# LECTURE NOTES
# ON
# FLUID MECHANICS
# AND
# INDUSTRIAL FLUID POWER

PREPARED BY
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# <u>PROPERTIES OF A FLUID AND HYDROSTATICS</u>

## Introduction:

### Fluid:
A fluid is a substance that can flow and take the shape of its container. It includes liquids and gases. Fluids cannot resist shear stress and deform continuously when subjected to it.

**Fluid Mechanics:**
Fluid Mechanics is the branch of physics that deals with the **study of fluids (liquids and gases)** and the **forces acting on them**. It includes the behavior of fluids at rest (fluid statics) and in motion (fluid dynamics).

- **Fluid Statics** – the study of fluids at rest.
- **Fluid Dynamics** – the study of fluid in motion where pressure force are consider.
- **Fluid Kinematics** – the study of fluid in motion without considering forces.

### <u>Classification:</u>

A fluid can be classified as follows:

- Liquid
- Gas

### Liquid:
It is a fluid which possesses a definite volume and assumed as incompressible

**GAS:**
It possesses no definite volume and is compressible.

Fluids are broadly classified into two types.

- Ideal fluids
- Real fluids

### <u>Ideal fluid:</u>

An ideal fluid is one which has no viscosity and surface tension and is incompressible actually no ideal fluid exists.

### <u>Real fluids:</u>

A real fluid is one which has viscosity, surface tension and compressibility in addition to the density.

## Newtonian Fluid

- A real fluid where the shear stress τ is directly proportional to the rate of shear strain (or velocity gradient, $\frac{du}{dy}$).
- The proportionality constant, dynamic viscosity (μ), is constant.

***Formula (Newton's Law of Viscosity):***

$$\tau = \mu \left( \frac{du}{dy} \right)$$

- **τ** = Shear stress (Pa or N/m²)
- μ = Dynamic viscosity (Pa·s)
- $\frac{du}{dy}$ = Velocity gradient or shear rate ($s^{-1}$)

***Explanation:***

- The shear stress is directly proportional to the shear rate.
- The fluid flows in layers, and the resistance to flow is only due to viscosity.

***Examples:***

- Water, Air, Kerosene, Alcohol, Mineral oil (at low shear rates)

## 2. Non-Newtonian Fluid

A **Non-Newtonian fluid** is a fluid whose **viscosity changes with the applied shear rate**.

***General Formula:***

$$\tau = k \left( \frac{du}{dy} \right)^{n}$$

- **k** = Flow consistency index
- $\boldsymbol{n}$ = Flow behavior index
- $\boldsymbol{n}$ **=1**: Newtonian
- $\boldsymbol{n}$ **<1**: Pseudoplastic (shear-thinning)
- $\boldsymbol{n}$ **>1**: Dilatant (shear-thickening)

***Explanation:***

- The relationship between shear stress and shear rate is **nonlinear**.
- Some fluids become thicker or thinner when stirred or shaken.

***Examples:***

- **Pseudoplastic (n < 1):** Paint, blood, toothpaste
- **Dilatant (n > 1):** Corn starch in water
- **Bingham plastic:** Mud, mayonnaise

## 3. Ideal Plastic Fluid

An **Ideal Plastic fluid** behaves like a solid until a certain **yield stress** is applied. Once that yield stress is exceeded, it flows like a Newtonian fluid.

***Formula:***

$$\tau = \tau_y + \mu \frac{du}{dy}$$

- $\tau_y$ = Yield stress (Pa)

***Explanation:***

- No flow occurs until $\tau \geq \tau_y$
- After yield stress, it behaves like a Newtonian fluid.

***Examples:***

- Toothpaste
- Bingham plastic model materials like **mud**, **ketchup**, and **slurries**

## PROPERTIES OF FLUIDS:

1. **Density or mass density: (S)**

   Density of a fluid is defined as the ratio of the mass of a fluid to its volume. It is denoted by $\rho$(rho) the density of liquids are considered as constant while that of gases changes with pressure & temperature variations.

   Mathematically

$$\rho = \frac{mass}{volume}$$

$$\text{Unit} = \frac{kg}{m^3}$$

$$\rho_{water} = 1000 \frac{kg}{m^3}$$

$$\text{Or } \frac{gm}{cm^3}$$

2. **Specific weight or weight density ((W):**

   Specific weight of a fluid is defined as the ratio between the weights of a fluid to its volume. It is denoted by W.

Mathematically (W)= $\dfrac{weight\ of\ fluid}{volume\ of\ fluid}$

= mg/v

W = $\rho g$

$$\text{Unit} = \frac{N}{m}$$

The weight density of water = 9.81×1000 $N/m^3$

3. **Specific volume:**

Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

Mathematically

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume}}} = \frac{1}{\rho}$$

Unit: $\frac{m^3}{kg}$

**4. Specific gravity:**

Specific gravity is defined as the ratio of the weight density of a fluid to the density or weight density standard fluid.

For liquids the standard fluid is water.

For gases the standard fluid is air.

It is denoted by the symbol **S**

$$\text{Mathematically,}\quad S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

$$\begin{aligned}
\text{Thus weight density of a liquid} &= S \times \text{Weight density of water} \\
&= S \times 1000 \times 9.81\ \text{N/m}^3 \\
\text{The density of a liquid} &= S \times \text{Density of water} \\
&= S \times 1000\ \text{kg/m}^3.
\end{aligned}$$

- it is a unit less quantity.

# Simple Problems:

## Problem: - 1

Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7N.

**Solution.** Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left( \because \ 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3 \right)$$

$$\text{Weight} = 7 \text{ N}$$

(*i*) Specific weight ($w$) $= \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{7 \text{ N}}{\left(\dfrac{1}{1000}\right) \text{m}^3} = $ **7000 N/m³. Ans.**

(*ii*) Density ($\rho$) $= \dfrac{w}{g} = \dfrac{7000}{9.81}$ kg/m³ = **713.5 kg/m³. Ans.**

(*iii*) Specific gravity $= \dfrac{\text{Density of liquid}}{\text{Density of water}} = \dfrac{713.5}{1000}$ $\{\because$ Density of water = 1000 kg/m³$\}$

= **0.7135. Ans.**

## Problem: - 2

Calculate the density, specific weight and specific gravity of one litre of petrol of specific gravity = 0.7

**Solution.** Given : Volume = 1 litre = $1 \times 1000 \text{ cm}^3 = \dfrac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity $\quad S = 0.7$

(*i*) *Density* ($\rho$)

Using equation (1.1.A),

*Density* ($\rho$) $= S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 =$ **700 kg/m³. Ans.**

(*ii*) *Specific weight* ($w$)

Using equation (1.1), $\quad w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 =$ **6867 N/m³. Ans.**

(*iii*) *Weight (W)*

We know that specific weight $= \dfrac{\text{Weight}}{\text{Volume}}$

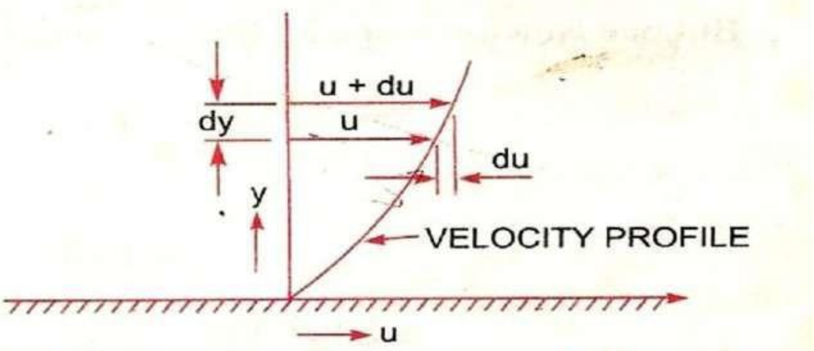
$$w = \frac{W}{0.001} \text{ or } 6867 = \frac{W}{0.001}$$

$\therefore \quad W = 6867 \times 0.001 =$ **6.867 N. Ans.**

**Viscosity:**

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

Let two layers of a fluid at a distance dy apart, move one over the other at different velocities u and u + du.




*Velocity variation near a solid boundary.*

The viscosity together with the with the relative velocity between the two layers while causes a shear stress acting between the fluid layers, the top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

This shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by $\tau$.

.

Mathematically $$\tau \;\alpha\; \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

Where $\mu$ = co-efficient of dynamic viscosity or constant of proportionality or viscosity

$$\frac{du}{dy} = \text{rate of shear strain or velocity gradient}$$

Then $\mu$ = τ

Viscosity is defined as the shear stress required to produce unit rate of shear strain.

Unit of viscosity in S.I system - $\frac{Ns}{m}$

In C.G.S = $\frac{Dyne\ \ s}{cm^2}$

In M.K.S. = $\frac{k\text{gf}s}{m^2}$

$\frac{Dyne\ s}{cm^2}$ = 1 Poise

$1\ \frac{Ns}{m^2} = 10$ poise

1 Centipoise = $\frac{1}{100}$ poise

**Kinematic Viscosity:**

It is defined as the ratio between the dynamic viscosity and density of fluid.

It is denoted by $\boldsymbol{\vartheta}$.

Mathematically

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} \qquad ...(1.4)$$

The units of kinematic viscosity is obtained as

$$\nu = \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}}$$

$$= \frac{\text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\frac{\text{Mass}}{\text{Length}}\right)} \qquad \left\{\because \text{Force} = \text{Mass} \times \text{Acc.} = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2}\right\}$$

$$= \frac{(\text{Length})^2}{\text{Time}}.$$

In MKS and SI, the unit of kinematic viscosity is metre$^2$/sec or m$^2$/sec while in CGS units it is written as cm$^2$/s. In CGS units, kinematic viscosity is also known stoke.

Thus, one stoke $= \text{cm}^2/\text{s} = \left(\frac{1}{100}\right)^2 \text{m}^2/\text{s} = 10^{-4}\ \text{m}^2/\text{s}$

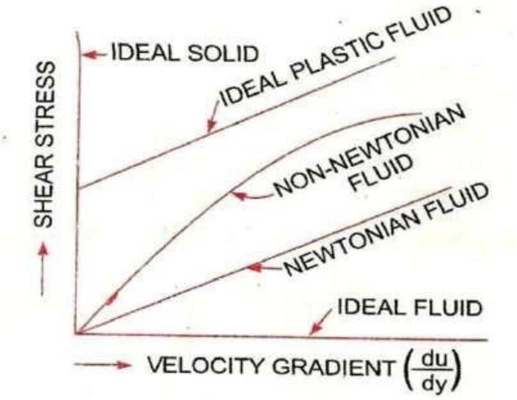
Centistoke means $= \frac{1}{100}$ stoke.

**Newton's law of viscosity:**

It states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity.

Mathematically $\quad \tau = \mu \frac{du}{dy}$

Fluids which obey the above equation or law are known as Newtonian fluids & the fluids which do not obey the law are called Non-Newtonian fluids.




- **Increase in temp. The viscosity of fluid decreases.**

**<u>Surface tension:</u>**

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a stretched membrane under tension. The magnitude of this force per unit length of the free will has the same value as the surface energy per unit area.

It is denoted by $\sigma$

Mathematically $\sigma = \frac{F}{L}$

Unit in si system is N/m

CGS system is Dyne/cm

MKS system is kgf/m

**1.6.1 Surface Tension on Liquid Droplet.** Consi
radius '$r$'. On the entire surface of the droplet, the tensile
Let $\sigma$ = Surface tension of the liquid
$p$ = Pressure intensity inside the droplet (in excess
$d$ = Dia. of droplet.
Let the droplet is cut into two halves. The forces actin
($i$) tensile force due to surface tension acting around th
in Fig. 1.11 ($b$) and this is equal to

$$= \sigma \times \text{Circumference}$$
$$= \sigma \times \pi d$$

(*ii*) pressure force on the area $\frac{\pi}{4} d^2 = p \times \frac{\pi}{4} d^2$ as shown in Fig. 1.11 (*c*). These two forces will be equal and opposite under equilibrium conditions, *i.e.*,

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

or
$$p = \frac{\sigma \times \pi d}{\frac{\pi}{4} \times d^2} = \frac{4\sigma}{d} \quad \text{...(1.14)}$$

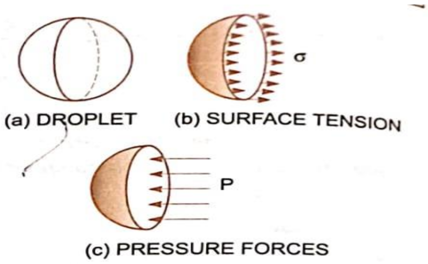


**Fig. 1.11** *Forces on droplet.*

Equation (1.14) shows that with the decrease of diameter of the droplet, pressure intensity inside the droplet increases.

**1.6.2 Surface Tension on a Hollow Bubble.** A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected to surface tension. In such case, we have

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$$\therefore \quad p = \frac{2\sigma\pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d} \quad \text{...(1.15)}$$

**1.6.3 Surface Tension on a Liquid Jet.** Consider a liquid jet of diameter '*d*' and length 'L' as shown in Fig. 1.12.

Let $p$ = Pressure intensity inside the liquid jet above the outside pressure

$\sigma$ = Surface tension of the liquid.

Consider the equilibrium of the semi jet, we have

Force due to pressure $= p \times$ area of semi jet

$= p \times L \times d$

Force due to surface tension $= \sigma \times 2L.$

Equating the forces, we have

$$p \times L \times d = \sigma \times 2L$$

$$\therefore \quad p = \frac{\sigma \times 2L}{L \times d} \quad \text{...(1.16)}$$

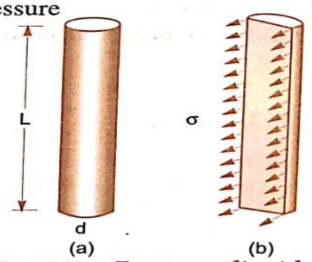


**Fig. 1.12** *Forces on liquid jet.*

## NUMERICALS

**1. The surface tension of water in contact with air at 20°C is 0.0725 *N/m*. the pressure inside a droplet is to be 0.02 $N/cm^2$ greater than the outside pressure. Calculate the diameter of the droplet of water.**

Surface tension, σ = 0.0725 N/m

Pressure intensity, P = 0.02 N/m$^2$

P = 4σ/d

Hence, the Diameter of the dropd = 4 x 0.0725/200 = **1.45 mm**

**2. Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5$N/m^2$above atmospheric pressure.**

d = 40 mm = 40 x $10^{-3}$m

P = 2.5 N/$m^2$

P = $\frac{8\sigma}{d}$ or 2.5 = $\frac{8\sigma}{40 \times 10^{-3}}$ σ = $\frac{2.5 \times 40 \times 10^{-3}}{8}$ = **0.0125 *N/m***

**3. The pressure outside droplet of water of diameter 0.04 mm is 10.32 $N/cm^2$.calculate the presure within the droplet if surface Tension is given bar has 0.725 N/m of water.**

**Solution.** Given :

Dia. of droplet, $d = 0.04 \text{ mm} = .04 \times 10^{-3} \text{ m}$

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

The pressure inside the droplet, in excess of outside pressure is given by equation (1.14)

or $$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = \frac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$$

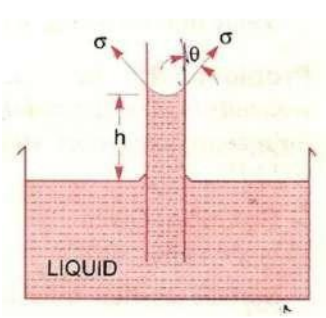
$\therefore$ Pressure inside the droplet $= p +$ Pressure outside the droplet

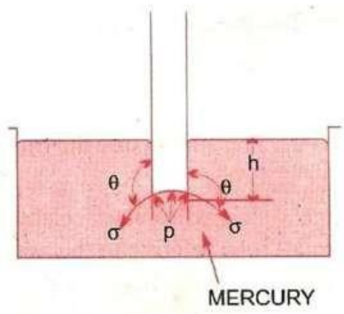
$= 0.725 + 10.32 = \mathbf{11.045 \text{ N/cm}^2}$. **Ans.**

## Capillarity:

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is is known as capillary rise while the fall of the liquid surface is known as capillary depression.

It is expressed in terms of cm or mm of liquid



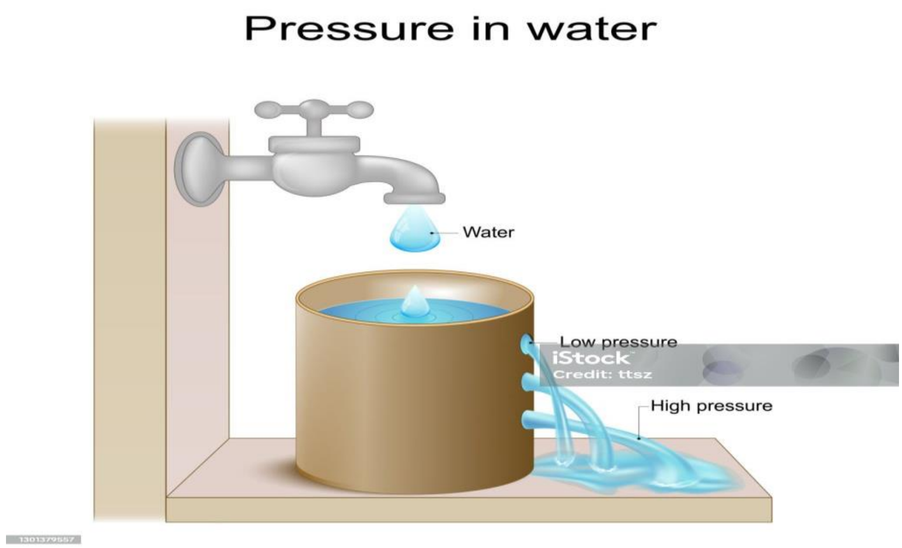

Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

## Fluid pressure:

**Fluid pressure** is the force **exerted by a fluid per unit area** on the walls of its container or any surface it touches.

## Hydrostatic pressure:

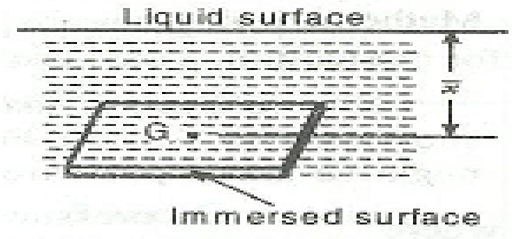
Hydrostatic pressure is the pressure exerted by a fluid at rest due to gravity. It increases with depth and is caused by the weight of the fluid above a given point. Essentially, it's the pressure you feel when submerged in water, and it's why you feel more pressure at the bottom of a pool than near the surface.




## Total Pressure and Centre of Pressure:

The **total pressure** is defined as the force exerted by a static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface. This force is always normal to the surface. The **center of pressure** is defined as the point of application of the resultant pressure on the surface.

The total pressure and center of pressure on the immersed surfaces are as follows:




ırved. Fig. 3.2

**1. Horizontally immersed surface.** The total pressure on a horizontally immersed surface, as shown in Fig. 3.2, is given by

$$P = wA.\bar{x}$$

where

w = Specific weight of the liquid,

A = Area of the immersed surface, and
x (bar) = Depth of the centre of gravity of the immersed surface from the liquid surface.

The above expression holds good for all surfaces whether flat or curved.

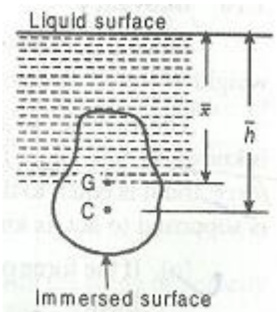


Fig. 3.3

**2. Vertically immersed surface.** The total pressure on a vertically immersed surface, as shown in Fig. 3.3 , is given by

$$P = wA.\bar{x}$$

and the depth of centre of pressure from the liquid surface,

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

where

A = Area of immersed surface,

x (bar) = Depth of centre of gravity of the immersed surface from the liquid surface, and

Ig = Moment of inertia of immersed surface about the horizontal axis through its centre of gravity.

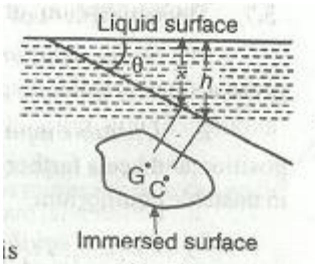


Fig. 3.4

**3. Inclined immersed surface.** The total pressure on an inclined surface, as shown in Fig. 3.4, is given by

$$P = w.A.\bar{x}$$

and the depth of centre of pressure from the liquid surface,

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

where

θ = Angle at which the immersed surface is inclined with the liquid surface.

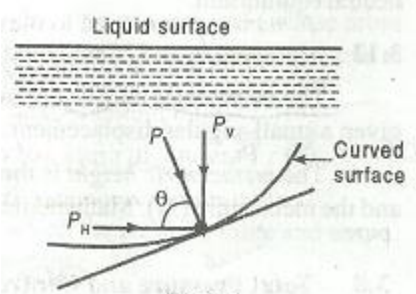


Fig. 3.5

**4. Curved immersed surface.** The total force on the curved surface, as shown in Fig. 3.5, is given by

$$P = \sqrt{(P_H)^2 + (P_V)^2}$$

and the direction of the resultant force on the curved surface with the horizontal is given by

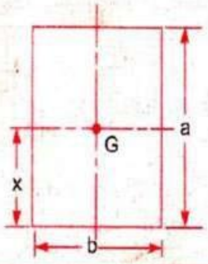
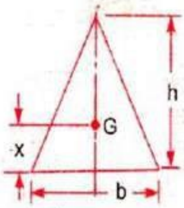
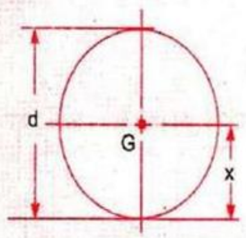
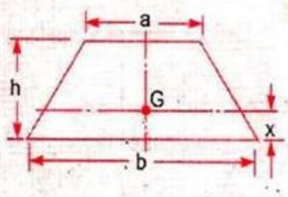
$$\tan\theta = \frac{P_V}{P_H} \quad \text{or} \quad \theta = \tan^{-1}\left[\frac{P_V}{P_H}\right]$$

where

$P_H$ = Horizontal force on the curved surface and is equal to the total pressure on the projected area of the curved surface on the vertical plane, and

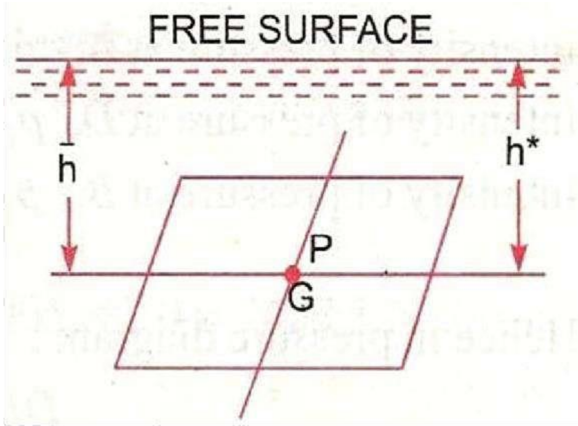
$P_V$ = Vertical force on the curved surface and is equal to the weight of the liquid supported by the curved surface upto the liquid surface.

