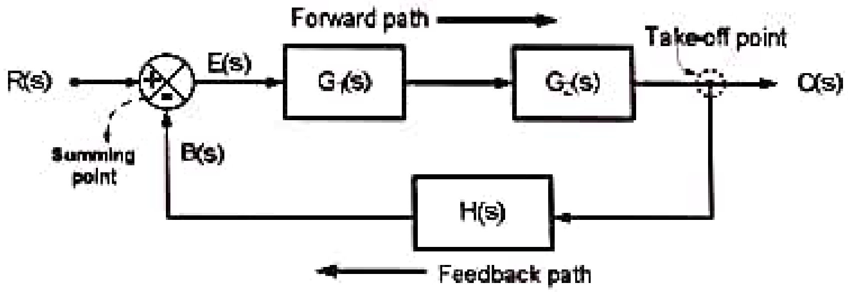


Fig.5.2. A block diagram representation of a system showing its different components

Take-off point: It is the component of a block diagram model at which a signal can be taken directly and supplied to one or more points as shown in Fig.5.2.

Forward path: It is the direction of signal flow from input towards output.

Feedback path: It is the direction of signal flow from output towards input.

5.2. Developing Block Diagram model from mathematical model:

Let's discuss this concept with the following example.

Example: A system is described by following mathematical equations. Find its corresponding block diagram model.

$$\dot{x}_1 = 3x_1 + 2x_2 + 5x_3 \tag{5.3}$$

$$\dot{x}_2 = x_1 + 4x_2 + 3x_3 \tag{5.4}$$

$$\dot{x}_3 = 2x_1 + x_2 + x_3 \qquad (5.5)$$

Example: Eq (5.3), (5.4) and (5.5) are combiningly results in the following block diagram model.

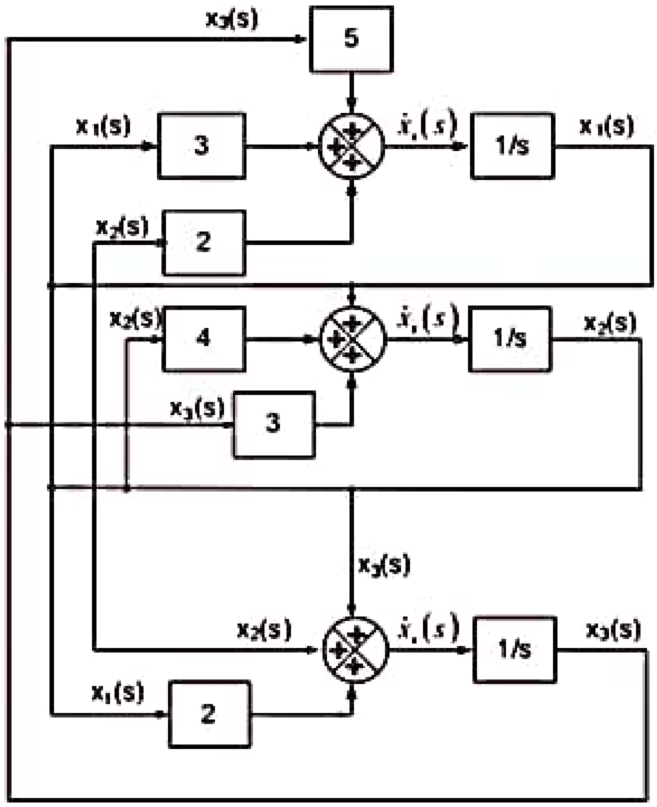
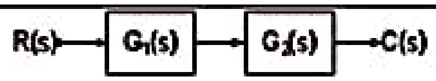
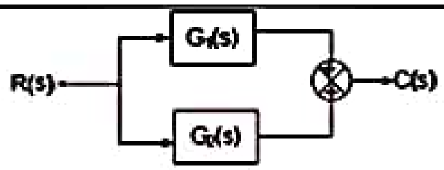
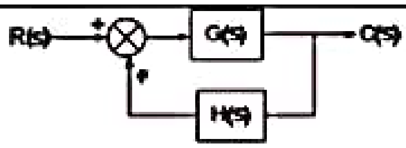
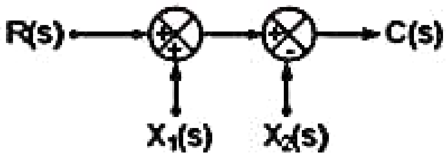
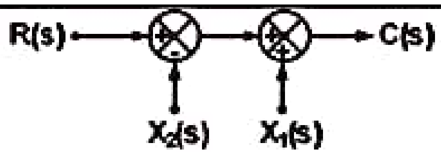
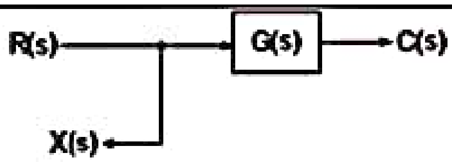
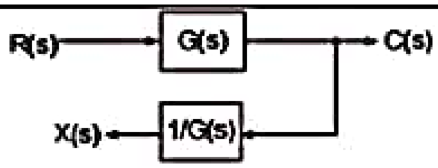
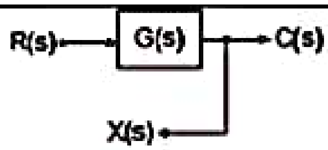
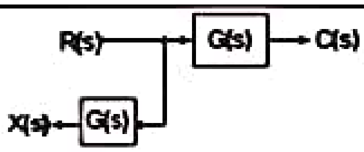
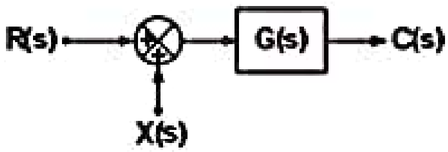
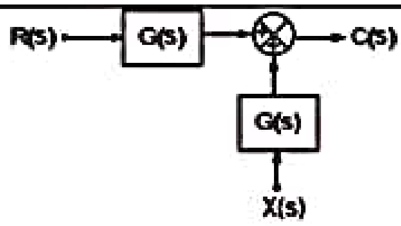
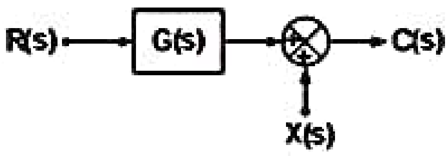
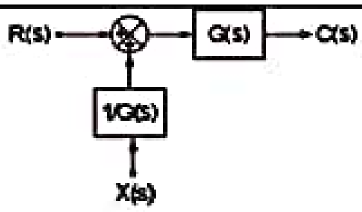


Fig.5.3. A block diagram representation of the above example

5.3. Rules for reduction of Block Diagram model:

Sl. No.	Rule No.	Configuration	Equivalent	Name
1	Rule 1			Cascade
2	Rule 2			Parallel
3	Rule 3			Loop
4	Rule 4			Associative Law
5	Rule 5			Move take-off point after a block
6	Rule 6			Move take-off point before a block
7	Rule 7			Move summing-point point after a block
8	Rule 8			Move summing-point point before a block

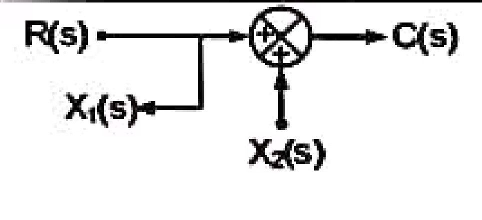
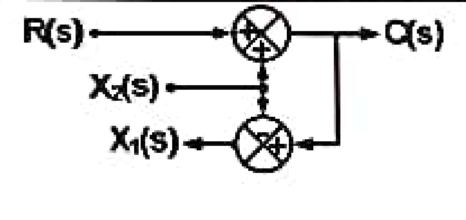
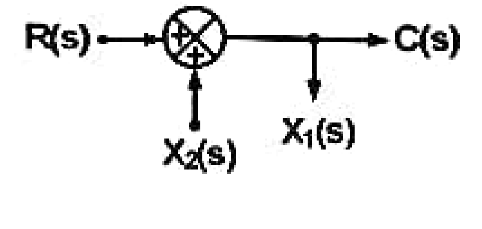
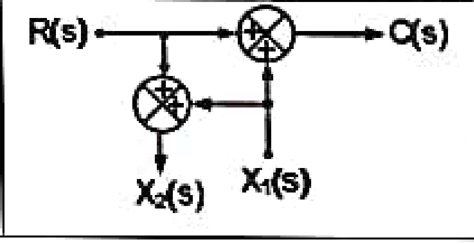
9	Rule 9			Move take-off point after a summing-point
10	Rule 10			Move take-off point before a summing-point