## The moments of inertia and other geometric properties of some important plane surface

| Plane surface | C.G. from the base | Area | Moment of inertia about an axis passing through C.G. and parallel to base ($I_G$) | Moment of inertia about base ($I_0$) |
|---|---|---|---|---|
| 1. Rectangle  | $x = \frac{d}{2}$ | $bd$ | $\frac{bd^3}{12}$ | $\frac{bd^3}{3}$ |
| 2. Triangle  | $x = \frac{h}{3}$ | $\frac{bh}{2}$ | $\frac{bh^3}{36}$ | $\frac{bh^3}{12}$ |
| 3. Circle  | $x = \frac{d}{2}$ | $\frac{\pi d^2}{4}$ | $\frac{\pi d^4}{64}$ | — |
| 4. Trapezium  | $x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$ | $\frac{(a+b)}{2} \times h$ | $\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$ | — |

## Horizontal plane surface submerged in liquid:

Consider a plane horizontal surface immersed in a static fluid as every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface.



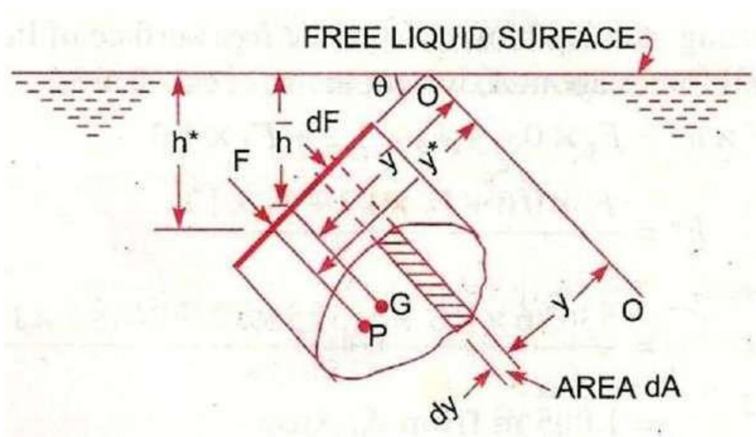

$P = qg\bar{h}$

A = total area

$F = P \times A$

$= qgA\bar{h}$

## Inclined plane surface submerged in liquid:



Let A = total area of the inclned surface

H = depth of C.G. of inclined area from free surface.

h* = distance of center of pressure from free surface of liquid.

8 = angle made by the plane of surface with free liquid surface.

Let the plane of the surface if produced meet the free liquid surface at 0. Then 0-0 is the axis parallel to the plane of the surface

$\bar{y}$ = distance of C.G of the inclined surface from 0-0.

y* = distance of the centre of pressure from 0-0.

Consider a small strip of area dA at a depth ‘h’ from free surface & at a distance y from axis 0-0.

P = qgh

dF = pdA

= qgh dA

Total pressure force

F = ∫ dF = ∫ qgh dA

h = ysin8

F = ∫qgysin8 dA

= qgsin8 ∫ydA

= qgsin8 Io

= qgsin8A $\bar{y}$

= qgA $\bar{y}$ sin8

= qgA$\bar{h}$

**Centre of pressure:**

Pressure force on the strip dF = qgh dA

$$= qgysin8\ dA$$

Moment of the force dF about 0 – 0

$$= dF \times y = qgy^2 sin8\ dA$$

Sum of moments of all such forces about 0 – 0

$$= qgsin8\ y^2 dA$$

$\int y^2\, dA$ = moment of inertia of the surface about 0 - 0 = Io

$$= qgsin8 Io$$

Moment of total force about 0 – 0

$$= F\, y^*$$

$F\, y^* = qgsin8\ Io$

$$qgA\bar{h} \times \frac{h^*}{sin8} = qgsin8\ Io$$

$$h^* = \frac{cin^2 8}{A\bar{h}}\, Io$$

$$= \frac{cin^2 8}{A\bar{h}} [I_G + A \times (\bar{y})^2]$$

Here $\frac{\bar{h}}{\bar{y}} = sin8$

$$\bar{y} = \frac{\bar{h}}{sin8}$$

$$h^* = \frac{cin^2 8}{A\bar{h}} \left[I_G + A \times \left(\frac{\bar{h}}{sin8}\right)^2\right]$$

$$h^* = \frac{I_G cin^2 8}{A\bar{h}} + \bar{h}$$

## Archimedes principle:

When a body is immersed in a fluid either wholly or partially, it is buoyed or lifted up by a force, which is equal to the weight of fluid displaced by the body.

## Buoyancy:

Whenever a body is immersed wholly or partially in a fluid it is subjected to an upword force which tends to lift itup. This tendency for an immersed body to be lifted up in the fluid due to an upward force opposite to action of gravity is known as buoyancy this upward force is known as force of buoyancy.

## Centre of Buoyancy:

It is defined as the point through which the forced of buoyancy is supposed to act. The force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body.

Canter of buoyancy will be the centre of gravity of the fluid displaced.

## Problem-1:

Find the volume of the water displaced & position of centre of duoyancy for a wooden block of width 2.5m & of depth 1.5m when it flats horizontally in water. The density of wooden block is 6540 kg/m3.& its length 6.0m.

Solution:

Width = 2.5 m

Density of wooden block = 650kg/$m^3$

Depth = 1.5m

Length = 6m

Volume of the block

$= 2.5 \times 1.5 \times 6$

$= 22.50 m^3$

Volume of the block = Wt of water displaced

$= W \times V$

$= qg \times V$

$= 650 \times 9.81 \times 6$

$= 143471$ N

Volume of water displaced

$$= \frac{\text{weight}}{qw \times g}$$

$$= \frac{143471}{1000 \times 9.81}$$

$= 14.625 m^3$

Position of centre of buoyancy

Volume of wooden block in water = volume of water displaced

$$2.5 \times 6 \times h = 14.625$$

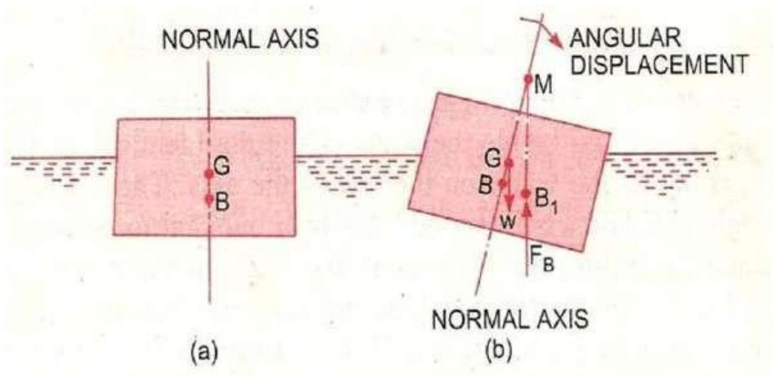
$$\Rightarrow h = \frac{14.625}{2.5 \times 6}$$

$= 0.975$m

$$\text{Centre of buoyancy} = \frac{0.975}{2}$$

= 0.4875 m from base

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The mate centre may also be defined as the point at which the lme of action of the force of buoyancy will melt the normal axis. Of the body when the body is given a small angular displacement.




## Mate centre height:

The distance between the meta centre of a floating body and the centre of gravity of the body is called meta-centric height i.e the distance MG.

## Concept of flotation:

### Flotation:

When a body is immersed in any fluid, it experiences two forces. First one is the weight of body W acting vertically downwards, second is the buoyancy force $F_b$ acting vertically upwards in case W is greater than $F_b$, the weight will cause the body to sink in the fluid. In case W = $F_b$ the body will remain in equilibrium at any level. In case W is small than $F_b$ the body will move upwards in fluid. The body moving up will come to rest or top moving up in fluid when the fluid displaced by it's submerged part is equal to its weight W, the body in this situation is said to be floating and this phenomenon is known as flotation.

## Principle of flotation:

The principle of flotation states that the weight of the floating body isequal to the weight of the fluid displaced by the body.

Consider a body floating at the free surface of the liwuid. The shaded part of the body is inside the fluid and it has volume $V_1$ The other part of the body is in air and it has volume $V_2$. Now the body can be considered to be in two fluids viz. air and liquid. Hence buoyant force

$$F_b = q_{Siquid}V_1g_1 + q_{air}V_2\,g_2 = W$$

Since $q_{air} \ll q_{Siquid}$

$$F_b = q_{Siquid}V_1g = W$$

Buoyancy force is equal to weight of the liquid displaced

## The ways to make the body float:

The body can be made to float:

1. Decreasing the weight of the body while keeping the volume same. For example, making body hollow.
2. Increasing the volume of the body while keeping the body same. For example, attaching live jacket to a person fixed the person floating.

Chapter-4

# KINEMATICS

| |
|---|
| **Types of fluid flow**<br>**Continuity equation (Statement and proof for one dimensional flow)**<br>**Bernoulli's theorem (Statement and proof)**<br>**Applications and limitations of Bernoulli's theorem (Venturimeter, pitot tube)**<br>**(Simple Numerical)** |

**Syllabus:**

**Introduction:-**

This chapter includes the study of forces causing fluid flow. The dynamics of fluid flow is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton second law of motion, which relates the acceleration with the forces. The fluid is assumed to be incompressible and non-viscous.

## TYPES OF FLOW:-

The fluid flow is classified as follows:

- **STEADY AND UNSTEADY FLOW**
- **UNIFORM AND NON- UNIFORM FLOWS**
- **LAMINAR AND TURBULANT FLOWS**
- **COMPRESSIBLE AND INCOMPRESSIBLE FLOWS**
- **ROTATIONAL AND IRROTATIONAL FLOWS**
- **ONE, TWO, THREE DIMENSIONAL FLOW**

### ➢ STEADY AND UNSTEADY FLOW:-

**1. Steady flow:-**

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density at a point do not change with time.

Thus, mathematically

$$\left(\frac{6v}{6t}\right)_{0,y_0,z_0} = 0$$

$$\left(\frac{6p}{6t}\right)_{0,y_0,z_0} = 0$$

$$\left(\frac{6\rho}{6t}\right)_{0,y_0,z_0} = 0$$

Where $x_0, y_0, z_0$ is a point in fluid flow .

**2. Unsteady flow:-**

Unsteady flow is defined as that type of flow in which the velocity, pressure, and density at a point changes w.r.t time.

Thus, mathematically

$$\left(\frac{6v}{6t}\right)_{0,y_0z_0} \neq 0,$$

$$\left(\frac{6p}{6t}\right)_{0,y_0z_0} \neq 0,$$

$$\left(\frac{6\rho}{6t}\right)_{0,y_0z_0} \neq 0$$

- **<u>UNIFORM AND NON- UNIFORM FLOWS:-</u>**

1. **<u>Uniform flow</u>**:-

It is defined as the flow in which velocity of flow at any given time does not change w.r.t length of flow or space.

Mathematically,

$$\left(\frac{dv}{ds}\right)_{=constant} = 0$$

where $\partial v$ = velocity of flow ,

$\partial s$ = length of flow ,

T = time

2. **<u>Non- uniform flows</u>:-**

It is defined as the flow in which velocity of flow at any given time changes w.r.t length of flow.

Mathematically,

$$\left(\frac{dv}{ds}\right)_{=constant} \neq 0$$

- **LAMINAR AND TURBULANT FLOWS**:-

1. **Laminar flow**:-

Laminar flow is that type of flow in which the fluid particles are moved in a well defined path called streamlines. The paths are parallel and straight to each other.

2. **Turbulent flow**:-

Turbulent flow is that type of flow in which the fluid particles are moved in a zig-zag manner.

For a pipe flow the type of flow is determined by Reynolds number ($R_e$)

Mathematically

$$R_e = \frac{VD}{v}$$

Where V = mean velocity of flow

D = diameter of pipe

V = kinematic viscosity

If $R_e$< 2000, then flow is laminar flow.

If $R_e$> 4000, then flow is turbulent flow.

If $R_e$ lies in between 2000 and 4000, the flow may be laminar or turbulent.

- **COMPRESSIBLE AND INCOMPRESSIBLE FLOWS** :-

1. **Compressible flow**:-

Compressible flow is that type of flow in which the density of fluid changes from point to point.

So, $\partial \neq$ constant.

2. **Incompressible flow:-**

Incompressible flow is that type of flow in which the density is constant for the fluid flow.

So, $\partial =$constant

- **ROTATIONAL AND IRROTATIONAL FLOWS**:-

1. **Rotational flow:-**

Rotational flow is that of flow in which the fluid particles while flowing along stream lines also rotate about their own axis.

2. **Ir-rotational flow**:-

Irrotational flow is that type of flow in which the fluid particles while flowing along streamlines do not rotate about their own axis.

- **ONE, TWO, THREE DIMENSIONAL FLOW**:-

1. **One dimensional flow**:-

One dimension flow is defined as that type of flow in which velocity is a function of time and one space co-ordinate only.

For a steady one dimensional flow, the velocity is a function of one space co-ordinate only.

So, $U = f(x)$,

$V = 0$,

$W = 0$

U, V, W are velocity components in x, y, z direction respectively.

2. **Two-dimensional flow**:-

Two-dimensional flow is the flow in which velocity is a function of time and 2- space co- ordinates only. For a steady 2- dimensional flow the velocity is a function of two – space co-ordinate only.

So, $U = f_1(x,y)$ ,

$V = f_2(x,y)$ ,

$W = 0$

3. **Three-dimensional flow**:-

Three – dimensional flow is the flow in which velocity is a function of time and 3- space co-ordinates only. For steady three- dimensional flow, the velocity is a function of three space co-ordinates only.

So $U = f_1(x, y, z)$

$V = f_2(x, y, z)$

$W = f_3(x, y, z)$

**RATE OF FLOW OR DISCHARGE**

It is defined as the quantity of a fluid flowing per second through a section of pipe.

For an incompressible fluid the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second.

For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

$$Q = A.V$$

Where A = cross sectional area of the pipe

V = velocity of fluid across the section

Unit:-

1. For incompressible fluid

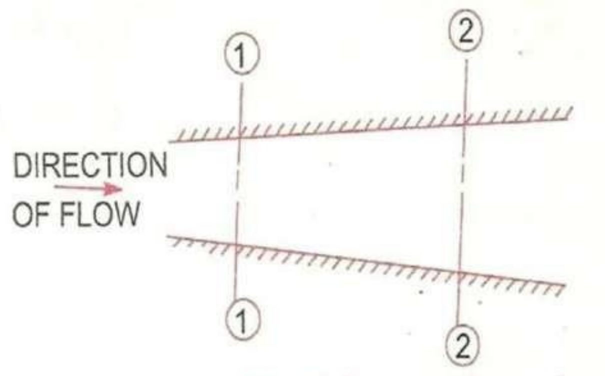
$$\frac{m3}{sec} \text{ or } \frac{litre}{sec}$$

2. For compressible fluid:

$$\frac{newton}{sec} \text{ (S.I units), } \frac{kgf}{sec} \text{ (M.K.S units)}$$

EQUATION OF CONTINUITY:-

It is based on the principle of conservation of mass. For a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.



*Fluid flowing through a pipe.*

Let $V_1$= average velocity at cross-section 1-1.

$\rho_1$ = density at cross-section 1-1

$A_1$ = area of pipe at section 1-1

$V_2$= average velocity at cross-section 2-2

$\rho_2$ = density at cross-section 2-2

$A_2$ = area of pipe at section 2-2

The rate of flow at section 1-1 =$\rho_1 A_1 V_1$

The rate of flow at section 2-2 =$\rho_2 A_2 V_2$

According to laws of conservation of mass rate of flow at section 1-1 is equal to the rate of flow at section 2-2 ,

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

This is called continuity equation.

If the fluid is compressible, then $\rho_1 = \rho_2$,

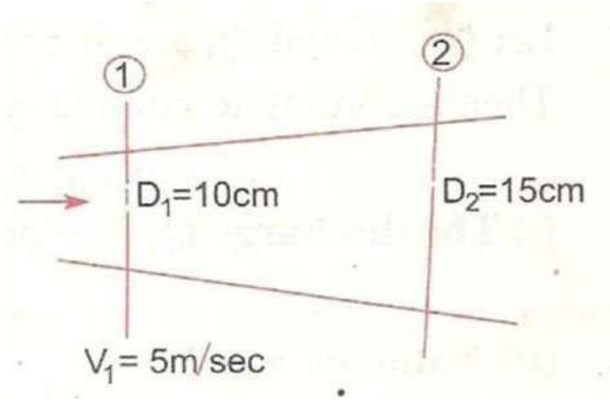
so $A_1 V_1 = A_2 V_2$

"If no fluid is added removed from the pipe in any length then the mass passing across different sections shall be same"

# Simple Problems

**Problem:-1**

The diameters of a pipe at the sections 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of the water flowing through the pipe at section 1 is 5m/s. Determine also the velocity at section 2.




**Solution.** Given :

At section 1, $D_1 = 10 \text{ cm} = 0.1 \text{ m}$

$$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$$

$$V_1 = 5 \text{ m/s.}$$

At section 2, $D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$

(*i*) Discharge through pipe is given by equation (5.1)

or $$Q = A_1 \times V_1$$

$$= .007854 \times 5 = \mathbf{0.03927 \text{ m}^3\text{/s.}} \textbf{ Ans.}$$

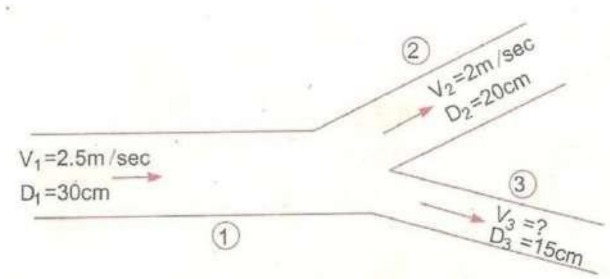
Using equation (5.3), we have $A_1V_1 = A_2V_2$

(*ii*) $\therefore$ $$V_2 = \frac{A_1V_1}{A_1} = \frac{.007854}{.01767} \times 5.0 = \mathbf{2.22 \text{ m/s.}}$$

**Problem:-2**

A 30m diameter pipe conveying water branches into two pipes of diameter 20cm and 15cm respectively. If the average velocity in the 340cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15cm pipe if the average velocity in 20cm diameter pipe is 2m/s

**Solution:**




**Given Data:**

$D_1$ = 30cm = 0.30m

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.3)^2 = 0.07068\ m^2$

$V_1$ = 2.5 m/s

$D_2$ = 20cm = 0.2m

$A_2 = \frac{\pi}{4} 0.2^2 = 0.0314\ m^2$

$V_2$ = 2m/s

$D_3$ = 15cm = 0.15m

$A_3 = \frac{\pi}{4} 0.15^2 = 0.01767\ m^2$

Let $Q_1$, $Q_2$, $Q_3$ are discharges in pipe 1, 2, 3 respsctively

$$Q_1 = Q_2 + Q_3$$

The discharge $Q_1$ in pipe 1 is given as

$Q_1 = A_1 V_1$

$= 0.07068 \times 2.5\ m^3/s$

$Q_2 = A_2 V_2$

$= 0.0314 \times 2.0\ 0.0628\ m^3/s$

Substituting the values of $Q_1$ and $Q_2$ on the above equation we get

$0.1767 = 0.0628 + Q_3$

$Q_3 = 0.1767 – 0.0628$

$= 0.1139$ m3/s

Again $Q_3 = A_3 V_3$

$= 0.01767 \times V_3$

Or $0.1139 = 0.01767 \times V_3$

$$V_3 = \frac{0.1139}{0.01767}$$

$= 6.44$m/s

**Problem:-3**

A 25 cm diameter pipe carries oil of sp. Gr. 0.9 at a velocity of 3m/s. At another section the diameter is 20cm. Find the velocity at this section and also mass rater of flow of oil.

**Solution.** Given :

at section 1, $D_1 = 25\text{ cm} = 0.25\text{ m}$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .25^2 = 0.049\text{ m}^3$$

$$V_1 = 3\text{ m/s}$$

at section 2, $D_2 = 20\text{ cm} = 0.2\text{ m}$

$$A_2 = \frac{\pi}{4} (.2)^2 = 0.0314\text{ m}^2$$

$$V_2 = ?$$

Mass rate of flow of oil = ?

Applying continuity equation at sections 1 and 2,

$$A_1V_1 = A_2V_2$$

or $$0.049 \times 3.0 = 0.0314 \times V_2$$

$\therefore$ $$V_2 = \frac{0.049 \times 3.0}{.0314} = \mathbf{4.68\text{ m/s. Ans.}}$$

Mass rate of flow of oil $= \text{Mass density} \times Q = \rho \times A_1 \times V_1$

Sp. gr. of oil $= \dfrac{\text{Densit of oil}}{\text{Densit of water}}$

$\therefore$ Density of oil = Sp. gr. of oil $\times$ Density of water

$$= 0.9 \times 1000\text{ kg/m}^3 = \frac{900\text{ kg}}{\text{m}^3}$$

$\therefore$ Mass rate of flow $= 900 \times 0.049 \times 3.0\text{ kg/s} = \mathbf{132.23\text{ kg/s. Ans.}}$

## Bernoulli's equation:

**Statement:** It states that in a steady ideal flow of an in compressible fluid, the total energy at any point of flow is constant.

The total energy consists of pressure energy, kinetic energy & potential energy or datum energy. These energies per unit weight are

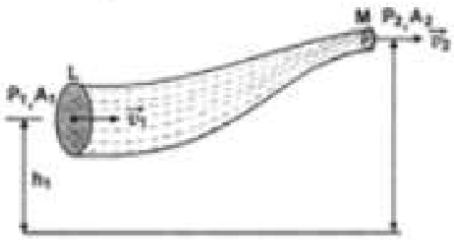
Pressure energy = $\frac{P}{\rho g}$

Kinetic energy = $\frac{v^2}{\rho g}$

Datum energy = z

Mathematically

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{Constant}$$



Proof: Let us consider the ideal liquid of density ρ flowing through the pipe LM of varying cross-section. Let $P_1$ and $P_2$ be the pressures at ends L and M and $A_1$ and $A_2$ be the areas of cross-sections at ends L and M respectively. Let the liquid enter with velocity $V_1$ and leave with velocity $v_2$. Let $A_1 > A_2$. By equation of continuity,

$$A_1 v_1 = A_2 v_2$$

Since $A_1 > A_2$,

$\therefore \quad v_2 > v_1 \quad$ and $\quad P_1 > P_2$

Let $m$ be mass of liquid entering at end L in time $t$. In time $t$, the liquid will cover a distance of $v_1 t$.