Fig.5.4. Rules for reduction of Block Diagram model

5.4. Procedure for reduction of Block Diagram model:

Step 1: Reduce the cascade blocks.

Step 2: Reduce the parallel blocks.

Step 3: Reduce the internal feedback loops.

Step 4: Shift take-off points towards right and summing points towards left.

Step 5: Repeat step 1 to step 4 until the simple form is obtained.

Step 6: Find transfer function of whole system as $\frac{C(s)}{R(s)}$.

5.5. Procedure for finding output of Block Diagram model with multiple inputs:

Step 1: Consider one input taking rest of the inputs zero, find output using the procedure described in section 4.3.

Step 2: Follow step 1 for each inputs of the given Block Diagram model and find their corresponding outputs.

Step 3: Find the resultant output by adding all individual outputs.

CHAPTER#6

6. Signal Flow Graphs (SFGs)

It is a pictorial representation of a system that graphically displays the signal transmission in it.

6.1. Basic Definitions in SFGs:

Input or source node: It is a node that has only outgoing branches i.e. node 'r' in Fig.6.1.

Output or sink node: It is a node that has only incoming branches i.e. node 'c' in Fig.6.1.

Chain node: It is a node that has both incoming and outgoing branches i.e. nodes 'x_1', 'x_2','x_3','x_4','x_5'and 'x_6' in Fig.6.1.

Gain or transmittance: It is the relationship between variables denoted by two nodes or value of a branch. In Fig.6.1, transmittances are 't_1', 't_2','t_3','t_4','t_5'and 't_6'.

Forward path: It is a path from input node to output node without repeating any of the nodes in between them. In Fig.6.1, there are two forward paths, i.e. path-1:'r-x_1-x_2-x_3-x_4-x_5-x_6-c' and path-2:'r-x_1-x_3-x_4-x_5-x_6-c'.

Feedback path: It is a path from output node or a node near output node to a node near input node without repeating any of the nodes in between them (Fig.6.1).

Loop: It is a closed path that starts from one node and reaches the same node after trading through other nodes. In Fig.6.1, there are four loops, i.e. loop-1:'x_2-x_3-x_4-x_2', loop-2:'x_5-x_6-x_5', loop-3:'x_1-x_2-x_3-x_4-x_5-x_6-x_1' and loop-4:'x_1-x_3-x_4-x_5-x_6-x_1'.

Self Loop: It is a loop that starts from one node and reaches the same node without trading through other nodes i.e. loop in node 'x_4' with transmittance 't_{55}' in Fig.6.1.

Path gain: It is the product of gains or transmittances of all branches of a forward path. In Fig.6.1, the path gains are $P_1 = t_1t_2t_3t_4t_5$ (for path-1) and $P_2 = t_9t_3t_4t_5$ (for path-2).

Loop gain: It is the product of gains or transmittances of all branches of a loop In Fig.6.1, there are four loops, i.e. $L_1 = -t_2t_3t_6$, $L_2 = -t_5t_7$, $L_3 = -t_1t_2t_3t_4t_5t_8$, and $L_4 = -t_9t_3t_4t_5t_8$.

Dummy node: If the first node is not an input node and/or the last node is not an output node than a node is connected before the existing first node and a node is connected after the existing last node with unity transmittances. These nodes are called dummy nodes. In Fig.6.1, 'r' and 'c' are the dummy nodes.

Non-touching Loops: Two or more loops are non-touching loops if they don't have any common nodes between them. In Fig.6.1, L_1 and L_2 are non-touching loops

Example:

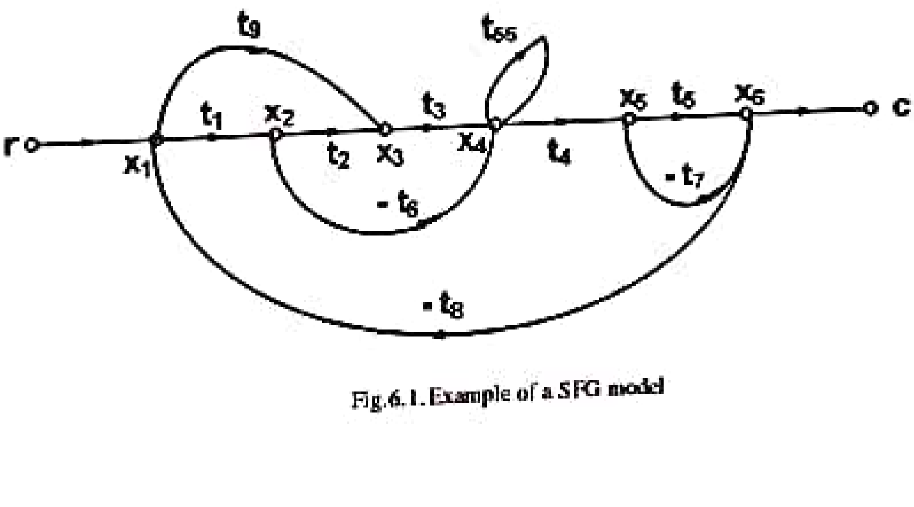


Fig.6.1. Example of a SFG model

6.2. Properties SFGs:

- Applied to linear system
- Arrow indicates signal flow
- Nodes represent variables, summing points and take-off points
- Algebraic sum of all incoming signals and outgoing nodes is zero
- SFG of a system is not unique
- Overall gain of an SFG can be determined by using Mason's gain formula

6.3. SFG from block diagram model:

Let's find the SFG of following block diagram model shown in Fig.6.2.

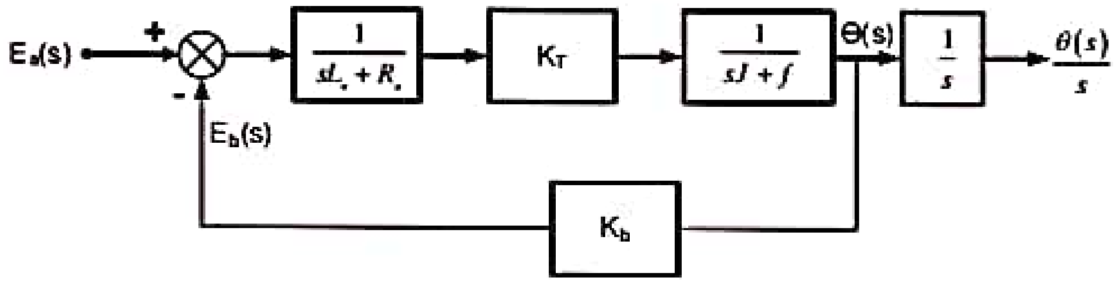


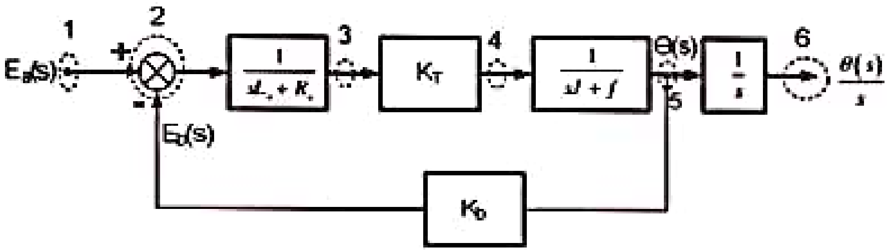
Fig.6.2. Armature type speed control of a DC motor

Step-1: All variables and signals are replaced by nodes.