Therefore the work done by pressure on the liquid at end L in time $t$ is

$$W_1 = \text{force} \times \text{displacement}$$
$$= P_1 A_1 v_1 t \qquad ...(1)$$

Since same mass $m$ leaves the pipe at end M in same time $t$, in which liquid will cover the distance $v_2 t$, therefore work done by liquid against the force due to pressure $P_2$ is

$$W_2 = P_2 A_2 v_2 t \qquad ...(2)$$

Net work done by pressure on the liquid in time $t$ is,

$$W = W_1 - W_2 = P_1 A_1 v_1 t - P_2 A_2 v_2 t \qquad ...(3)$$

This work done on liquid by pressure increases its kinetic and potential energy.

Increase in kinetic energy of liquid is,

$$\Delta K = \frac{1}{2} m (v_2^2 - v_1^2) \qquad ...(4)$$

According to work-energy relation,

$$P_1 A_1 v_1 t - P_2 A_2 v_2 t = \frac{1}{2} m (v_2^2 - v_1^2) + mg(h_2 - h_1) \quad ...(6)$$

If there is no source and sink of liquid, then mass of liquid entering at end L is equal to the mass of liquid leaving the pipe at end M and is given by

$$A_1 v_1 \rho t = A_2 v_2 \rho t = m$$

or $$A_1 v_1 t = A_2 v_2 t = \frac{m}{\rho} \qquad ...(7)$$

From (6) and (7)

$$P_1 \frac{m}{\rho} - P_2 \frac{m}{\rho} = \frac{1}{2} m (v_2^2 - v_1^2) + mg(h_2 - h_1)$$

or $$P_1 \frac{m}{\rho} + \frac{1}{2} m v_1^2 + mgh_1 = P_2 \frac{m}{\rho} + \frac{1}{2} m v_2^2 + mgh_2$$

or $$\frac{P}{\rho} + gh + \frac{1}{2} v^2 = \text{Constant}$$

**Problem:- 5**

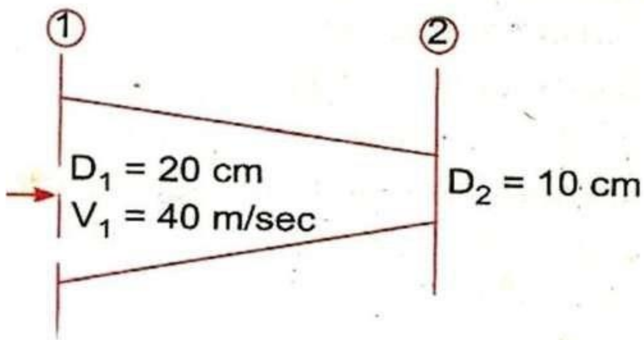
Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm2 (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

**Solution.** Given :

| | |
|---|---|
| Diameter of pipe | $= 5 \text{ cm} = 0.5 \text{ m}$ |
| Pressure, | $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$ |
| Velocity, | $v = 2.0 \text{ m/s}$ |
| Datum head, | $z = 5 \text{ m}$ |
| Total head | $=$ pressure head + kinetic head + datum head |
| Pressure head | $= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\left\{\rho \text{ for water} = 1000 \frac{\text{kg}}{\text{m}^3}\right\}$ |
| Kinetic head | $= \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$ |
| $\therefore$ Total head | $= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5 = \mathbf{35.204 \text{ m. Ans.}}$ |

**Problem:- 6**

A pipe, through which water is flowing, is having diameters, 20cm and 10cm at the cross sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find thevelocity head at sections 1 and 2 and also rate of discharge.



**Solution.** Given :

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$\therefore$ Area, $$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$$V_1 = 4.0 \text{ m/s}$$

$$D_2 = 0.1 \text{ m}$$

$\therefore$ $$A_2 = \frac{\pi}{4} (.1)^2 = .00785 \text{ m}^2$$

(*i*) Velocity head at section 1

$$= \frac{V_1^2}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = \textbf{0.815 m. Ans.}$$

(*ii*) Velocity head at section 2 = $V_2^2/2g$

To find $V_2$, apply continuity equation at 1 and 2

$\therefore$ $$A_1 V_1 = A_2 V_2 \quad \text{or} \quad V_2 = \frac{A_1 V_1}{A_2} = \frac{.0314}{.00785} \times 4.0 = 16.0 \text{ m/s}$$

$\therefore$ Velocity head at section 2 $= \frac{V_2^2}{2g} = \frac{16.0 \times 16.0}{2 \times 9.81} = \textbf{83.047 m. Ans.}$

(*iii*) Rate of discharge $= A_1 V_1$ or $A_2 V_2$

$$= 0.0314 \times 4.0 = 0.1256 \text{ m}^3/\text{s}$$

**Application of Bernoulli's equation:**

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy consideration are involved. It is also applied to following measuring devices

1. **Venturimeter**
2. **Pitot tube**

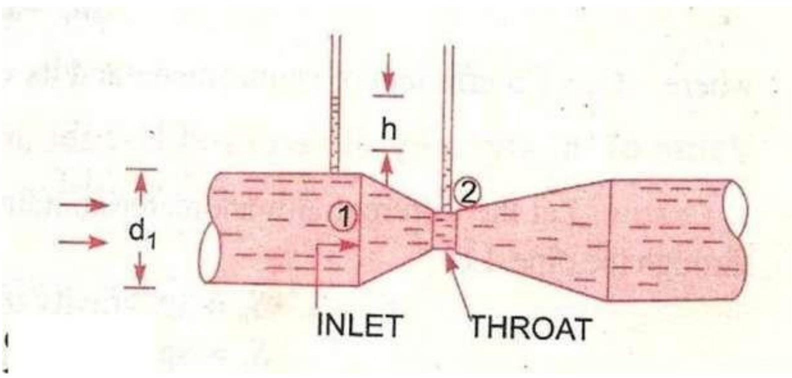
**Venturimeter:**

A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe it consists of three parts.

I. **Short converging part**
II. **Throat**
III. **Diverging part**

**Expression for rate of flow through venturimeter:**

Consider a venturimeter is fitted in a horizontal pipe through which a fluid flowing



Let $d_1$ = diameter at inlet or at section (i)-(ii)

$P_1$ = pressure at section (1)-(1)

$V_1$ = velocity of fluid at section (1) – (1)

$A_1$= area at section (1) – (1) $= \frac{\pi}{4} d_1^2$

$D_2$, $p_2$ , $v_2$ , $a_2$ are corresponding values at section 2 applying Bernouli's equation at sections 1 and 2 we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{P_1 - P_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h

So, $h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$

Now applying continuity equation at sections 1 & 2 $a_1 v_1 = a_2 v_2$

Or $v_1 = \frac{a_2 v_2}{a_1}$

Substituting this value

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$v_2{}^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

$$Q = a_2 v_2$$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Where Q = Theoretical discharge

Actual discharge will be less than theoretical discharge

$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Where $C_d$ = co-efficient of venturimetre and value is less than 1

## <u>Value of 'h' given by differential U-tube manometer:</u> <u>Case-i:</u>

Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe

Let $S_h$ = Sp. Gravity of the heavier liquid

$S_0$ = Sp. Gravity of the liquid flowing through pipe

x = difference of the heavier liquid column in U-tube

$P_A - P_B = gx(\rho_g - \rho_0)$

$$\frac{P_A - P_B}{\rho_0 g} = x\left(\frac{\rho_g}{\rho_0} - 1\right)$$

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

**Case-ii**

If the differential manometer contains a liquid lighter than the liquid flowing through the pipe

Where $S_l$ = Specific gravity of lighter liquid in U-tube nanometre

So = Specific gravity of fluid flowing through in U-tube nanometre

x = Difference of lighter liquid columns in U- tube

The value of h is given by

$$h = x\left[1 - \frac{Sl}{S_0}\right]$$

**Case-iii:**

Inclined venturimetre with differential U-tube manometre

Let the differential manometer contains heavier liquid

Then h is given as

$$h = \left[\frac{P1}{\rho g} + z_1\right] - \left[\frac{P2}{\rho g} + z_2\right]$$

$$= x\left[\frac{Sh}{S_0} - 1\right]$$

**Case-iv:**

Similarly for inclined venturimetre in which differential manometer contaoins a liquid which is kighter than the liquid flowing through the pipe. Then

$$\mathbf{h} = \left[\frac{P1}{\rho g} + z_1\right] - \left[\frac{P2}{\rho g} + z_2\right]$$

$$\mathbf{h} = \mathbf{x}\left[1 - \frac{}{S_0}\right]$$

**Limitations:**

- Bernoulli's equation has been derived underthe assumption that no external force except the gravity force is acting on the liquid. But in actual practice some external forces always acting on the liquid when effect the flow of liquid
- If the liquid is flowing in a curved path the energy due to centrifugal force should also be taken into account.

**Pitot-tube:**

It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

It is based on the principle that if the velocity flow at a point becomes zero, the pressure there is increased due to conversion of the kinetic energy into pressure energy.

The pitot-tube consists of a glass tube, bent an right angles

Consider two points 1 and 2 at te same level. Such a ay that 2 is at he inlet of pitot tube and one is the far away from the tube

Let $P_1$ = pressure at point 1

$V_1$ = velocity of fluid at point 1

$P_2$ = pressure at 2

$V_2$ = velocity of fluid at point 2

H = Depth of tube in the liquid

h = Rise of the liquid in the tube above the free surface

Applying Bernoulli's theorm

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1}{\rho g} = H \qquad \frac{P_2}{\rho g} = (h + H)$$

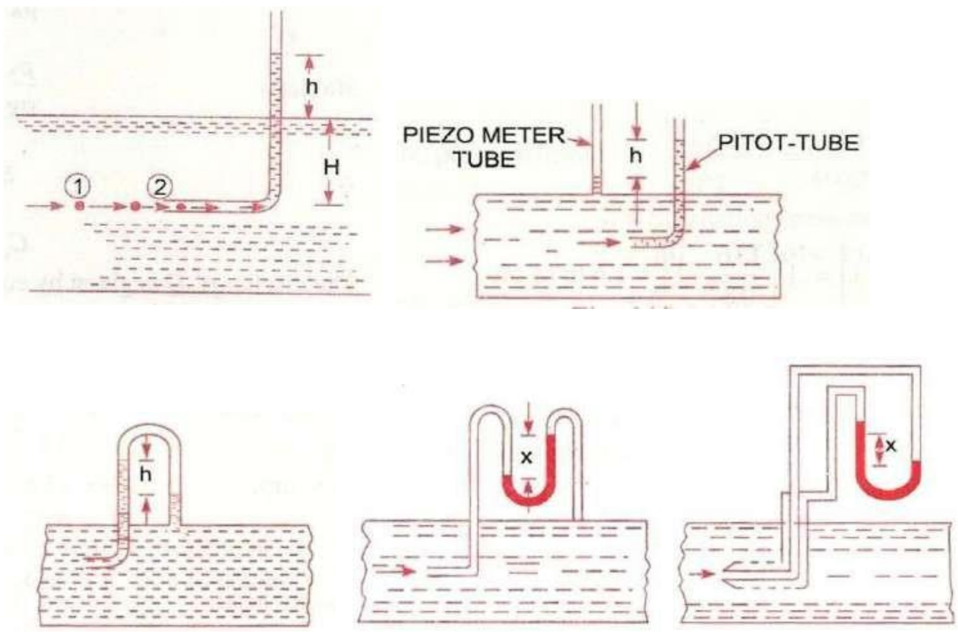
$$H + \frac{V_1^2}{2g} = h + H$$

$$V_1 = \sqrt{2gh}$$

Actual velocity, $V_{act} = C_v \sqrt{2gh}$

$C_v$ = co-efficient of Pitot-tube

## **Different Arrangement of Pitot tubes**



## Numerical Problems:

**Problem:- 7**

Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm$^2$ (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

**Solution.** Given :

Diameter of pipe $= 5 \text{ cm} = 0.5 \text{ m}$

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0 \text{ m/s}$

Datum head, $z = 5 \text{ m}$

Total head $=$ pressure head $+$ kinetic head $+$ datum head

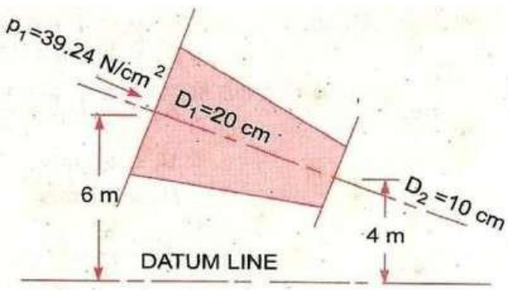
Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\left\{ \rho \text{ for water} = 1000 \dfrac{\text{kg}}{\text{m}^3} \right\}$

Kinetic head $= \dfrac{v^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

$\therefore$ Total head $= \dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

**Problem:- 8**

The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35lit/s. The section 1 is 6m above datum and sedction 2 is 4m aboved datum. If the pressure at section 1 is 39.24 N/cm$^2$. Find the intensity of pressure at section 2



**Solution:**

Given

At section 1, $D_1 = 20 \text{ cm} = 0.2 \text{ m}$

$$A_1 = \frac{\pi}{4}(.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2$$
$$= 39.24 \times 10^4 \text{ N/m}^2$$
$$z_1 = 6.0 \text{ m}$$

At section 2, $D_2 = 0.10 \text{ m}$

$$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$$
$$z_2 = 4 \text{ m}$$
$$p_2 = ?$$

Rate of flow, $Q = 35 \text{ lit/s} = \frac{35}{1000} = .035 \text{ m}^3\text{/s}$

Now $Q = A_1V_1 = A_2V_2$

$\therefore$ $$V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = 1.114 \text{ m/s}$$

and $$V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = 4.456 \text{ m/s}$$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or $$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

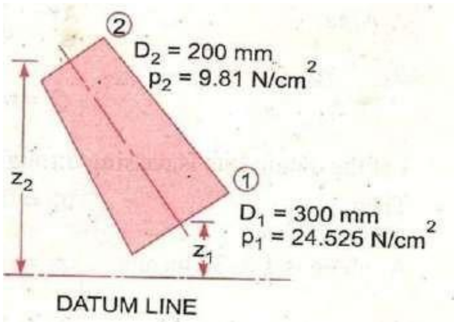
or $$46.063 = \frac{p_2}{9810} + 5.012$$

$\therefore$ $$\frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$\therefore$ $$p_2 = 41.051 \times 9810 \text{ N/m}^2$$
$$= \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2}.$$

**Problem:- 9**

Water is flowing through a pipe having diameter 300mm and 200 mm at the buttom and upper end respectively. The intensity of pressure at the bottom end is 9.81N/m$^2$. Determine the difference in datum head if the rate of flow through pipe is 40 lit/s




**Solution.** Given :

**Section 1,** $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

**Section 2,** $D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow $= 40$ lit/s

or $$Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$$

Now $$A_1V_1 = A_2V_2 = \text{rate of flow} = 0.04$$

$\therefore$ $$V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4}D_1^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.5658 \text{ m/s}$$

$$\simeq 0.566 \text{ m/s}$$

$$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4}(D_2)^2} = \frac{0.04}{\frac{\pi}{4}(0.2)^2} = 1.274 \text{ m/s}$$

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

or $$25 + .32 + z_1 = 10 + 1.623 + z_2$$

or $$25.32 + z_1 = 11.623 + z_2$$

$\therefore$ $$z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$$

$\therefore$ Difference in datum head $= z_2 - z_1 =$ **13.70 m. Ans.**

**Problem:- 10**

A horizontal venturimetre with inlet and throat diameters 10cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and throat is 20cm of mercury. Determine the rate of flow. Take $C_d = 0.98$

**Solution.** Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer $= x = 20$ cm of mercury.

$\therefore$ Difference of pressure head is given by (6.9)

or $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where $S_h$ = Sp. gravity of mercury = 13.6, $S_0$ = Sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s.}}$$

**Problem:- 11**

An oil of Sp.gr. 0.8 is flowing through a horizontal venturimrtre having inlet diameter 20cm and throaty diameter 10 cm. The oil mercury differential manometer shows a reading of 25cm. Calculate the discharge of oil through the horizontal venturimetre. Take Cd = 0.98

**Solution.** Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

$\therefore$ Difference of pressure head, $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25\,[17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

$$\therefore \quad a_1 = \frac{\pi}{4} d_1{}^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$$

$d_2 = 10$ cm

$$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$C_d = 0.98$

$\therefore$ The discharge $Q$ is given by equation (6.8)

or
$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - 7a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3\text{/s}$$

$= 70465$ cm$^3$/s = **70.465 litres/s. Ans.**

**Problem:- 12**

A horizontal venturimrtre with inlet and throat diameters 20cm and 10 cm respectively is used to measure the flow of oil of Sp. gr. The discharge of oil through venturimetre is 60lit/s . Find thereading of oil-mercury differential manometer. Take $C_d$ = 0.98

**Solution.** Given : $d_1 = 20$ cm

$\therefore$ $$a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$$

$$d_2 = 10 \text{ cm}$$

$\therefore$ $$a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$$C_d = 0.98$$

$$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3\text{/s}$$

Using the equation (6.8), $$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

or $$60 \times 1000 = 9.81 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h}$$

$$= \frac{1071068.78\sqrt{h}}{304}$$

or $$\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$$

$\therefore$ $$h = (17.029)^2 = 289.98 \text{ cm of oil}$$

But $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where $S_h$ = Sp. gr. of mercury = 13.6

$S_o$ = Sp. gr. of oil = 0.8

$x$ = Reading of manometer

$\therefore$ $$289.98 = x\left[\frac{13.6}{0.8} - 1\right] = 16x$$

$\therefore$ $$x = \frac{289.98}{16} = 18.12 \text{ cm.}$$

$\therefore$ Reading of oil-mercury differential manometer = **18.12 cm.**

**Problem:-13**

A static pitot-tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and is perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60mm of water. Take $C_v = 0.98$

**Solution.** Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Diff. of pressure head, $h = 60 \text{ mm of water} = .06 \text{ m of water}$

$C_v = 0.98$

Mean velocity, $\overline{V} = 0.80 \times \text{Central velocity}$

Central velocity is given by equation (6.14)

$$= C_v\sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times .06} = 1.063 \text{ m/s}$$

$\therefore$ $$\overline{V} = 0.80 \times 1.063 = 0.8504 \text{ m/s}$$

Discharge, $$Q = \text{Area of pipe} \times \overline{V}$$

$$= \frac{\pi}{4}d^2 \times \overline{V} = \frac{\pi}{4}(.30)^2 \times 0.8504 = \mathbf{0.06\ m^3/s.\ Ans.}$$

# ( HYDRAULIC TURBINES)

## Introduction to Hydraulic machines:

- These are the machines in which force is transmitted by means of motion of fluid under pressure.
- These can convert hydraulic energy to mechanical energy or mechanical energy to hydraulic energy.
- The hydraulic system works on the principle of *Pascal's law***.** This law states that, the pressure in an enclosed fluid is uniform in all the directions.
- *Examples***:** Hydraulic turbines, Pumps, cranes, forklifts, bulldozers

## Hydraulic turbine:

- It is a hydraulic machine.
- It uses energy of flowing water (hydraulic energy) and converts it into mechanical energy (in the form of rotation of runner)
- Shaft power available at the shaft of the Turbine is utilized to run Generator to produce electricity.

## Classification of turbine:

- *According to the type of energy at inlet*
    - Impulse turbine
        - An impulse turbine is a turbine in which the water entering the runner possesses kinetic energy only. In this, the rotation of the runner occurs due to the impulse action of water. (Pelton Turbine)
    - Reaction turbine
        - A reaction turbine is a turbine in which the water entering the runner possesses pressure as well as kinetic energy. In this, the rotation of runner occurs due to the pressure difference between the inlet and outlet of the runner. (Francis and Kaplan Turbine)
- *According to the direction of flow through runner*
    - Tangential flow turbine
        - When the flow of water is tangential to the wheel circle, the turbine is called tangential flow turbine. (Pelton Turbine)
    - Radial flow turbine
        - When the water moves along the vanes towards the axis of rotation of the runner or away from it, the turbine is called radial flow turbine. When the flow is towards the axis of rotation, the turbine is called an inward flow turbine. When the flow is away from the axis of rotation, the turbine is called an outward flow turbine. (Francis Turbine)
    - Axial flow turbine
        - When the water flows parallel to the axis of rotation, the turbine is called an axial or parallel flow turbine. (Kaplan Turbine/Propeller Turbine)

- Mixed flow turbine
    - When the water enters radially inwards at inlet and discharge at outlet in a direction parallel to the axis of rotation of the runner, the turbine is called mixed flow turbine. (Moden Francis Turbine)

- *According to the head at the inlet of turbine*
    - High head turbine
        - When a turbine works under a head of more than 250 m. (Pelton Turbine)
    - Medium head turbine
        - When a turbine works under a head of 45 m – 250 m. (Francis Turbine)
    - Low head turbine
        - When a turbine works under a head of less than 45 m. (Kaplan Turbine)
- *According to the specific speed of the turbine*
    - Low specific speed turbine
        - The specific speed up to 30 (Pelton Turbine)
    - Medium specific speed turbine
        - The specific speed varies from 50 to 250 (Francis Turbine)
    - High specific head turbine
        - specific speed is more than 250 (Kaplan Turbine)

## Impulse Turbine – Pelton Wheel:

- Pelton turbine is a *tangential flow* impulse turbine.
- It works at high head and requires low flow of water.
- It converts pressure energy into kinetic energy in in one or more nozzles.
- It is driven by high velocity jets of water coming out from a nozzle directed on to vanes or buckets attached to a wheel.
- The impulse provided by the jets is used to spins the turbine wheel and removes kinetic energy from the fluid flow.
- Pressure of water remains atmospheric inside the turbine.

## Construction of Pelton Wheel:

Major Components component of Pelton wheel are described below.

- Casing:
    - Casing prevents the splashing of water and helps in discharge of water from the nozzle to the tailrace. It protects the turbine from dust and dirt.
- Nozzle and Spear Mechanism:
    - Nozzle produces high velocity jets of water and converts pressure energy into kinetic energy.

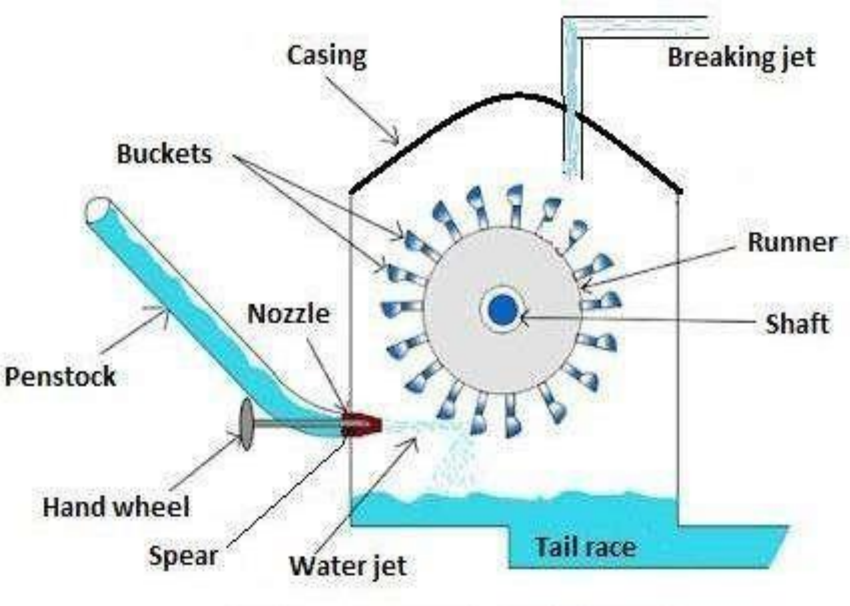
- The spear mechanism controls the water flow into the turbine and control the turbine speed according to load. It minimizes energy loss at inlet and provides smooth flow.

- Break Nozzle:
  - It is used to produce and supply breaking jet of water. It directs the water on the bucket to stop the runner to rest in a short time

- Runner/Rotor:
  - It is a circular disc mounted by a number of equally spaced buckets which are fixed on its periphery. Each bucket consists of two symmetrical halves having shape of semi-ellipsoidal cup.
  - It provides rotational energy when jet of water having kinetic energy strike the buckets.

- Penstock:
  - It is the channel or pipeline that connect the high head source water to the power station

- Governing Mechanism:
  - It controls the speed and power output of the turbine by controlling the flow of water



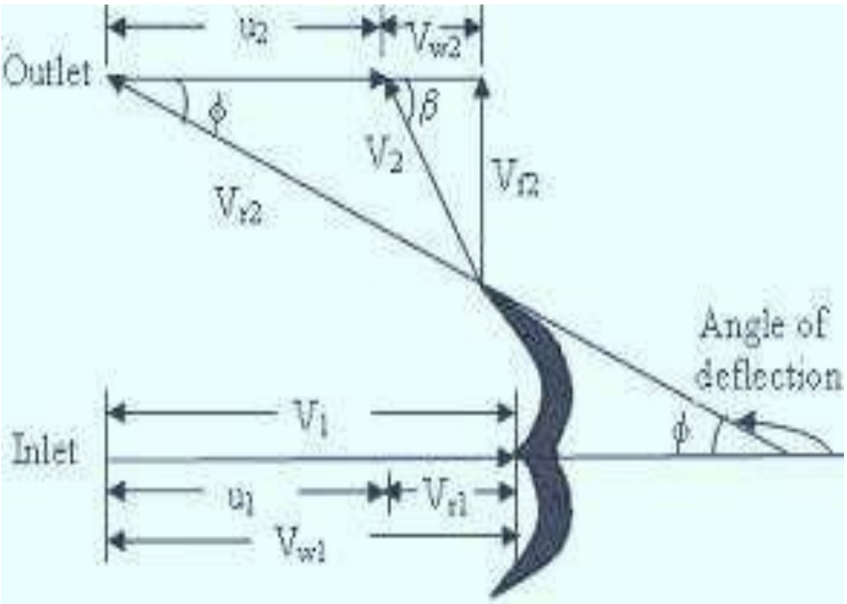

## Working Principle:

- Water is coming from the storage reservoir through a penstock to the inlet of the nozzle.
- Nozzle converts the hydraulic energy of the water into kinetic energy and produces high velocity of jet.
- The jet of water released from the nozzle strikes on the buckets mounted on the runner.
- Water jet strikes over the runner bucket and imparts a very high impulsive force on the buckets for a small amount of time to rotate the runner and so mechanical energy develops.
- Pressure of water remains atmospheric inside the turbine.

## Velocity triangle of Impulse turbine:

Consider the following terms for understanding the velocity triangle.

| *At inlet velocity triangle:* | *At outlet velocity triangle:* |
|---|---|
| $V_1$ = absolute velocity of water | $V_2$ = absolute velocity of water |
| $u_1$ = peripheral velocity of runner (bucket speed) | $u_2$ = peripheral velocity of runner (bucket speed) |
| $Vr_1$ = relative velocity of water | $Vr_2$ = relative velocity of water |
| $Vw_1$ = velocity of whirl | $Vw_2$ = velocity of whirl |
| $Vf_1$ = velocity of flow | $Vf_2$ = velocity of flow |
| $\alpha$ = angle between the direction of the jet and the direction of motion of the vane (*guide blade angle*) | $\beta$ = angle between the direction of the jet and the direction of motion of the vane (*guide blade angle*) |
| $\theta$ = angle made by the relative velocity $Vr_1$ with the direction of motion (*vane angle*) | $\varphi$ = angle made by the relative velocity Vr2 with the direction of motion (*vane angle*) |
| *From inlet velocity triangle we obtain:* | *From outlet velocity triangle we obtain:* |
| $\alpha = 0$, $\theta = 0$, $Vf_1 = 0$<br>$V_1 = Vw_1 = u_1 + Vr_1$ and $Vr_1 = V_1 - u_1$ | $Vr_2 = Vr_1$<br>$Vw_2 = Vr_2 \cos \varphi - u_2$ |
| $u = u_1 = u_2 = \pi DN/60$, where D = diameter of wheel, N = speed in r.p.m | |




(Velocity triangle of Pelton turbine)

## Work done and power developed of a Pelton wheel:

Let, F = force exerted by the jet of water in the direction of motion

= mass x change in velocity in the direction of force

= m x ($Vw_1$ + $Vw_2$) = $\rho aV_1$ x ($Vw_1$ + $Vw_2$)

Where: ρ = density of water;

a = area of jet = $\frac{\pi}{4} \times d^2$

d = diameter of jet

Let, W = Net work done by the jet on runner per second = F x u

= $\rho aV_1$ x ($Vw_1$ + $Vw_2$) x u

Work done per second per unit weight of water striking = $\frac{\rho a V_1 \times (V_{w_1}+V_{w_2})\times u}{\rho a V_1 \times g} = \frac{(V_{w_1}+V_{w_2})\times u}{g}$

## NOTE:

Gross head ($H_g$): - Difference between the water level at head race and tail race

Net head (H): - Head available at the inlet (Effective head)

Absolute velocity can be obtained as: $V_1 = C_V \sqrt{2gH}$

Cv = coefficient of velocity of the nozzle

Velocity of wheel (bucket speed) = $u = \emptyset \times \sqrt{2gH}$

∅ is the speed ratio

## Efficiencies of turbine:

### 1) Hydraulic efficiency ($\eta_h$):

$$\eta_h = \frac{Work\ done\ per\ second}{Kinetic\ energy} = \frac{\rho a V_1 \times (Vw_1 \pm Vw_2)\times u}{\frac{1}{2}\times(\rho a V_1)\times V_1^2} = \frac{2\times(Vw_1 \pm Vw_2)\times u}{V_1^2}$$

It can also be obtained as:

$$\eta_h = \frac{Runner\ Power}{Water\ Power} = \frac{\rho a V_1 \times \frac{(Vw_1 \pm Vw_2)\times u}{1000}\ kW}{\frac{\rho g\ Q\ H}{1000}\ kW} = \frac{\rho \times Q \times (Vw_1 \pm Vw_2)\times u}{\rho g\ Q\ H}$$

$$= \frac{(Vw_1 \pm Vw_2)\times u}{g\ H}$$

**2) Mechanical efficiency ($\eta_m$):**

$$\eta_m = \frac{Shaft\ \ Power}{Runner\ \ Power} = \frac{P}{\rho a V_1 \times (Vw_1 \pm Vw_2) \times u}$$

**3) Volumetric efficiency** ($\eta_m$):

$$\eta_V = \frac{volume of\ \ water\ actually\ striking\ the\ runner}{total\ water\ \ given by the\ \ jet\ to\ the turbine} = \frac{Q_a}{Q}$$

**4) Overall efficiency** ($\eta_m$):

$$\eta_h = \frac{Shaft\ Power}{Water\ \ Power} = \frac{P}{\rho g\, Q\, H}$$

**Relationship between efficiencies:**

$$\eta_o = \eta_h \times \eta_m \times \eta_v$$

## Problem from Pelton Turbine:

**Problem-(1)** *A Pelton wheel has a mean bucket speed of 10 metres per second with a jet of water flowing at the rate of 700 litres/s under a head of 30 metres. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.*

**Solution:** Given :

Speed of bucket, $u = u_1 = u_2 = 10$ m/s

Discharge, $Q = 700$ litres/s $= 0.7$ m$^3$/s, Head of water, $H = 30$ m

Angle of deflection $= 160°$

$\therefore$ Angle, $\phi = 180° - 160° = 20°$

Co-efficient of velocity, $C_v = 0.98$.

The velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77$ m/s

$\therefore$ $V_{r_1} = V_1 - u_1 = 23.77 - 10$

$= 13.77$ m/s

$V_{w_1} = V_1 = 23.77$ m/s

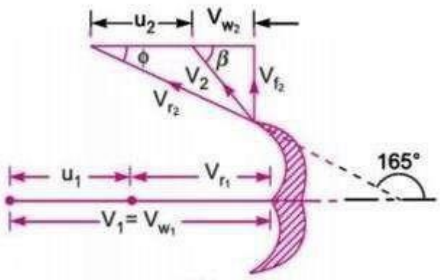


Fig. 18.6

From outlet velocity triangle,

$V_{r_2} = V_{r_1} = 13.77$ m/s

$V_{w_2} = V_{r_2} \cos\phi - u_2$

$= 13.77 \cos 20° - 10.0 = 2.94$ m/s

Work done by the jet per second on the runner is given by equation (18.9) as

$$= \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u$$

$$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10 \qquad (\because\ \ aV_1 = Q = 0.7\ \text{m}^3/\text{s})$$

$$= 186970\ \text{Nm/s}$$

$\therefore$ Power given to turbine $= \frac{186970}{1000} =$ **186.97 kW. Ans.**

The hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77}$$

$$= \mathbf{0.9454} \quad \text{or} \quad \mathbf{94.54\%.\ Ans.}$$

**Problem (2)** *A Pelton wheel is to be designed for the following specifications : Shaft power = 11,772 kW ; Head = 380 metres ; Speed = 750 r.p.m. ; Overall efficiency = 86% ; Jet diameter is not to exceed one-sixth of the wheel diameter. Determine :*

*(i) The wheel diameter,* *(ii) The number of jets required, and*

*(iii) Diameter of the jet.*

*Take $K_{v_1} = 0.985$ and $K_{u_1} = 0.45$*

**Solution.** Given :

Shaft power, S.P. = 11,772 kW

Head , $H = 380$ m

Speed, $N = 750$ r.p.m.

Overall efficiency, $\eta_0 = 86\%$ or 0.86

Ratio of jet dia. to wheel dia. $= \dfrac{d}{D} = \dfrac{1}{6}$

Co-efficient of velocity, $K_{v_1} = C_v = 0.985$

Speed ratio, $K_{u_1} = 0.45$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.985\sqrt{2 \times 9.81 \times 380} = 85.05$ m/s

The velocity of wheel, $u = u_1 = u_2$

$$= \text{Speed ratio} \times \sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 380} = 38.85 \text{ m/s}$$

But $$u = \frac{\pi DN}{60} \quad \therefore \quad 38.85 = \frac{\pi DN}{60}$$

or $$D = \frac{60 \times 38.85}{\pi \times N} = \frac{60 \times 38.85}{\pi \times 750} = \mathbf{0.989\ m.\ Ans.}$$

But $$\frac{d}{D} = \frac{1}{6}$$

$\therefore$ Dia. of jet, $$d = \frac{1}{6} \times \text{D} = \frac{0.989}{6} = \mathbf{0.165\ m.\ Ans.}$$

Discharge of one jet, $q$ = Area of jet × Velocity of jet

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4}(.165) \times 85.05 \text{ m}^3\text{/s} = 1.818 \text{ m}^3\text{/s} \qquad ...(i)$$

Now $$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{11772}{\dfrac{\rho g \times Q \times H}{1000}}$$

$$0.86 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 380}, \text{ where } Q = \text{Total discharge}$$

$\therefore$ Total discharge, $$Q = \frac{11772 \times 1000}{1000 \times 9.81 \times 380 \times 0.86} = 3.672 \text{ m}^3\text{/s}$$

$\therefore$ Number of jets $$= \frac{\text{Total discharge}}{\text{Discharge of one jet}} = \frac{Q}{q} = \frac{3.672}{1.818} = \mathbf{2\ jets.\ Ans.}$$

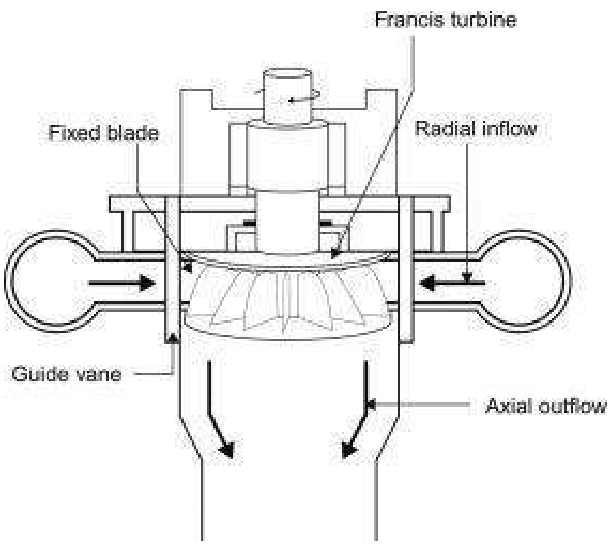
## Reaction Turbine – Francis Turbine:

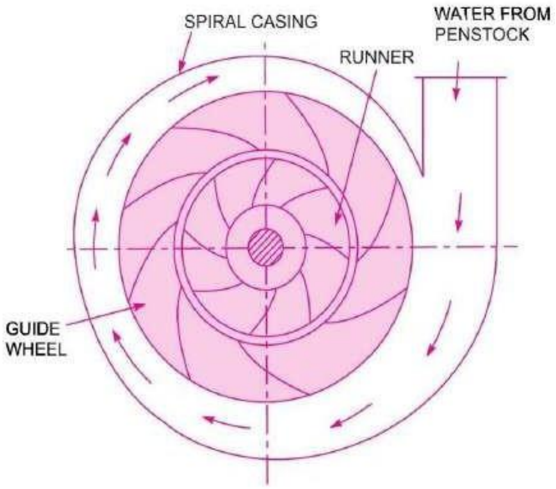
- Francis turbine is a medium head inward radial flow reaction turbine.
- Modern Francis turbine is an inward mixed flow reaction turbine. In this turbine, the water under pressure enters radially to the impeller blades while exits axially.
- When water flows radially from outward to inward, the turbine is called inward radial flow turbine
- When water flows radially from inward to outward, the turbine is called Outward radial flow turbine.
- An inward mixed flow reaction turbine, is a combination of impulse and reaction turbine where blades rotate using both reaction and impulse force of water flowing through them.

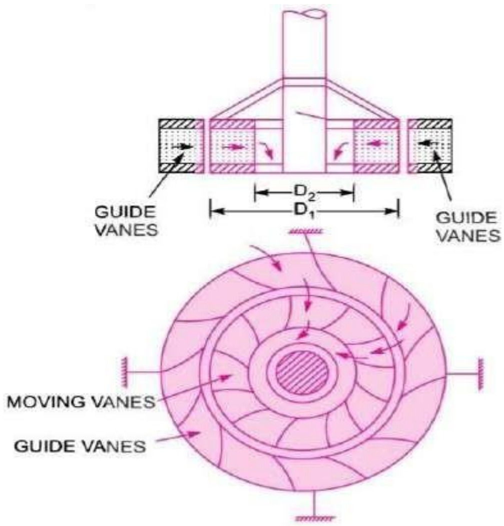
## Construction of Francis Turbine:

Major Components component of Francis turbine are described below.

- **Spiral/Scroll Casing:**
  - Its cross-sectional area is maximum at inlet and minimum at exit.
  - It encloses the turbine runner completely and prevents the splashing of water.
  - It maintains constant velocity throughout the circumference.
- **Runner with fixed blades:**
  - It is a circular wheel with a series of radially curved vanes which are fixed on its periphery.
  - It provides rotational energy due to impulse and reaction effects on runner.
- **Penstock:**
  - It is the channels or pipelines which conveys water from source to the power station
- **Governing Mechanism:**
  - It controls the speed and power output of the turbine by changing the position of guide blades to vary the water flow rate at variation of loads.








(FRANCIS TURBINE)

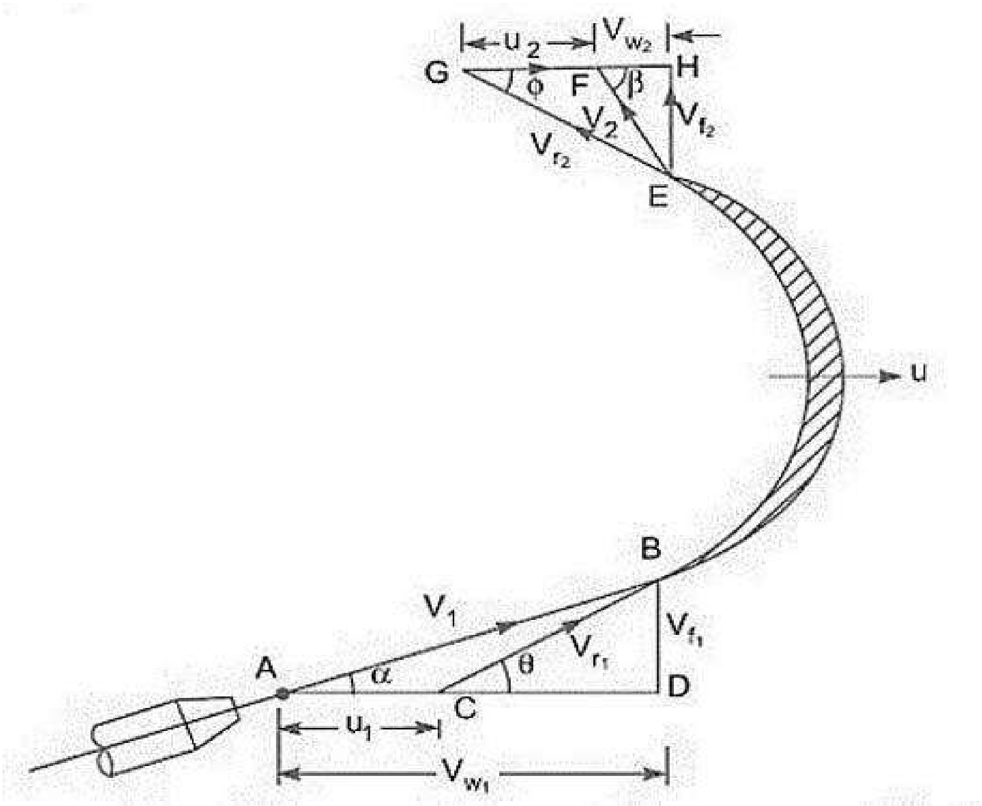
## Working Principle:

- In modern Francis turbine; water enters into the turbine with both pressure and kinetic energy.
- When water flows through the stationary parts, a part of its pressure energy is converted into kinetic energy. When water flows over the moving parts, there is change in pressure, absolute velocity and direction.
- The pressure difference between the blade and runner is known as the reaction pressure. This pressure results the motion of the runner.