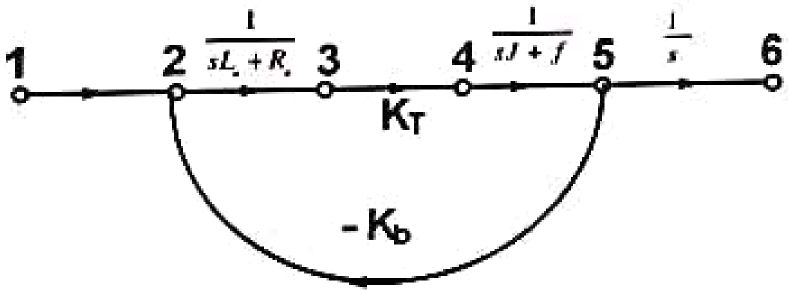
Step-2: Connect all nodes according to their signal flow.

Step-3: Each of gains is replaced by transmittances of the branches connected between two nodes of the forward paths.

Step-4: Each of gains is replaced by transmittances multiplied with (-1) of the branches connected between two nodes of the forward paths.



(a)



(b)

Fig.6.3. Armature type speed control of a DC motor

6.4. Mason's gain formula:

Transfer function of a system=

$$G(s)=\frac{C(s)}{R(s)}=\frac{\sum_{k=1}^{N} P_k \Delta_k}{\Delta} \qquad (6.1)$$

Where,

N= total number of forward paths

P_k= path gain of k^{th} forward path

Δ= 1 - ($\sum$loop gains of all individual loops) + ($\sum$gain product of loop gains of all possible two non-touching loops) - ($\sum$gain product of loop gains of all possible three non-touching loops) + ...

Δ_k= value of Δ after eliminating all loops that touches k^{th} forward path

Example:

Find the overall transfer function of the system given in Fig.6.1 using Mason's gain formula.

Solution:

In Fig.6.1,

No. of forward paths: $N=2$

Path gain of forward paths: $P_1 = t_1 t_2 t_3 t_4 t_5$ and $P_2 = t_6 t_3 t_4 t_5$

Loop gain of individual loops: $L_1 = -t_2 t_3 t_6$, $L_2 = -t_3 t_7$, $L_3 = -t_1 t_2 t_3 t_4 t_5 t_8$ and $L_4 = -t_6 t_3 t_4 t_5 t_8$

No. of two non-touching loops = 2 i.e. L_1 and L_2

No. of more than two non-touching loops = 0

$$\Delta = 1 - \left(L_1 + L_2 + L_3 + L_4\right) + \left(L_1 L_2\right) - 0 = 1 - L_1 - L_2 - L_3 - L_4 + L_1 L_2$$

$$\Delta_1 = 1 - 0 = 1 \text{ and } \Delta_2 = 1 - 0 = 1$$

$$G(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\Rightarrow G(s) = \frac{\left(t_1 t_2 t_3 t_4 t_5\right)(1) + \left(t_6 t_3 t_4 t_5\right)(1)}{1 + t_2 t_3 t_6 + t_5 t_7 + t_1 t_2 t_3 t_4 t_5 t_8 + t_9 t_3 t_4 t_5 t_8 + t_2 t_3 t_5 t_6 t_7}$$

$$\Rightarrow G(s) = \frac{t_1 t_2 t_3 t_4 t_5 + t_6 t_3 t_4 t_5}{1 + t_2 t_3 t_6 + t_5 t_7 + t_1 t_2 t_3 t_4 t_5 t_8 + t_9 t_3 t_4 t_5 t_8 + t_2 t_3 t_5 t_6 t_7}$$