## Velocity triangle of Francis turbine:

Consider the following terms for understanding the velocity triangle.

| *At inlet velocity triangle:* | *At outlet velocity triangle:* |
|---|---|
| $V_1$ = absolute velocity of water | $V_2$ = absolute velocity of water |
| $u_1$ = peripheral velocity of runner (bucket speed) | $u_2$ = peripheral velocity of runner (bucket speed) |
| $Vr_1$ = relative velocity of water | $Vr_2$ = relative velocity of water |
| $Vw_1$ = velocity of whirl | $Vw_2$ = velocity of whirl |
| $Vf_1$ = velocity of flow | $Vf_2$ = velocity of flow |
| $\alpha$ = angle between the direction of the jet and the direction of motion of the vane (*guide blade angle*) | $\beta$ = angle between the direction of the jet and the direction of motion of the vane (*guide blade angle*) |
| $\theta$ = angle made by the relative velocity $Vr_1$ with the direction of motion (*vane angle*) | $\varphi$ = angle made by the relative velocity Vr2 with the direction of motion (*vane angle*) |




(Velocity triangle of Francis turbine)

| *From inlet velocity triangle we obtain:* | *From outlet velocity triangle we obtain:* |
|---|---|
| $u_1 = \pi D_1 N_1/60$ | $u_2 = \pi D_2 N_2/60$ |
| where D = diameter of wheel, N = speed in r.p.m | |

## Work done and power developed of a Pelton wheel:

Let, F = force exerted by the jet of water in the direction of motion

= mass x change in velocity in the direction of force

= m x ($Vw_1 + Vw_2$) = $\rho a V_1$ x ($Vw_1 + Vw_2$)

Where: ρ = density of water;

a = area of jet = $\frac{\pi}{4} \times d^2$

d = diameter of jet

Let, W = Net work done by the jet on runner per second

= $\rho a V_1$ x ($Vw_1$ x $u_1$ + $Vw_2$ x $u_2$)

Work done per second per unit weight of water striking = $\frac{\rho a V_1 \times (V_{w_1} u_1 + V_{w_2} u_2)}{\rho a V_1 \times g} = \frac{(V_{w_1} u_1 + V_{w_2} u_2)}{g}$

For radial discharge: $\beta = 90^0$ and $Vw_2 = 0$; Output is maximum

Therefore: Work done per second per unit weight of water striking = $\frac{V_{w_1} u_1}{g}$

## Hydraulic efficiency:

$$\eta_h = \frac{Runner\ Power}{Water\ Power} = \frac{\rho a V_1 \times \left(Vw_1\, u_1 \pm Vw_2\, u_2\right)}{\rho g\, Q\, H} = \frac{Vw_1\, u_1 \pm Vw_2\, u_2}{g\ H}$$

For radial discharge: when $Vw_2 = 0$;

$$\eta_h = \frac{Vw_1 u_1}{g\ H}$$

**NOTE:**

$$speed\ ratio = \frac{u_1}{\sqrt{2g\ H}}$$

$$flow\ ratio = \frac{V_{f_1}}{\sqrt{2g\ H}}$$

Discharge of the turbine = Q = π x $D_1$ x $B_1$ x $Vf_1$ = π x $D_2$ x $B_2$ x $Vf_2$

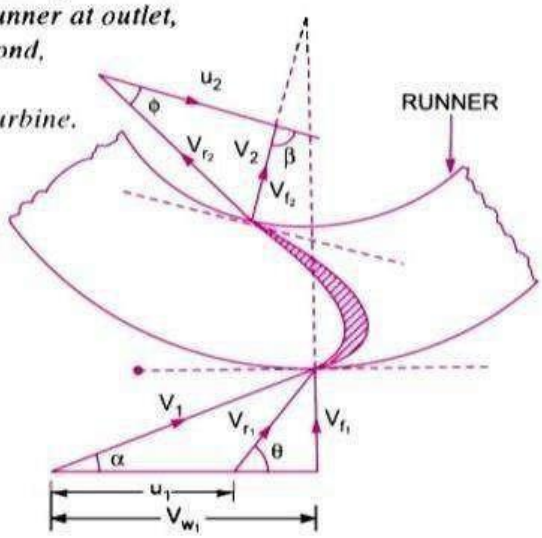
$D_1$ and $D_2$ are the diameter of runner at inlet and outlet respectively $B_1$ and $B_2$ are the width of runner at inlet and outlet respectively

$Vf_1$ and $Vf_2$ are the velocity of flow at the inlet and outlet respectively

## Problems from Francis Turbine

**Problem (3)** *An inward flow reaction turbine has external and internal diameters as 0.9 m and 0.45 m respectively. The turbine is running at 200 r.p.m. and width of turbine at inlet is 200 mm. The velocity of flow through the runner is constant and is equal to 1.8 m/s. The guide blades make an angle of 10° to the tangent of the wheel and the discharge at the outlet of the turbine is radial. Draw the inlet and outlet velocity triangles and determine:*

*(i) The absolute velocity of water at inlet of runner,*
*(ii) The velocity of whirl at inlet, (iii) The relative velocity at inlet,*
*(iv) The runner blade angles, (v) Width of the runner at outlet,*
*(vi) Mass of water flowing through the runner per second,*
*(vii) Head at the inlet of the turbine,*
*(viii) Power developed and hydraulic efficiency of the turbine.*




**Solution.** Given :

| | |
|---|---|
| External Dia., | $D_1 = 0.9$ m |
| Internal Dia., | $D_2 = 0.45$ m |
| Speed, | $N = 200$ r.p.m. |
| Width at inlet, | $B_1 = 200$ mm $= 0.2$ m |
| Velocity of flow, | $V_{f_1} = V_{f_2} = 1.8$ m/s |
| Guide blade angle, | $\alpha = 10°$ |
| Discharge at outlet | = Radial |
| ∴ | $\beta = 90°$ and $V_{w_2} = 0$ |

Tangential velocity of wheel at inlet and outlet are :

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times .9 \times 200}{60} = 9.424 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times .45 \times 200}{60} = 4.712 \text{ m/s.}$$

*(i) Absolute velocity of water at inlet of the runner i.e.,* $V_1$

From inlet velocity triangle,

$$V_1 \sin \alpha = V_{f_1}$$

$$\therefore \quad V_1 = \frac{V_{f_1}}{\sin \alpha} = \frac{18}{\sin 10^\circ} = \mathbf{10.365 \text{ m/s. Ans.}}$$

*(ii) Velocity of whirl at inlet, i.e.,* $V_{w_1}$

$$V_{w_1} = V_1 \cos \alpha = 10.365 \times \cos 10^\circ = \mathbf{10.207 \text{ m/s. Ans.}}$$

*(iii) Relative velocity at inlet, i.e.,* $V_{r_1}$

$$V_{r_1} = \sqrt{V_{r_1}{}^3 + (V_{w_1} - u_1)^2} = \sqrt{1.8^2 + (10.207 - 9.424)^2}$$

$$= \sqrt{3.24 + .613} = \mathbf{1.963 \text{ m/s. Ans.}}$$

*(iv) The runner blade angles* means the angle $\theta$ and $\phi$

Now $$\tan \theta = \frac{V_{f_1}}{\left(V_{w_1} - u_1\right)} = \frac{1.8}{(10.207 - 9.424)} = 2.298$$

$$\therefore \quad \theta = \tan^{-1} 2.298 = \mathbf{66.48^\circ} \text{ or } \mathbf{66^\circ\ 29'}. \textbf{ Ans.}$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2} = \frac{1.8}{4.712} = \tan 20.9^\circ$$

$$\therefore \quad \phi = \mathbf{20.9^\circ} \text{ or } \mathbf{20^\circ\ 54.4'}. \textbf{ Ans.}$$

*(v) Width of runner at outlet, i.e.,* $B_2$

From equation (18.21), we have

$$\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2} \text{ or } D_1 B_1 = D_2 B_2 \qquad \left(\because \quad \pi V_{f_1} = \pi V_{f_2} \text{ as } V_{f_1} = V_{f_2}\right)$$

$$\therefore \quad B_2 = \frac{D_1 B_1}{D_2} = \frac{0.90 \times 0.20}{0.45} = 0.40 \text{ m} = \mathbf{400 \text{ mm. Ans.}}$$

*(vi) Mass of water flowing through the runner per second.*

The discharge, $$Q = \pi D_1 B_1 V_{f_1} = \pi \times 0.9 \times 0.20 \times 1.8 = 1.0178 \text{ m}^3\text{/s.}$$

$$\therefore \quad \text{Mass} = \rho \times Q = 1000 \times 1.0178 \text{ kg/s} = \mathbf{1017.8 \text{ kg/s. Ans.}}$$

*(vii) Head at the inlet of turbine, i.e., H.*

Using equation (18.24), we have

$$H - \frac{V_2^2}{2g} = \frac{1}{g}(V_{w_1} u_1 \pm V_{w_2} u_2) = \frac{1}{g}(V_{w_1} u_1) \qquad (\because \text{ Here } V_{w_2} = 0)$$

$$H = \frac{1}{g} V_{w_1} u_1 + \frac{V_2^2}{2g} = \frac{1}{9.81} \times 10.207 \times 9.424 + \frac{1.8^2}{2 \times 9.81} \quad \left(\because V_2 = V_{f_2}\right)$$

$$= 9.805 + 0.165 = \mathbf{9.97 \text{ m. Ans.}}$$

*(viii) Power developed, i.e.,* $$P = \frac{\text{Work done per second on runner}}{1000}$$

$$= \frac{\rho Q \left[V_{w_1} u_1\right]}{1000} \qquad \text{[Using equation (18.18)]}$$

$$= 1000 \times \frac{1.0178 \times 10.207 \times 9.424}{1000} = \mathbf{97.9 \text{ kW. Ans.}}$$

Hydraulic efficiency is given by equation (18.20*B*) as

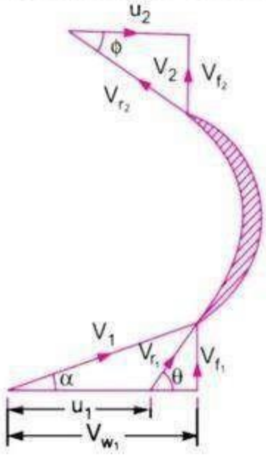
$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{10.207 \times 9.424}{9.81 \times 9.97} = 0.9834 = \mathbf{98.34\%. \text{ Ans.}}$$

**Problem (4)** *A reaction turbine works at 450 r.p.m. under a head of 120 metres. Its diameter at inlet is 120 cm and the flow area is 0.4 m². The angles made by absolute and relative velocities at inlet are 20° and 60° respectively with the tangential velocity. Determine :*

*(a) The volume flow rate, (b) The power developed, and*

*(c) Hydraulic efficiency.*

*Assume whirl at outlet to be zero.*




**Solution.** Given :

Speed of turbine, $N = 450$ r.p.m.

Head, $H = 120$ m

Diameter at inlet, $D_1 = 120$ cm $= 1.2$ m

Flow area, $\pi D_1 \times B_1 = 0.4\ \text{m}^2$

Angle made by absolute velocity at inlet, $\alpha = 20°$

Angle made by the relative velocity at inlet, $\theta = 60°$

Whirl at outlet, $V_{w_2} = 0$

Tangential velocity of the turbine at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 \text{ m/s}$$

From inlet velocity triangle,

$$\tan \alpha = \frac{V_{f_1}}{V_{w_1}} \text{ or } \tan 20° = \frac{V_{f_1}}{V_{w_1}} \text{ or } \frac{V_{f_1}}{V_{w_1}} = \tan 20° = 0.364$$

$$\therefore \quad V_{f_1} = 0.364\, V_{w_1}$$

Also $$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{0.364\, V_{w_1}}{V_{w_1} - 28.27} \qquad (\because\ V_{f_1} = 0.364\, V_{w_1})$$

or $$\frac{0.364\, V_{w_1}}{V_{w_1} - 28.27} = \tan \theta = \tan 60° = 1.732$$

$$\therefore \quad 0.364 V_{w_1} = 1.732(V_{w_1} - 28.27) = 1.732 V_{w_1} - 48.96$$

or $$(1.732 - 0.364)\, V_{w_1} = 48.96$$

$$\therefore \quad V_{w_1} = \frac{48.96}{(1.732 - 0.364)} = 35.789 = 35.79 \text{ m/s.}$$

From equation (*i*), $$V_{f_1} = 0.364 \times V_{w_1} = 0.364 \times 35.79 = 13.027 \text{ m/s.}$$

(*a*) Volume flow rate is given by equation (18.21) as $Q = \pi D_1 B_1 \times V_{f_1}$

But $\pi D_1 \times B_1 = 0.4\ \text{m}^2$ (given)

$$Q = 0.4 \times 13.027 = \mathbf{5.211\ m^3/s.\ Ans.}$$

(*b*) Work done per sec on the turbine is given by equation (18.18),

$$= \rho Q\, [V_{w_1} u_1] \qquad (\because\ V_{w_2} = 2)$$

$$= 1000 \times 5.211\, [35.79 \times 28.27] = 5272402 \text{ Nm/s}$$

$$\therefore \quad \text{Power developed in kW} = \frac{\text{Work done per second}}{1000} = \frac{5272402}{1000} = \mathbf{5272.402\ kW.\ Ans.}$$

(*c*) The hydraulic efficiency is given by equation (18.20*B*) as

$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{35.79 \times 28.27}{9.81 \times 120} = 0.8595 = \mathbf{85.95\%.\ Ans.}$$

**Problem (5)** *As inward flow reaction turbine has external and internal diameters as 1.0 m and 0.6 m respectively. The hydraulic efficiency of the turbine is 90% when the head on the turbine is 36 m. The velocity of flow at outlet is 2.5 m/s and discharge at outlet is radial. If the vane angle at outlet is 15° and width of the wheel is 100 mm at inlet and outlet, determine : (i) the guide blade angle, (ii) speed of the turbine, (iii) vane angle of the runner at inlet, (iv) volume flow rate of turbine and (v) power developed.*

**Solution.** Given :

External diameter, $D_1 = 1.0$ m

Internal diameter, $D_2 = 0.6$ m

Hydraulic efficiency, $\eta_h = 90\% = 0.90$

Head, $H = 36$ m

Velocity of flow at outlet, $V_{f_2} = 2.5$ m/s

Discharge is radial, $V_{w_2} = 0$

Vane angle at outlet, $\phi = 15°$

Width of wheel, $B_1 = B_2 = 100$ mm $= 0.1$ m

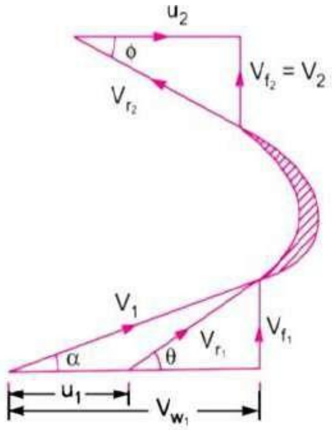


Fig. 18.14

Using equation (18.20 *B*) for hydraulic efficiency as

$$\eta_h = \frac{V_{w_1} u_1}{gH} \text{ or } 0.90 = \frac{V_{w_1} . u_1}{9.81 \times 36}$$

$$\therefore \quad V_{w_1} u_1 = 0.90 \times 9.81 \times 36 = 317.85 \quad ...(i)$$

From outlet velocity triangle, $\tan \phi = \frac{V_{f_2}}{u_2} = \frac{2.5}{u_2}$

$$\therefore \quad u_2 = \frac{2.5}{\tan \phi} = \frac{2.5}{\tan 15^\circ} = 9.33$$

But

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times N}{60}$$

$$\therefore \quad 9.33 = \frac{\pi \times 0.6 \times N}{60} \text{ or } N = \frac{60 \times 9.33}{\pi \times 0.6} = \mathbf{296.98.\ Ans.}$$

$$\therefore \quad u_1 = \frac{\pi \times D_1 \times N}{60} = \frac{\pi \times 1.0 \times 296.98}{60} = 15.55 \text{ m/s.}$$

Substituting this value of '$u_1$' in equation (*i*),

$$V_{w_1} \times 15.55 = 317.85$$

$$\therefore \quad V_{w_1} = \frac{317.85}{15.55} = 20.44 \text{ m/s}$$

Using equation (18.21), $\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$ or $D_1 V_{f_1} = D_2 V_{f_2}$ $\quad (\because \; B_1 = B_2)$

$$\therefore \quad V_{f_1} = \frac{D_2 \times V_{f_2}}{D_1} = \frac{0.6 \times 2.5}{1.0} = 1.5 \text{ m/s.}$$

(*i*) *Guide blade angle* ($\alpha$).

From inlet velocity triangle, $\tan \alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{1.5}{20.44} = 0.07338$

$\therefore$ $\alpha = \tan^{-1} 0.07338 = \mathbf{4.19°}$ or $\mathbf{4° \; 11.8'}$. **Ans.**

(*ii*) *Speed of the turbine,* $N =$ **296.98 r.p.m. Ans.**

(*iii*) *Same angle of runner at inlet* ($\theta$)

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{1.5}{(20.44 - 15.55)} = 0.3067$$

$\therefore$ $\theta = \tan^{-1} .3067 = 17.05°$ or $\mathbf{17° \; 3'}$. **Ans.**

(*iv*) *Volume flow rate of turbine* is given by equation (18.21) as

$$= \pi D_1 B_1 V_{f_1} = \pi \times 1.0 \times 0.1 \times 1.5 = \mathbf{0.4712 \; m^3/s}. \textbf{ Ans.}$$

(*v*) *Power developed* (in kW)

$$= \frac{\text{Work done per second}}{1000} = \frac{\rho Q \left[V_{w_1} u_1\right]}{1000}$$

[Using equation (18.18) and $V_{w_2} = 0$]

$$= 1000 \times \frac{0.4712 \times 20.44 \times 15.55}{1000} = \mathbf{149.76 \; kW}. \textbf{ Ans.}$$

## Axial flow reaction Turbine:

- In an axial flow reaction turbine water flows parallel to the axis of rotation of the shaft of the turbine.
- It has a vertical shaft with larger lower end known as hub/boss.
- Vanes are fixed on the hub and so the hub works like a runner.
- It requires large quantity of water at low head

## Classification of Axial flow reaction Turbine:

Axial flow reactions are classified as:

- Propellor turbine
  - Propeller turbine is the axial flow reaction turbine which has not adjustable fixed vanes.
- Kaplan turbine
  - Kaplan turbine is the axial flow reaction turbine which has adjustable vanes.

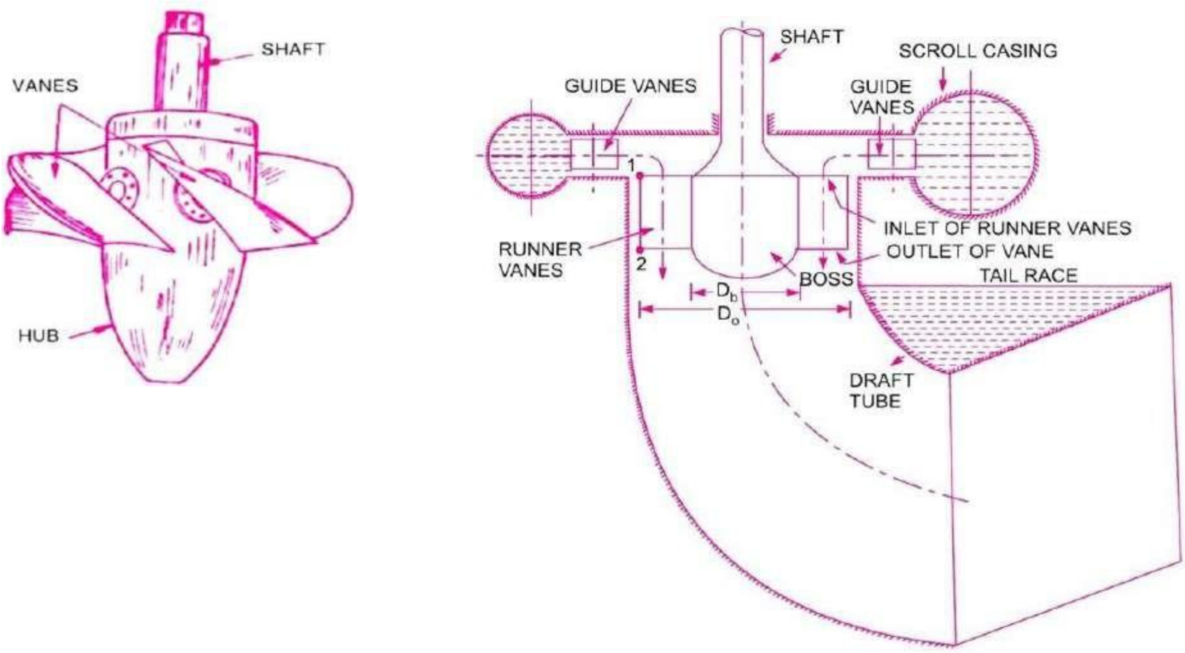
## Kaplan Turbine:

- It is an axial flow reaction turbine in which water flows parallel to the axis of rotation of the shaft of the turbine. The water enters the runner of turbine in an axial direction and leaves the runner axially.
- It has a vertical shaft with larger lower end known as hub/boss.
- Vanes are fixed on the hub and so the hub works like a runner.
- It requires large quantity of water at low head

## Construction of Kaplan Turbine:

It consists of the following major parts.

- Scroll Casing:
    - It encloses the turbine runner completely and prevents the splashing of water.
    - Cross-section of scroll casing decreases uniformly to maintain the pressure of water such that the flow pressure is not lost.
    - From the scroll casing the guide vanes direct the water to the runner.
- Guide vanes mechanism:
    - The guide vanes are adjustable and can be adjusted to meet the required flow rate.
    - Guide vanes also control the swirl of the water flow.
- Hub with vanes:
    - The vanes are fixed on the hub and hence hub acts as a runner for the axial flow reaction turbine.
- Draft tube:
    - The draft tube is a connecting pipe whose inlet is fitted at the outlet of the turbine.
    - The diameter of the draft tube is small near its inlet and large near its outlet. The outlet of the draft tube is always submerged in water.
    - It **c**onverts the kinetic energy of the water to static pressure at the outlet of the turbine**.** So pressure of the exit fluid increases. This helps to avoid the dissipation of the kinetic energy of the exit water. It improves the capacity of the turbine.




(KAPLAN TURBINE)

**Key points for Kaplan Turbine:**

1. Discharge through the runner: $Q = \frac{\pi}{4} \times (D_o^2 - D_b^2) \times V_{f1}$

Where: $D_o$ = diameter of the runner $D_b$ =

diameter of the hub/boss

$V_{f1}$ = velocity of flow at inlet

2. Area of flow at inlet = Area of flow at outlet = $A = \frac{\pi}{4} \times (D_o^2 - D_b^2)$
3. Peripheral velocity at inlet and outlet are equal: $u_1 = u_2 = \frac{\pi D_o N}{60}$
4. Velocity of flow at inlet ($V_{f1}$) = Velocity of flow at outlet ($V_{f2}$)
5. Speed ratio = $\frac{u_1}{\sqrt{2gH}}$
6. Flow ratio = $\frac{V_{f1}}{\sqrt{2gH}}$
7. Water power = $\frac{\rho g Q H}{1000}$
8. Runner Power = $\frac{1}{g} \times \frac{V_{w1} u_1 + V_{w1} u_2}{1000}$
9. Hydraulic efficiency = $\frac{V_{w1} u_1}{gH}$ (for radial discharge)
10. Overall efficiency = $\frac{SP}{WP}$
11. Specific speed pf turbine = $N_s = \frac{N\sqrt{P}}{H^{5/4}}$

## Problems from Kaplan Turbine:

**Problem (6)** *A Kaplan turbine working under a head of 20 m develops 11772 kW shaft power. The outer diameter of the runner is 3.5 m and hub diameter is 1.75 m. The guide blade angle at the extreme edge of the runner is 35°. The hydraulic and overall efficiencies of the turbines are 88% and 84% respectively. If the velocity of whirl is zero at outlet, determine :*

*(i) Runner vane angles at inlet and outlet at the extreme edge of the runner, and*

*(ii) Speed of the turbine.*

**Solution.** Given :

| | |
|---|---|
| Head, | $H = 20$ m |
| Shaft power, | S.P. = 11772 kW |
| Outer dia. of runner, | $D_o = 3.5$ m |
| Hub diameter, | $D_b = 1.75$ m |
| Guide blade angle, | $\alpha = 35°$ |
| Hydraulic efficiency, | $\eta_h = 88\%$ |
| Overall efficiency, | $\eta_o = 84\%$ |
| Velocity of whirl at outlet | = 0. |

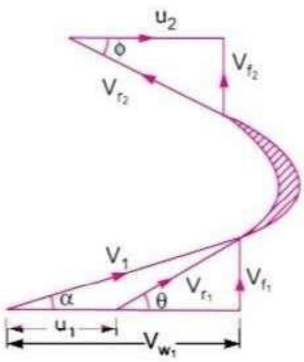


Fig. 18.27

Using the relation, $\eta_o = \frac{\text{S.P.}}{\text{W.P.}}$

$$\text{W.P.} = \frac{\text{W.P.}}{1000} = \frac{\rho \times g \times Q \times H}{1000}, \text{ we get}$$

$$0.84 = \frac{11772}{\frac{\rho \times g \times Q \times H}{1000}}$$

$$= \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 20} \qquad (\because \ \rho = 1000)$$

$$\therefore \qquad Q = \frac{11772 \times 1000}{0.84 \times 1000 \times 9.81 \times 20} = 71.428 \text{ m}^3\text{/s.}$$

Using equation (18.25), $Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$

or $$71.428 = \frac{\pi}{4}(3.5^2 - 1.75^2) \times V_{f_1} = \frac{\pi}{4}(12.25 - 3.0625)\, V_{f_1}$$

$$= 7.216\, V_{f_1}$$

$$\therefore \qquad V_{f_1} = \frac{71.428}{7.216} = 9.9 \text{ m/s.}$$

From inlet velocity triangle, $\tan \alpha = \frac{V_{f_2}}{V_{w_1}}$

$$\therefore \qquad V_{w_1} = \frac{V_{f_1}}{\tan \alpha} = \frac{9.9}{\tan 35^\circ} = \frac{9.9}{.7} = 14.14 \text{ m/s}$$

Using the relation for hydraulic efficiency,

$$\eta_h = \frac{V_{w_1} u_1}{gH} \qquad \left(\because \ V_{w_2} = 0\right)$$

$$0.88 = \frac{14.14 \times u_1}{9.81 \times 20}$$

$$\therefore \qquad u_1 = \frac{0.88 \times 9.81 \times 20}{14.14} = 12.21 \text{ m/s.}$$

(*i*) Runner vane angles at inlet and outlet at the extreme edge of the runner are given as :

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{9.9}{(14.14 - 12.21)} = 5.13$$

$$\therefore \qquad \theta = \tan^{-1} 5.13 = \mathbf{78.97^\circ} \text{ or } \mathbf{78^\circ\ 58'}. \ \textbf{Ans.}$$

For Kaplan turbine, $u_1 = u_2 = 12.21$ m/s and $V_{f_1} = V_{f_2} = 9.9$ m/s

$\therefore$ From outlet velocity triangle, $\tan \phi = \frac{V_{f_2}}{u_2} = \frac{9.9}{12.21} = 0.811$

$$\therefore \qquad \phi = \tan^{-1} .811 = \mathbf{39.035^\circ} \text{ or } \mathbf{39^\circ\ 2'}. \ \textbf{Ans.}$$

(*ii*) Speed of turbine is given by $u_1 = u_2 = \frac{\pi D_o N}{60}$

$$12.21 = \frac{\pi \times 3.5 \times N}{60}$$

$$\therefore \qquad N = \frac{60 \times 12.21}{\pi \times 3.50} = \mathbf{66.63\ r.p.m.}\ \textbf{Ans.}$$

**Problem (7)** *A Kaplan turbine develops 24647.6 kW power at an average head of 39 metres. Assuming a speed ratio of 2, flow ratio of 0.6, diameter of the boss equal to 0.35 times the diameter of the runner and an overall efficiency of 90%, calculate the diameter, speed and specific speed of the turbine.*

Using the relation, $\eta_o = \frac{\text{S.P.}}{\text{W.P.}}$, where W.P. $= \frac{\rho \times g \times Q \times H}{1000}$

$$\therefore \quad 0.90 = \frac{24647.6}{\frac{\rho \times g \times Q \times H}{1000}} = \frac{24647.6 \times 1000}{1000 \times 9.81 \times Q \times 39}$$

$$\therefore \quad Q = \frac{24647.6 \times 1000}{0.9 \times 1000 \times 9.81 \times 39} = 71.58 \text{ m}^3/\text{s.} \quad 5.32 \text{ m/s}$$

Flow ratio, $\frac{v_{f_1}}{\sqrt{2gH}} = 0.6$

$$\therefore \quad V_{f_1} = 0.6 \times \sqrt{2gH} = 0.6 \times \sqrt{2 \times 9.81 \times 39} = 16.59 \text{ m/s}$$

Diameter of boss $= 0.35 \times$ Diameter of runner

$$\therefore \quad D_b = 0.35 \times D_o$$

Overall efficiency, $\eta_o = 90\% = 0.90$

But from equation (18.25), we have

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$$

$$\therefore \quad 71.58 = \frac{\pi}{4}\left[D_o^2 - (.35\, D_o)^2\right] \times 16.59 \qquad \left(\because \; D_b = 0.35\, D_{o,}\; V_{f_1} = 16.59\right)$$

$$= \frac{\pi}{4}\left[D_o^2 - .1225\, D_o^2\right] \times 16.59$$

$$= \frac{\pi}{4} \times .8775\, D_o^2 \times 16.59 = 11.433\, D_o^2$$

(*i*) $\therefore \quad D_o = \sqrt{\frac{71.58}{11.433}} = \mathbf{2.5 \text{ m. Ans.}}$

$$\therefore \quad D_b = 0.35 \times D_o = 0.35 \times 2.5 = \mathbf{0.875 \text{ m. Ans.}}$$

(*ii*) Speed of the turbine is given by $u_1 = \frac{\pi D_o N}{60}$

$$\therefore \quad 55.32 = \frac{\pi \times 2.5 \times N}{60}$$

$$\therefore \quad N = \frac{60 \times 55.32}{\pi \times 2.5} = \mathbf{422.61 \text{ r.p.m. Ans.}}$$

(*iii*) Specific speed * is given by $N_s = \frac{N\sqrt{P}}{H^{5/4}}$, where $P$ = Shaft power in kW

$$\therefore \quad N_s = \frac{422.61 \times \sqrt{24647.6}}{(39)^{5/4}} = \frac{422.61 \times 156.99}{97.461} = \mathbf{680.76 \text{ r.p.m. Ans.}}$$

**Problem (8)** *The hub diameter of a Kaplan turbine, working under a head of 12 m, is 0.35 times the diameter of the runner. The turbine is running at 100 r.p.m. If the vane angle of the extreme edge of the runner at outlet is 15° and flow ratio is 0.6, find :*

*(i) Diameter of the runner,* *(ii) Diameter of the boss, and*

*(iii) Discharge through the runner.*

*The velocity of whirl at outlet is given as zero.*

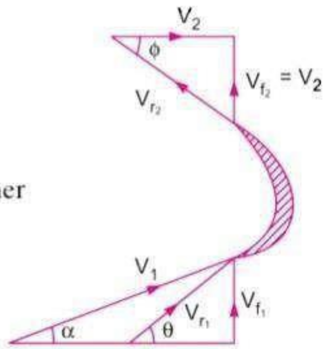



Fig. 18.28

**Solution.** Given :

Head, $H = 12$ m

Hub diameter, $D_b = 0.35 \times D_o$, where $D_o$ = Dia. of runner

Speed, $N = 100$ r.p.m.

Vane angle at outlet, $\phi = 15°$

Flow ratio $= \dfrac{V_{f_1}}{\sqrt{2gH}} = 0.6$

$$\therefore \quad V_{f_1} = 0.6 \times \sqrt{2gH} = 0.6 \times \sqrt{2 \times 9.81 \times 12}$$
$$= 9.2 \text{ m/s.}$$

From the outlet velocity triangle, $V_{w_2} = 0$

$$\tan \phi = \frac{V_{f_2}}{u_2} = \frac{V_{f_1}}{u_2}$$

$$\therefore \quad \tan 15° = \frac{9.2}{u_2}$$

$$\therefore \quad u_2 = \frac{9.2}{\tan 15°} = 34.33 \text{ m/s.}$$

But for Kaplan turbine, $u_1 = u_2 = 34.33$

Now, using the relation, $u_1 = \dfrac{\pi D_o \times N}{60}$ or $34.33 = \dfrac{\pi \times D_o \times 100}{60}$

$$D_o = \frac{60 \times 34.33}{\pi \times 100} = \textbf{6.55 m. Ans.}$$

$$\therefore \quad D_b = 0.35 \times D_o = 0.35 \times 6.35 = \textbf{2.3 m. Ans.}$$

Discharge through turbine is given by equation (18.25) as

$$Q = \frac{\pi}{4}\left[D_o^2 - D_b^2\right] \times V_{f_1} = \frac{\pi}{4}\left[6.55^2 - 2.3^2\right] \times 9.2$$

$$= \frac{\pi}{4}(42.9026 - 5.29) \times 9.2 = \textbf{271.77 m}^3\textbf{/s. Ans.}$$

## Difference between Impulse and Reaction turbine:

| **Impulse Turbine** | **Reaction Turbine** |
|---|---|
| The water flows through the nozzles and impinges on the moving blades | The water flows first through guide mechanism and then through the moving blades |
| The water impinges on the buckets with kinetic energy | The water glides over the moving vanes with pressure and kinetic energy |
| The water may or may not be admitted over the whole circumference. | The water must be admitted over the whole circumference |
| The water pressure remains constant during its flow through the moving blades. | The water pressure is reduced during its flow through the moving blades. |
| The relative velocity of water while gliding over the blades remains constant. | The relative velocity of water while gliding over the moving blades increase |
| The blades are symmetrical | The blades are not symmetrical |
| The number of stages required is less for the same power developed. | The number of stages required is more for the same power developed |

# (CENTRIFUGAL PUMPS)

## Introduction:

- It is a hydraulic machine in which force is transmitted by means of motion of fluid under pressure.
- In this machine, mechanical energy is converted into hydraulic energy in the form of pressure energy by the action of centrifugal force on the fluid.
- Its main purpose is to transfer fluids through an increase in pressure.
- It acts as a reverse of an inward flow reaction turbine.
- It is used in the field of agriculture, municipality, industries, power plants, petrochemicals, mining etc.

## Construction:

Major components of Centrifugal pump are:

- **Casing:**

It is an air tight passage which surrounds the impeller. It converts kinetic energy of water into pressure energy with its special design. It is classified as:

- Volute casing
  - *Volute casing* is the spiral casing in which the area of flow increases from inlet to outlet. This gradual increase in area helps to reduce the velocity of flow and increase the pressure at outlet. Due to formation of eddies, there is a limitation of energy loss.
- Vortex casing
  - In *Vortex casing* a circular chamber is provided in between the impeller and casing. This decreases the energy loss formation of eddies. It helps to increase the efficiency of the pump.
- Casing with guide blades
  - In *Casing with guide blades* a series of guide blades mounted on a ring surrounds the impeller. This helps to control the velocity and pressure of water by adjusting the guide blades.

- **Impeller:**

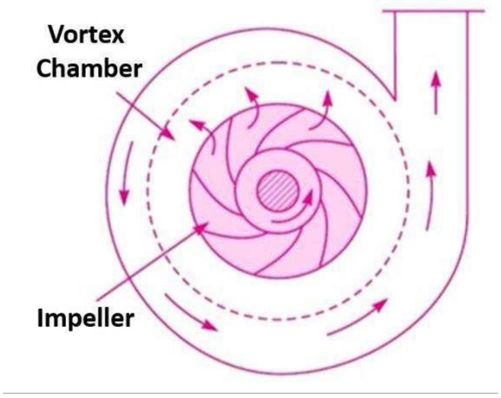
It is a wheel or rotor which is provided with a series of backward curved blades or vanes. It is mounted on the shaft powered by motor.

- **Suction pipe with foot valve and strainer:**

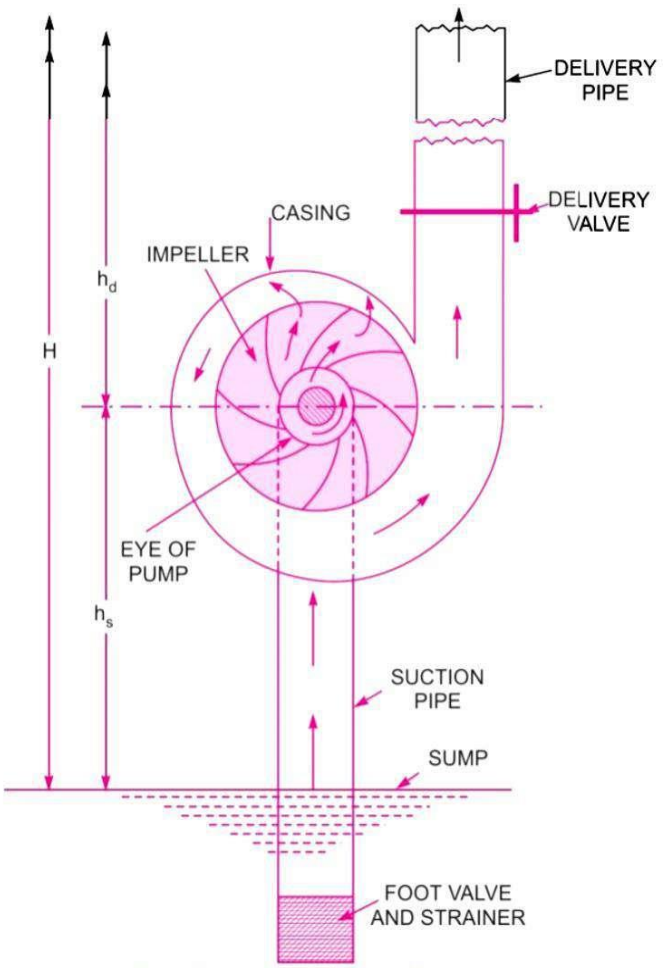
It's one end connects the inlet of the impeller and the other end is dipped into the sump of water. The foot valve fitted to the bottom of suction pipe is a one way valve that opens in the upward direction. The strainer fitted to the bottom of suction pipe is used to filter the unwanted particle present in the water to prevent the centrifugal pump from blockage.

- **Delivery pipe and delivery valve:**

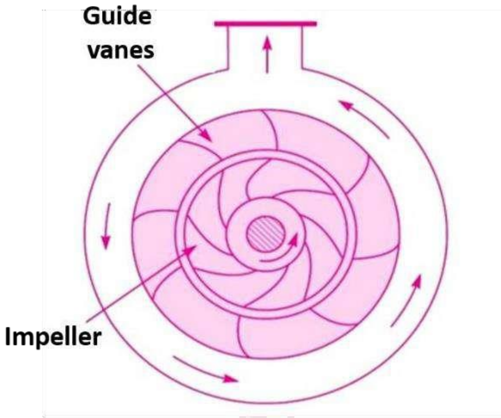
It's one end connects the outlet of the pump and other end connects the point where water is delivered. A delivery valve is fitted with the outlet controls the flow from the pump into delivery pipe.




Vortex Casing

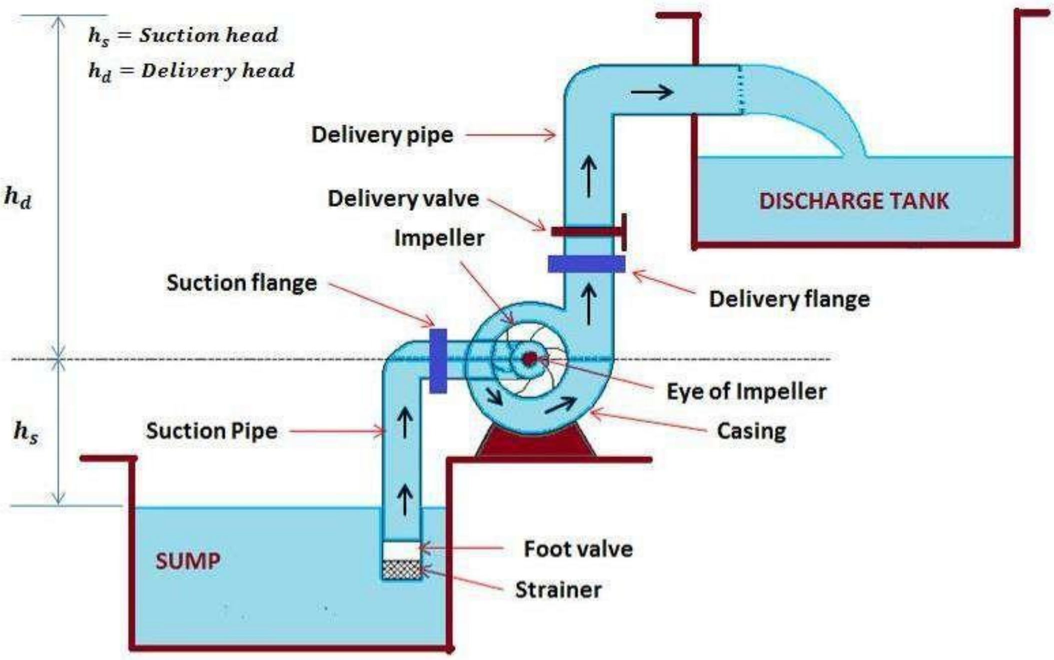



Centrifugal pump with Volute Casing




Casing with guide vanes

## Working of Centrifugal Pump:

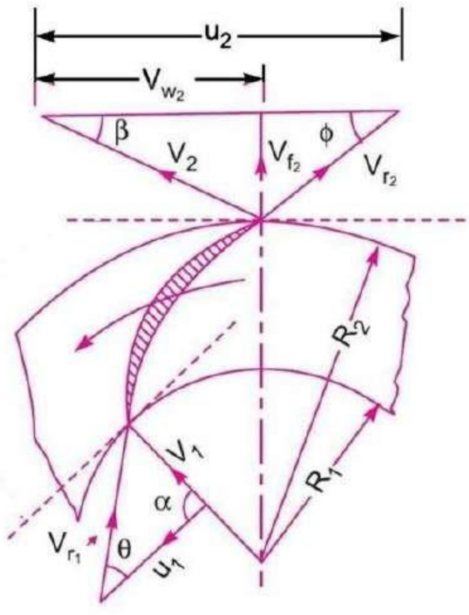


- When the electric motor starts, the shaft of the pump coupled with the motor shaft rotates. It gives rotational motion to the impeller mounted on the shaft.
- The rotating impeller drives the water inside it and produce centrifugal force. This creates velocity difference between the inlet and outlet.
- It causes the rising of water from sump through suction pipe to eye of the impeller.
- When water gets pressurized, the delivery valve opens to discharge water to desired height.
- **Priming** is the operation in which water is feed into the casing and suction pipeline keeping the delivery valve closed, so that all the air from the pump is driven out and no air is left.

## Velocity triangle of Centrifugal pump:

Consider the following terms for understanding the velocity triangle.

| *At inlet velocity triangle:* | *At outlet velocity triangle:* |
|---|---|
| $V_1$ = absolute velocity of water | $V_2$ = absolute velocity of water |
| $u_1$ = peripheral velocity of runner (bucket speed) | $u_2$ = peripheral velocity of runner (bucket speed) |
| $Vr_1$ = relative velocity of water | $Vr_2$ = relative velocity of water |
| $Vw_1$ = velocity of whirl | $Vw_2$ = velocity of whirl |
| $Vf_1$ = velocity of flow | $Vf_2$ = velocity of flow |
| $\alpha$ = angle between the direction of the jet and the direction of motion of the vane (*guide blade angle*) | $\beta$ = angle between the direction of the jet and the direction of motion of the vane (*guide blade angle*) |
| $\theta$ = angle made by the relative velocity $Vr_1$ with the direction of motion (*vane angle*) | $\varphi$ = angle made by the relative velocity Vr2 with the direction of motion (*vane angle*) |



When water enters the impeller radially. at inlet

**$\alpha = 90^0$ , $Vw_1 = 0$, $Vf_1 = V_1$**

$$u_1 = \frac{\pi D_1 N_1}{60} \quad and \quad u_2 = \frac{\pi D_2 N_2}{60}$$

$$Volume of water per\ (Q) = \pi D_1 N_1 V_{f_1} = \pi D_2 N_2 V_{f_2}$$

Let, W = Net work done by the jet on runner per second = $\rho a V_1$ x ($Vw_2$ x $u_2$)

Work done per second per unit weight of water striking = ($Vw_2$ x $u_2$)/g

## Heads of Centrifugal pump:

### Suction head ($h_s$)

It is the vertical distance between the centre line of pump and the water surface at sump level.

### Delivery head ($h_d$)

It is the vertical distance between the centre line of pump and the water surface at the discharge tank.

### Static head (H)

It is the sum of suction and delivery head.

### Manometric head ($H_m$)

It is the working head of the centrifugal pump.

It is given by:

$H_m$ **= (**Head imparted by impeller to the water – loss of head in the pump)

$$H_m = \frac{(Vw_2 \times u_2)}{g} - loss\ of\ head$$

*If loss of head is neglected*

$$H_m = \frac{(Vw_2 \times u_2)}{g}$$

## Efficiencies of Centrifugal pump:

### Manometric efficiency ($\eta_{man}$)

It is the ratio between manometric head and the head imparted by the impeller to the water.

$$\eta_{man} = \frac{H_m}{\left(\frac{Vw_2 \times u_2}{g}\right)} = \frac{g H_m}{Vw_2 \times u_2}$$

**Mechanical efficiency ($\eta_m$)**

It is the ratio between the power at the impeller and the power at the shaft.

$$\eta_m = \frac{\rho \times Q \times Vw_2 \times u_2}{S.P}$$

**Overall efficiency ($\eta_o$)**

It is the ratio between the power output of the pump and the power input of the pump.

$$\eta_o = \frac{S.P}{\left( \frac{g H_m}{Vw_2 \times u_2} \right)}$$

**Relation between $\eta_{man}$, $\eta_m$ & $\eta_o$**

$$\eta_o = \eta_{man}$$

## Problem from Pelton Turbine:

**Problem (1)** *The internal and external diameters of the impeller of a centrifugal pump are 200 mm and 400 mm respectively. The pump is running at 1200 r.p.m. The vane angles of the impeller at inlet and outlet are 20° and 30° respectively. The water enters the impeller radially and velocity of flow is constant. Determine the work done by the impeller per unit weight of water.*

**Solution.** Given :

Internal diameter of impeller, $D_1 = 200$ mm $= 0.20$ m

External diameter of impeller, $D_2 = 400$ mm $= 0.40$ m

Speed, $N = 1200$ r.p.m.

Vane angle at inlet, $\theta = 20°$

Vane angle at outlet, $\phi = 30°$

Water enters radially* means, $\alpha = 90°$ and $V_{w_1} = 0$

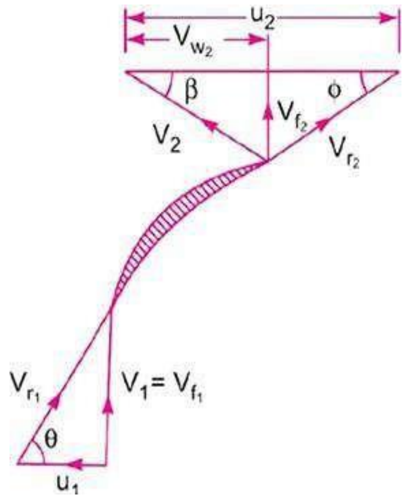
Velocity of flow, $V_{f_1} = V_{f_2}$

Tangential velocity of impeller at inlet and outlet are,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.20 \times 1200}{60} = 12.56 \text{ m/s}$$

and

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.13 \text{ m/s.}$$




From inlet velocity triangle, $\tan\theta = \frac{V_{f_1}}{u_1} = \frac{V_{f_1}}{12.56}$

$\therefore \quad V_{f_1} = 12.56 \tan\theta = 12.56 \times \tan 20° = 4.57$ m/s

$\therefore \quad V_{f_2} = V_{f_1} = 4.57$ m/s.

From outlet velocity triangle, $\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{4.57}{25.13 - V_{w_2}}$

or

$$25.13 - V_{w_2} = \frac{4.57}{\tan\phi} = \frac{4.57}{\tan 30°} = 7.915$$

$\therefore \quad V_{w_2} = 25.13 - 7.915 = 17.215$ m/s.

The work done by impeller per kg of water per second is given by equation (19.1) as

$$= \frac{1}{g} V_{w_2} u_2 = \frac{17.215 \times 25.13}{9.81} = \textbf{44.1 Nm/N. Ans.}$$

**Problem (2)** *A centrifugal pump is to discharge 0.118 m³/s at a speed of 1450 r.p.m. against a head of 25 m. The impeller diameter is 250 mm, its width at outlet is 50 mm and manometric efficiency is 75%. Determine the vane angle at the outer periphery of the impeller.*

**Solution.** Given :

Discharge, $Q = 0.118 \text{ m}^3/\text{s}$

Speed, $N = 1450$ r.p.m.

Head, $H_m = 25$ m

Diameter at outlet, $D_2 = 250 \text{ mm} = 0.25$ m

Width at outlet, $B_2 = 50 \text{ mm} = 0.05$ m

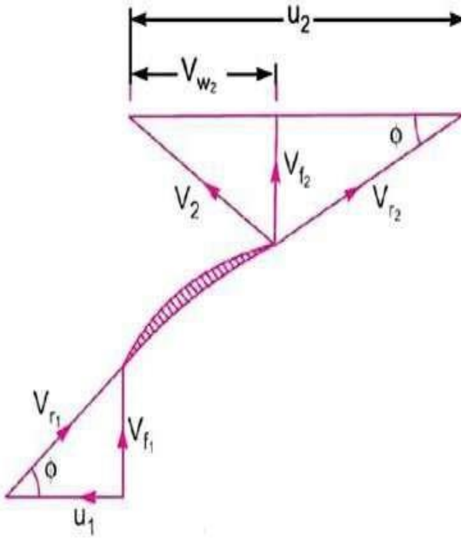
Manometric efficiency, $\eta_{man} = 75\% = 0.75.$

Let vane angle at outlet $= \phi$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1450}{60} = 18.98 \text{ m/s}$$

Discharge is given by $Q = \pi D_2 B_2 \times V_{f_2}$

$$\therefore \quad V_{f_2} = \frac{Q}{\pi D_2 B_2} = \frac{0.118}{\pi \times 0.25 \times .05} = 3.0 \text{ r}$$




Using equation (19.8), $$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 25}{V_{w_2} \times 18.98}$$

$$\therefore \quad V_{w_2} = \frac{9.81 \times 25}{\eta_{man} \times 18.98} = \frac{9.81 \times 25}{0.75 \times 18.98} = 17.23.$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{\left(u_2 - V_{w_2}\right)} = \frac{3.0}{(18.98 - 17.23)} = 1.7143$$

$$\therefore \quad \phi = \tan^{-1} 1.7143 = 59.74° \text{ or } \mathbf{59° \ 44'}. \ \textbf{Ans.}$$

**Problem (3)** *A centrifugal pump delivers water against a net head of 14.5 metres and a design speed of 1000 r.p.m. The vanes are curved back to an angle of 30° with the periphery. The impeller diameter is 300 mm and outlet width is 50 mm. Determine the discharge of the pump if manometric efficiency is 95%.*

**Solution.** Given :

Net head, $H_m = 14.5$ m

Speed, $N = 1000$ r.p.m.

Vane angle at outlet, $\phi = 30°$

Impeller diameter means the diameter of the impeller at outlet

∴ Diameter, $D_2 = 300 \text{ mm} = 0.30$ m

Outlet width, $B_2 = 50 \text{ mm} = 0.05$ m

Manometric efficiency, $\eta_{man} = 95\% = 0.95$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1000}{60} = 15.70 \text{ m/s.}$$

Now using equation (19.8), $$\eta_{man} = \frac{gH_m}{V_{w_2} \times u_2}$$

$$\therefore \quad 0.95 = \frac{9.81 \times 14.5}{V_{w_2} \times 15.70}$$

$$\therefore \quad V_{w_2} = \frac{0.95 \times 14.5}{0.95 \times 15.70} = 9.54 \text{ m/s.}$$

Refer to Fig. 19.5. From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} \text{ or } \tan 30° = \frac{V_{f_2}}{(15.70 - 9.54)} = \frac{V_{f_2}}{6.16}$$

$\therefore$ $V_{f_2} = 6.16 \times \tan 30° = 3.556$ m/s.

$\therefore$ Discharge, $Q = \pi D_2 B_2 \times V_{f_2}$

$= \pi \times 0.30 \times 0.05 \times 3.556 \text{ m}^3/\text{s} = \mathbf{0.1675 \text{ m}^3/\text{s}}$. **Ans.**

**Problem (4)** *A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 r.p.m. works against a total head of 40 m. The velocity of flow through the impeller is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at outlet is 50 mm, determine :*

*(i) Vane angle at inlet,* *(ii) Work done by impeller on water per second, and*

*(iii) Manometric efficiency.*

**Solution.** Given :

Speed, $N = 1000$ r.p.m.

Head, $H_m = 40$ m

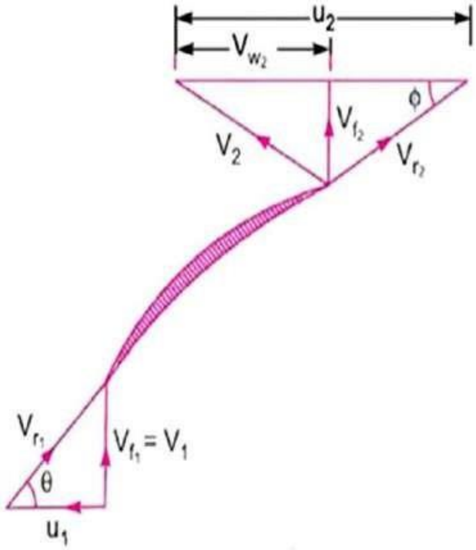
Velocity of flow, $V_{f_1} = V_{f_2} = 2.5$ m/s

Vane angle at outlet, $\phi = 40°$

Outer dia. of impeller, $D_2 = 500 \text{ mm} = 0.50$ m

Inner dia. of impeller, $D_1 = \frac{D_2}{2} = \frac{0.50}{2} = 0.25$ m

Width at outlet, $B_2 = 50 \text{ mm} = 0.05$ m




Tangential velocity of impeller at inlet and outlet are

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.50 \times 1000}{60} = 26.18 \text{ m/s.}$$

Discharge is given by, $Q = \pi D_2 B_2 \times V_{f_2} = \pi \times 0.50 \times .05 \times 2.5 = 0.1963 \text{ m}^3/\text{s}$.

**(*i*) Vane angle at inlet (θ).**

From inlet velocity triangle $\tan \theta = \frac{V_{f_1}}{u_1} = \frac{2.5}{13.09} = 0.191$

$\therefore$ $\theta = \tan^{-1} .191 = 10.81°$ or **10° 48′**. **Ans.**

(*ii*) **Work done by impeller on water per second** is given by equation (19.2) as

$$= \frac{W}{g} \times V_{w_2} u_2 = \frac{\rho \times g \times Q}{g} \times V_{w_2} \times u_2$$

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times V_{w_2} \times 26.18 \quad \ldots(i)$$

But from outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{2.5}{(26.18 - V_{w_2})}$$

$\therefore$ $26.18 - V_{w_2} = \frac{2.5}{\tan \phi} = \frac{2.5}{\tan 40°} = 2.979$

$\therefore$ $V_{w_2} = 26.18 - 2.979 = 23.2$ m/s.

Substituting this value of $V_{w_2}$ in equation (*i*), we get the work done by impeller as

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times 23.2 \times 26.18$$

$$= \mathbf{119227.9\ Nm/s.} \quad \textbf{Ans.}$$

---

(*iii*) **Manometric efficiency ($\eta_{man}$).** Using equation (19.8), we have

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = 0.646 = \mathbf{64.4\%.}\ \textbf{Ans.}$$

# (RECIPROCATING PUMPS)

## Introduction:

- It is a hydraulic machine which converts mechanical energy into hydraulic energy (pressure energy).
- It is a type of positive displacement pump.
- It is suitable where small amount of water is to be delivered at higher pressure.
- While working, it sucks water at low pressure into a cylinder containing a reciprocating piston. The piston exerts a thrust force on the water and increases its pressure.

## Advantages:

- It can deliver the required flow rate very precisely.
- It gives a continuous rate of discharge.
- It can deliver fluid at very high pressure.
- It provides high suction lift.
- No priming is needed.

## Disadvantages:

- It requires high maintenance
- It gives low flow rate i.e. it discharges low amount of water..
- These are heavy and bulky in size.
- It has high initial cost.

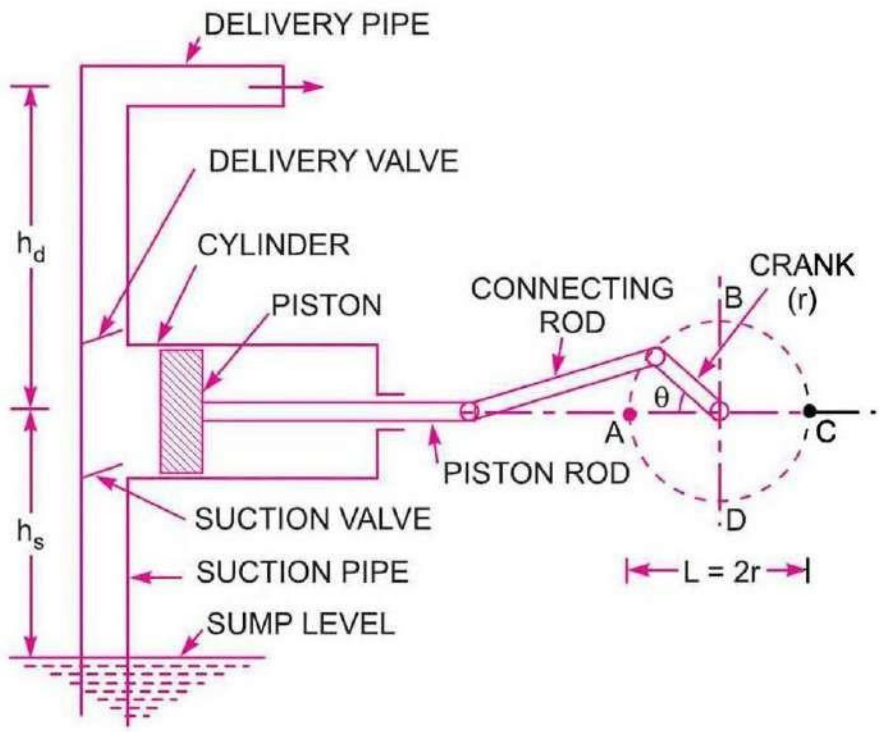
## Classification of Reciprocating pump:

- According to sides in contact with water:
  - Single acting reciprocating pump
    - *In single acting reciprocating pump* water comes in contact of only one side of the piston. Suction and delivery of water occurs at one side.
  - Double acting reciprocating pump
    - *In double acting reciprocating pump* water comes in contact of both sides of the piston. Suction and delivery of water occurs at both sides.
- According to number of cylinders used:
  - Single cylinder pump
  - Double cylinder pump
  - Multi cylinder pump

## Construction of Reciprocating pump:

Major components of Reciprocating pump are:

- A cylinder with piston, piston rod, connecting rod, crank and crank shaft
- Suction pipe
- Suction valve
- Delivery pipe
- Delivery valve




## Working of Single acting reciprocating pump:

- The above figure shows the single acting reciprocating pump.
- It works in two strokes such as suction and delivery strokes.
- During suction stroke, the piston moves backward and suction valve opens. So, water enter into the cylinder. During suction the delivery valve remains closed.
- During delivery stroke, the piston moves forward and delivery valve opens. Suction valve remains closed. Piston exerts thrust on the water and increases water pressure.
- Water with pressure energy escapes out of the cylinder through delivery pipe to the delivery point.

## Work done of Single acting reciprocating pump:

Consider the following terms:

D = diameter of the cylinder

A = cross-sectional area of the piston = $(\pi/4) \times D^2$

r = radius of crank

N =speed of crank in r.p.m L =

length of stroke = **2 × r** $H_s$ =

suction head

$H_d$ = delivery head

H = $\mathbf{H_s + H_d}$ = total head

Q = discharge of pump per second

ρ = density of water

Discharge of water in one revolution of crank = Volume of water delivered in one second = **A × L**

If, number of revolutions per sec = **N/60**

Discharge of pump per second, $\mathbf{Q = A \times L \times (N/60)}$

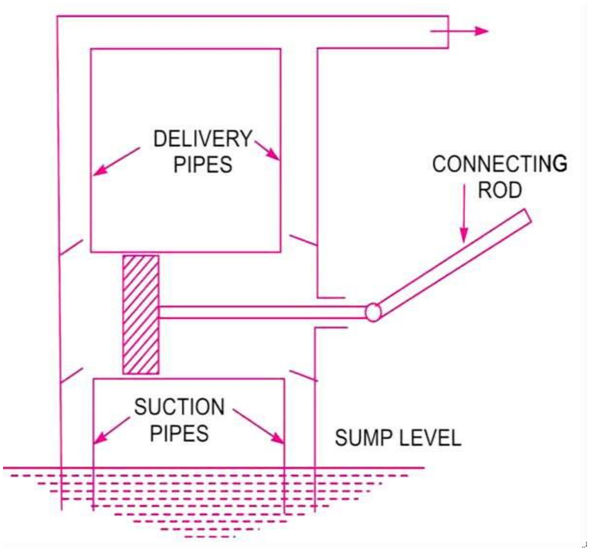
- Weight of water delivered /sec,

  $\mathbf{W = \rho \times g \times A \times L \times (N/60)}$

- Work done by the pump /sec,

$\mathbf{W \times (H_s + H_d)}$

## Working of Double acting reciprocating pump:




- When the piston moves right the suction valve of left side opens and suction valve of right side remains closed. The water is sucked into the cylinder at left side of piston.
- At this stroke delivery valve of left side remains closed and delivery valve of right side remains open. So, piston displaces the water with pressure energy at its right.
- Thus, suction occurs at left end of piston and discharge occurs at the right end.
- Similarly, when piston moves towards left, suction occurs at the right end and discharge occurs at the left end.

## Work done of Double acting reciprocating pump:

Consider the following terms:

D = diameter of the cylinder

A = cross-sectional area of the piston = $(\pi/4) \times D^2$

r = radius of crank

N =speed of crank in r.p.m L =

length of stroke = $\mathbf{2 \times r}$ $H_s$ =

suction head

$H_d$ = delivery head

H = $\mathbf{H_s + H_d}$ = total head

Q = discharge of pump per second

ρ = density of water

Discharge of water in one revolution of crank = Volume of water delivered in one second = 2×**A.L**

If, number of revolutions per sec = **N/60**

Discharge of pump per second, Q = 2×**A × L × (N/60)**

- Weight of water delivered /sec,

  **W = 2×ρ × g × A × L × (N/60)**

- Work done by the pump /sec,

$$\mathbf{W \times (H_s + H_d) = 2\times\rho \times g \times A \times L \times (N/60) \times (H_s + H_d)}$$

## Slip & Percentage of Slip:

*Slip* is the difference between the theoretical discharge ($Q_{th}$) and actual discharge ($Q_{act}$).

$$\mathbf{Slip = Q_{th} - Q_{act}}$$

This is known as *positive slip* when, Qth > Qact.

This is known as *negative slip* when, Qth < Qact

$$Percentage\ of\ slip = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

**Problem (1)** ***A single acting reciprocating pump, running at 50 r.p.m delivers 0.01 $m^3/s$ of water. The diameter of the piston is 200 mm and stroke length 400 mm. Determine: (i) the theoretical discharge of the pump, (ii) coefficient of discharge, (iii) slip and the percentage of slip of the pump.***

**Solution:** Given

Speed pf the pump, N = 50 r.p.m

Actual discharge, Qa = 0.01 $m^3/s$

Diameter of piston, D = 200 mm = 0.2 m

Stroke length, L = 400 mm = 0.4 m

Cross-sectional area of piston, $A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 0.2^2 = 0.031416\ m^2$

(i) Theoretical discharge of the pump, $Qth = \frac{A \times L \times N}{60} = \frac{0.031416 \times 0.4 \times 50}{60} = 0.01047\ m^3/s$

(ii) Coefficient of discharge, $C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{0.01047} = 0.955$

(iii) slip of the pump, $Q_{th} - Q_{act} = 0.01047 - 0.01 = 0.00047\ m^3/s$

Percentage of slip of the pump $= \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \frac{0.01047 - 0.01}{0.01047} \times 100 = 4.489\%$

***Problem (2) A double acting reciprocating pump, running at 40 r.p.m delivers 1 $m^3$ of water per minute. The diameter of the piston is 200 mm and stroke length 400 mm. The delivery and suction head are 20 m and 5 m respectively. Find the slip of the pump and power required to drive the pump.***

**Solution:** Given

Speed of the pump, N = 40 r.p.m

Actual discharge, $Q_{act} = 1 m^3/min = \frac{1}{60} m^3/s = 0.01666\ m^3/s$

Stroke length, L = 400 mm = 0.4 m

Diameter of piston, D = 200 mm = 0.2 m

Suction head, $H_s$ = 5 m

Delivery head, $H_d$ = 20 m

Cross-sectional area of piston, $A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 0.2^2 = 0.031416\ m^2$

Theoretical discharge of the pump, $Qth = \frac{2 \times A \times L \times N}{60} = \frac{2 \times 0.031416 \times 0.4 \times 40}{60} = 0.01675\ m^3/s$

Slip of the pump, $Q_{th} - Q_{act} = 0.01675 - 0.01666 = 0.00009\ m^3/s$

Power required to drive the pump,

$$P = \frac{2 \times \rho \times g \times A \times L \times N \times (H_s + H_d)}{60000} = \frac{2 \times 1000 \times 9.81 \times 0.031416 \times 0.4 \times 40 \times (5 + 20)}{60000} = 4.109\ kW$$

## Difference between Centrifugal pump and Reciprocating pump:

| **Centrifugal pump** | **Reciprocating pump** |
|---|---|
| 1. Simple in construction | 1. Complicated in construction |
| 2. Total weight of pump is less for a given discharge | 2. Total weight of pump is more for a given discharge |
| 3. Suitable for large discharge and smaller heads | 3. Suitable for less discharge and higher heads |
| 4. Required less floor area and simple foundation | 4. Required more floor area and heavy foundation |
| 5. Less wear and tear | 5. More wear and tear |
| 6. Maintenance cost is less | 6. Maintenance cost is high |
| 7. Can run at higher speeds | 7. Can’t run at higher speeds |
| 8. Its delivery is continuous | 8. Its delivery is pulsating |
| 9.Needs priming | 9. Doesn’t need priming |
| 10. It has less efficiency | 10. It has more efficiency |

# (CHAPTER -4) PNEUMATIC CONTROL SYSTEM

**Elements –filter-regulator-lubrication unit**

**FRL Unit: Filter, Regulator, & Lubricator - How They Work**

Filter, regulator, and lubricator (FRL) compressed air systems are used to deliver clean air, at a fixed pressure, and lubricated (if needed) to ensure the proper pneumatic component operation and increase their operational lifetime.

The air supplied by compressors is oftentimes contaminated, over-pressurized, and non-lubricated meaning that an FRL unit is required to prevent damage to equipment. Filters, regulators, and lubricators can be bought individually or as a package (as seen in Figure 1) depending on what is needed to ensure the proper air specifications are being met for downstream equipment.

It is recommended to install these devices if you:

- Use pneumatic tools and equipment;
- Are installing an HVAC system;
- Require clean air to be delivered to your facility or workplace;
- Require compliance to ISO, OSHA, ASHRA or other air quality standards;
- Want to improve the service life, safety and reliability of your air system.

What do FRL units do?
An FRL unit is comprised of a filter (F), regulator (R), and a lubricator (L). They are often used as one unit to ensure clean air in a pneumatic system but can also be used individually. Having a proper FRL unit installed in a pneumatic system provides higher reliability of the components downstream, reduced power waste due to over pressurization, and increased component lifetime. The three components work together to do the following:

- Filters remove water, dirt and other harmful debris from an air system. This is often the first step in improving the air quality.

- Regulators adjust and control the air pressure of a system to ensure that down-line components do not exceed their maximum operating pressures. This is the second step in the FRL system.
- Lubricators reduce the internal friction in tools or equipment by releasing a controlled mist of oil into the compressed air. This is often done last and/or right before the component needs lubrication.

Pneumatic filter selection

Pneumatic filter

- ➢ Filters remove water, dirt and other harmful debris from an air system (Figure 2). The type and size of contaminants present in the system and the air requirements for components will ultimately affect what micron size and bowl material is needed for the filter.
- ➢ Common applications generally only require a filter rated between 5-40 microns. However, ISO 8157 goes down to 0.1 micron and for special applications, like medical or pharmaceutical the specifications can be as low as .001 micron.
- ➢ The rating means that it doesn't allow bigger particles through. For example, if you have a 20 micron filter it will allow particles smaller than 20 microns to pass through. It should be noted that filters experience a small pressure drop across the inlet and outlet ports because of the flow restriction.
- ➢ A 0.1 micron filter will create a larger pressure drop than a 40 micron filter and will require more regular maintenance due to the easy build-up of contaminants. Therefore, do not oversize your filter by selecting the finest possible micron size.
- ➢ It will lead toto higher cost for the component, a larger pressure drop, and more maintenance time. Instead, select a filter that will remove only the smallest contaminant specific to your system.
- ➢ The bowl material and drainage type are also important. The bowl comes into contact with the contaminants and houses the filtered particles. The pressure, temperature, and chemicals present affect the bowl material selection.
- ➢ Filters also require drainage, which can be either accomplished by the filter as an automatic, semi-automatic, or manual drainage system or a condensate drain can be attached to the outlet to remove the filtered contaminants.

- **Automatic:** An automatic drain is a 2/2-way valve that closes when the system is pressurized. It has a float system in it that rises when the system is de-pressurized or when liquid accumulates, and this rise in the float causes the drain to open. Advised when the equipment is in continual use, requires frequent drainage, or in a hard to reach location.

- **Semi-automatic:** A semi-automatic drain automatically drains the system upon de-pressurization. It can drain the system when pressurized, but only by a manual process. If the system is not always under pressure, it is recommended to have a semi-automatic drainage filter.
- **Manual:** The filter can be manually drained when the system is depressurized. Not advised for a hard-to-reach location, if it requires frequent drainage, and if the system isn't regularly depressurized.
- **Condensate Drain:** A condensate drain can be attached to the outlet of the filter to accomplish the drainage. However, proper timing of the open/closing needs to be set.

PRESSURE CONTROL VALVE

- The valves enable the regulation of system pressure to adjust the force on a hydraulic piston rod or the torque on a hydraulic motor shaft. Pressure relief valves are used to set the maximum pressure in the circuit and protect it from overloading.

**Pressure Relief Valves**

- Most pneumatic and hydraulic power systems are designed to operate within a defined pressure range. This range is a function of the forces the actuators in the system must generate to do the required work.

- Without controlling these forces, the power components and expensive equipment could get damaged. Relief valves make it possible to avoid this hazard.

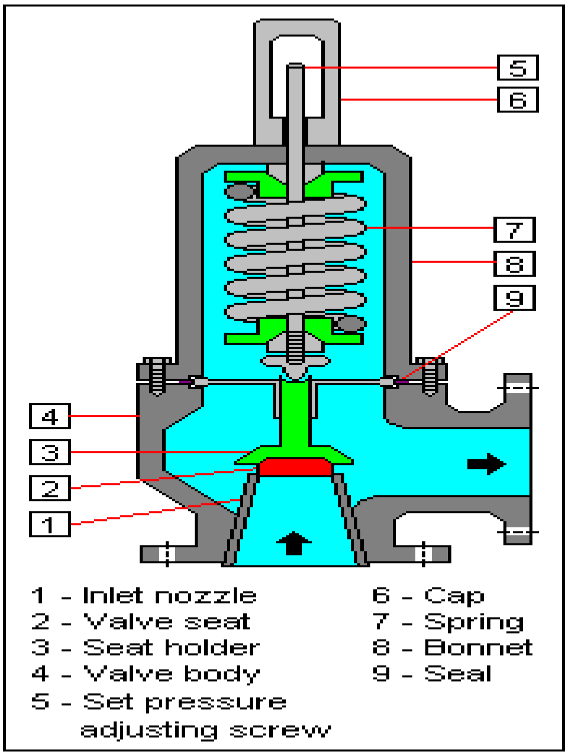
- They are the safeguards that limit maximum pressure in a system by diverting excess gases when pressure gets too high. The pressure at which a relief valve first opens to allow fluid to flow through is known as cracking pressure.

- When the valve is bypassing its full rated flow, it is in a state of full-flow pressure.

- The difference between full-flow and cracking pressure is sometimes known as the pressure differential, or the pressure override.

- In some cases, this pressure override is not objectionable. It can be a disadvantage if it wastes power via gas lost through the valve prior to reaching the maximum setting.

- This can allow maximum system pressure to exceed the ratings of the other components.



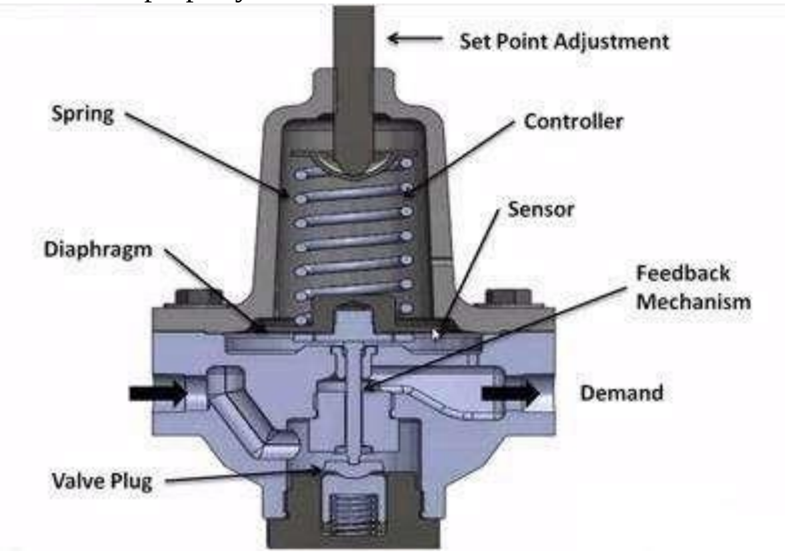

**Pressure-Reducing Valves**

The most practical components for maintaining lower pressure in a pneumatic system are pressure-reducing valves. Pressure-reducing valves are usually open two-way valves that close when subjected to sufficient downstream pressure. There are subcategories of pressure-reducing valves: direct acting and pilot operated.

**Direct-acting valves are** pressure-reducing valves that limit the maximum pressure available in the secondary circuit regardless of pressure changes in the main circuit.

- This assumes the workload generates no backflow into the reducing valve port, in which case the valve will close.
- The pressure-sensing signal comes from the secondary circuit. The valve operates in reverse from a relief valve because they are normally closed and sense the pressure from the inlet.
- When outlet pressure reaches the valve setting, the valve closes except for a small quantity of gas that bleeds from the low-pressure side of the valve, usually through an orifice in the spool.
- The spool in a pilot-operated, pressure-reducing valve is balanced hydraulically by downstream pressure at both ends. The pilot valve relieves enough gas to position the spool so that flow through the main valve equals the requirements of the reduced-pressure circuit.

- If no flow is required during the cycle, the main valve closes. Leakage of high-pressure gas into the reduced-pressure section of the valve then returns to the reservoir through the pilot-operated relief valve.
- This type of valve generally has a wider range of spring adjustments than direct-acting valves and provides better repetitive accuracy. However, in hydraulic applications, oil contamination can block flow to the pilot valve, and the main valve will fail to close properly.




DIRECTIONAL CONTROL VALVES

Directional control valves perform only three functions:

- stop fluid flow
- allow fluid flow, and
- change direction of fluid flow.

These three functions usually operate in combination.

- The simplest directional control valve is the 2-way valve. A 2-way valve stops flow or allows flow. A water faucet is a good example of a 2-way valve. A water faucet allows flow or stops flow by manual control.
- A single-acting cylinder needs supply to and exhaust from its port to operate. This requires a 3-way valve. A 3-way valve allows fluid flow to an actuator in one position and exhausts the fluid from it in the other position. Some 3-way valves have a third position that blocks flow at all ports.
- A double-acting actuator requires a 4-way valve. A 4-way valve pressurizes and exhausts two ports interdependently. A 3-position, 4-way valve stops an actuator or allows it to float. The 4-way function is a common type of directional control valve for both air and hydraulic circuits. A 3-position, 4-way valve is more common in hydraulic circuits.

**3/2-Way Pneumatic Valve - How They Work**

- A 3/2-way valve has three ports and two positions that can be driven pneumatically, mechanically, manually or electrically via a solenoid valve.
- They are used, for example, to control a single-action cylinder, driving pneumatic actuators, blow-off, pressure release and vacuum applications.
- A valve is used to fill the cylinder, and also to exhaust it afterwards, so that a new working stroke can be realized. Therefore, a valve with two ports would not be adequate. Venting requires a third port.

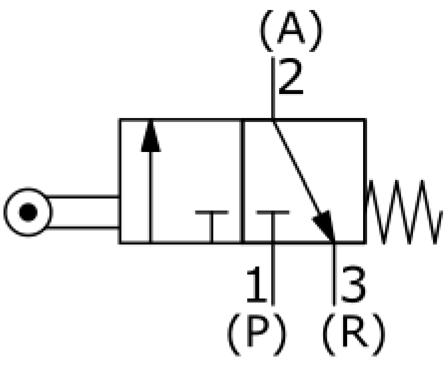
- There are two kinds of 3/2 valves: mono-stable and bi-stable. Mono-stable 3/2-way valves can also be normally closed or normally open, just like 2/2-way valves.

Circuit function of 3 way air valves

The 3/2-way pneumatic valve has three connection ports and two states. The three ports are:

- inlet (P, 1),
- outlet (A, 2)
- exhaust (R, 3)

The two states of the valve are open and closed. When the valve is open, air flows from the inlet (P, 1) to the outlet (A, 2). When the valve is closed, air flows from the outlet (A, 2) to the exhaust (R, 3). A valve that is closed in a non-actuated state is normally closed (N.C.), the opposite is called normally open (N.O.).



Circuit function 0f a monostable, normally closed 3/2 way valve

- Most valves are mono-stable and return to their default position when not actuated, this is achieved with a spring mechanism. Bi-stable 3/2-way valves retain their position during power loss and require a separate action to switch the valve state.
- Therefore, they cannot be designated as Normally Closed or Normally Open. Bi-stable pneumatic solenoid valves typically have a coil at each position and are pulse operated.

Summarized, the different functions of the 3/2-way valve are:

- 3/2-way mono-stable NC
- 3/2-way mono-stable NO
- 3/2-way bi-stable

The circuit functions can be shown with valve symbols. For the three above-mentioned functions, the symbol of an indirectly operated solenoid valve is shown below. You can find detailed information about other pneumatic valve symbols and their explanation in our valve symbol article.

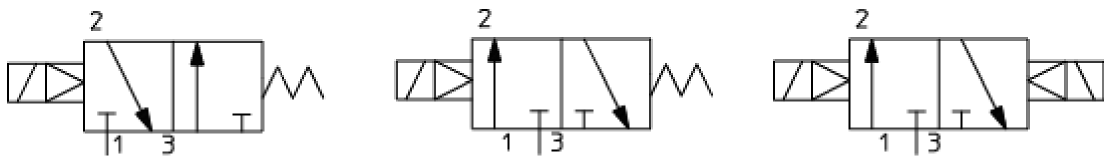



*Figure 3: Symbols of 3/2-way pneumatic solenoid valves, from left to right: normally open, monostable (left), normally closed, monostable (center), bi-stable (right).*

3/2-way valves can be actuated by different means such as:

- pneumatically
- manually
- mechanically
- electrically (solenoid valve)

Furthermore, the valves can be directly operated or indirect operated. With the indirect operation, the valve uses the inlet pressure to help switching the valve state.

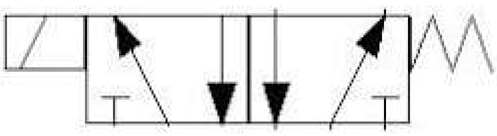
3/2-way valves are available in several designs. The sealing mechanism of the valves can be a poppet or a spool. The valve's main parts are the following: housing, seals, poppet (or spool) and an actuator

With direct operated valves, the spool or poppet is moved directly by the actuator. Several types of actuators are possible:

- Solenoid (coil)
- Push button

- Lever
- Foot pedal, etc.

**<u>5/2 DCV</u>**
**5 Port 2 Position Valve Working Principle**



A valve is a device that regulates the flow of fluid (gases, liquids, fluidized solids, or slurries) by opening and closing or partially obstructing passageways.

A 5/2 way directional valve from the name itself has 5 ports equally spaced and 2 flow positions. It can be used to isolate and simultaneously bypass a passageway for the fluid which for example should retract or extend a double-acting cylinder.

There are a variety of ways to have this valve actuated. A solenoid valve is commonly used, a lever can be manually twisted or pinched to actuate the valve, an internal or external hydraulic or pneumatic pilot to move the shaft inside, sometimes with a spring return on the other end so it will go back to its original position when pressure is gone or a combination of any of the mention above.

In the Illustration given, a single solenoid is used and a spring return is installed in the other end. The inlet pressure is connected to (P)1. (A)2 could possibly be connected to one end of the double-acting cylinder where the piston will retract while (B)4 is connected to the other end that will make the piston extend.

The normal position when the solenoid is de-energized is that the piston rod is blocking (B)4 and pressure coming from (P)1 passes through (A)2 that will make the cylinder normally retracted.

When the solenoid is energized, the rod blocks (A)2 and pressure from (P)1 passes through (B)4 and will extend the cylinder and when the solenoid is de-energized, the rod bounces back to its original position because of the spring return. (E)3 and (E)5 is condemned or used as exhaust.

**5/3 DCV**

What is a 5 Way 3 Position Pneumatic Valve?

The 5/3 or 4/3 series are directional-control valves. The 5/3 and 4/3 body designs allow compressed air to flow to one port of a double-acting air actuator while simultaneously allowing air to exhaust from the other port on the same air actuator at the same time.

By shifting the internal flow paths of the valve, the 5/3 and 4/3 air valve sends compressed air alternatively to each of the two actuator ports and exhaust from the other, thus allowing the double-acting air cylinder to function.

The valve shown in the following image has 5 airports. It may be a 5/3, or it may be a 5/2 configuration. You cannot tell the difference from looking at the valve body. The valve schematic, which is typically shown on the side of the valve, is the only way you can determine if the spool is two-position (a 5/2) or three-position, (a 5/3) unless it is identified as such by the vendor.

A 5/2 double solenoid air valve. The two actuator ports are visible. On the opposite side are three ports; 2 exhaust & 1 supply

What is a 3 Position Valve?

The "extra" position inside a 5/3 or 4/3 air valve means that the internal spool can be shifted to a center position. The typical spool movement is end to end inside the valve. With a two-position valve, the spool shifts from the end, across the middle, and to the other end. In the three-position body style, the spool can be positioned to stop in the middle location to accomplish a specific goal.

Each of the three spool positions is selected to accomplish the desired result in the action of the air cylinder.

Since valves with three-position spools are more expensive than their two-position counterparts, the selection of a three-position valve will be deliberate. The circuit designer will have a particular scenario in mind for the action of the air cylinder when the valve that controls it is shifted, and that circuit will require the selection of a specific three-position valve to accomplish the goal.

A 5/3 or 4/3 valve will normally have two internal spring actuators that, when the valve is not being operated by an external valve actuator, shift that valve spool to the center position automatically. It is normally when the 5/3 or 4/3 valve is “at rest” that the third of the three positions come into play.

## Flow-Control Valves

1. Flow-control valves include simple orifices to sophisticated closed-loop electrohydraulic valves that automatically adjust to variations in pressure and temperature.
2. The purpose of flow control in a hydraulic system is to regulate speed. All the devices discussed here control the speed of an actuator by regulating the flow rate. Flow rate also determines rate of energy transfer at any given pressure.
3. The two are related in that the actuator force multiplied by the distance through which it moves (stroke) equals the work done on the load. The energy transferred must also equal the work done. Actuator speed determines the rate of energy transfer (i.e., horsepower), and speed is thus a function of flow rate.
4. Directional control, on the other hand, does not deal primarily with energy control, but rather with directing the energy transfer system to the proper place in the system at the proper time. Directional control valves can be thought of as fluid switches that make the desired "contacts." That is, they direct the high-energy input stream to the actuator inlet and provide a return path for the lower-energy oil.
5. It is of little consequence to control the energy transfer of the system through pressure and flow controls if the flow stream does not arrive at the right place at the right time.
6. Thus, a secondary function of directional control devices might be defined as the timing of cycle events. Because fluid flow often can be throttled in directional-control valves, some measure of flow rate or pressure control can also be achieved with them.

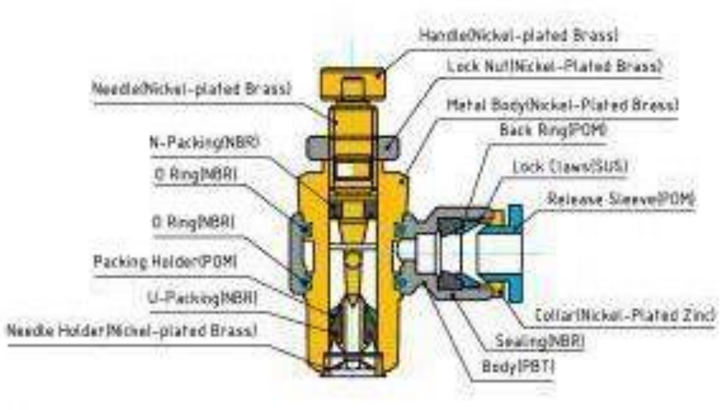
PNEUMATIC THROTTLE VALVE

**Pneumatic speed controller/Pneumatic throttle valve** is used for controlling the operation speed of a driving device and the movement of machines such as cylinder, pneumatic finger, etc. The flow rate of air from A side to B side can be control, whereas air entering from B side to A side is not under control.

FEATURES

- Mainly installed in the air actuator.
- Working Pressure: 0-150 Psi.
- Working Temperature: 32 -140°F / 0-60℃.

CASE IN USE




**METER OUT:**

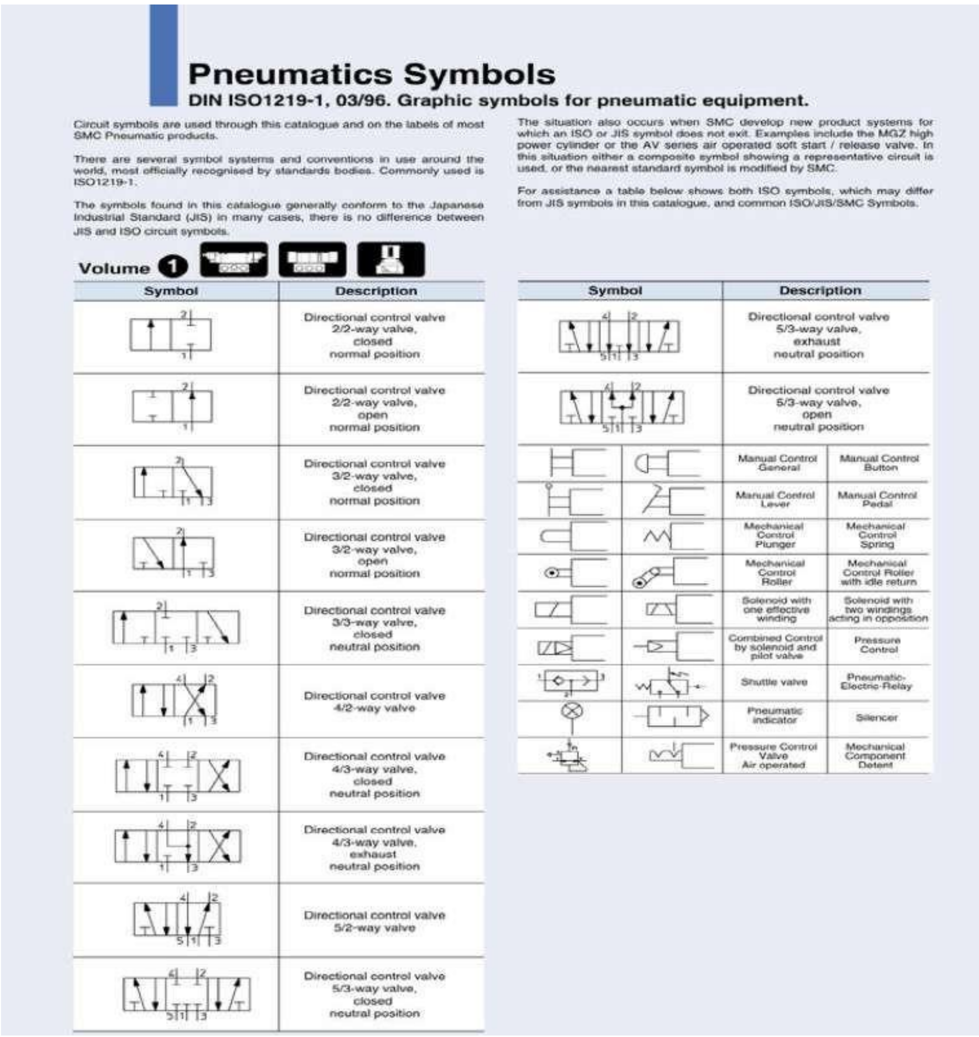
The flow rate of air entering from the thread side can be controlled, whereas air entering from joint side comes out from the thread side at the same flow rate (not controlled).

**METER IN:**

The flow rate of air entering from the joint side can be controlled, whereas air entering from thread side come out from joint side at the same speed (not controlled).

**FLAT TYPE:**

The way to control of flow or control flow upon piping in accordance with, the signal on the body. Air flows from each side of valve.



# Pneumatics Symbols

**DIN ISO1219-1, 03/96. Graphic symbols for pneumatic equipment.**

Circuit symbols are used through this catalogue and on the labels of most SMC Pneumatic products.

There are several symbol systems and conventions in use around the world, most officially recognised by standards bodies. Commonly used is ISO1219-1.

The symbols found in this catalogue generally conform to the Japanese Industrial Standard (JIS) in many cases, there is no difference between JIS and ISO circuit symbols.

The situation also occurs when SMC develop new product systems for which an ISO or JIS symbol does not exit. Examples include the MGZ high power cylinder or the AV series air operated soft start / release valve. In this situation either a composite symbol showing a representative circuit is used, or the nearest standard symbol is modified by SMC.

For assistance a table below shows both ISO symbols, which may differ from JIS symbols in this catalogue, and common ISO/JIS/SMC Symbols.

Volume 1

| Symbol | Description |
|---|---|
| | Directional control valve 2/2-way valve, closed normal position |
| | Directional control valve 2/2-way valve, open normal position |
| | Directional control valve 3/2-way valve, closed normal position |
| | Directional control valve 3/2-way valve, open normal position |
| | Directional control valve 3/3-way valve, closed neutral position |
| | Directional control valve 4/2-way valve |
| | Directional control valve 4/3-way valve, closed neutral position |
| | Directional control valve 4/3-way valve, exhaust neutral position |
| | Directional control valve 5/2-way valve |
| | Directional control valve 5/3-way valve, closed neutral position |

| Symbol | Description |
|---|---|
| | Directional control valve 5/3-way valve, exhaust neutral position |
| | Directional control valve 5/3-way valve, open neutral position |

| Symbol | Symbol | Description | Description |
|---|---|---|---|
| | | Manual Control General | Manual Control Button |
| | | Manual Control Lever | Manual Control Pedal |
| | | Mechanical Control Plunger | Mechanical Control Spring |
| | | Mechanical Control Roller | Mechanical Control Roller with idle return |
| | | Solenoid with one effective winding | Solenoid with two windings acting in opposition |
| | | Combined Control by solenoid and pilot valve | Pressure Control |
| | | Shuttle valve | Pneumatic-Electric-Relay |
| | | Pneumatic indicator | Silencer |
| | | Pressure Control Valve Air operated | Mechanical Component Detent |

**<u>Pneumatic circuits</u>**

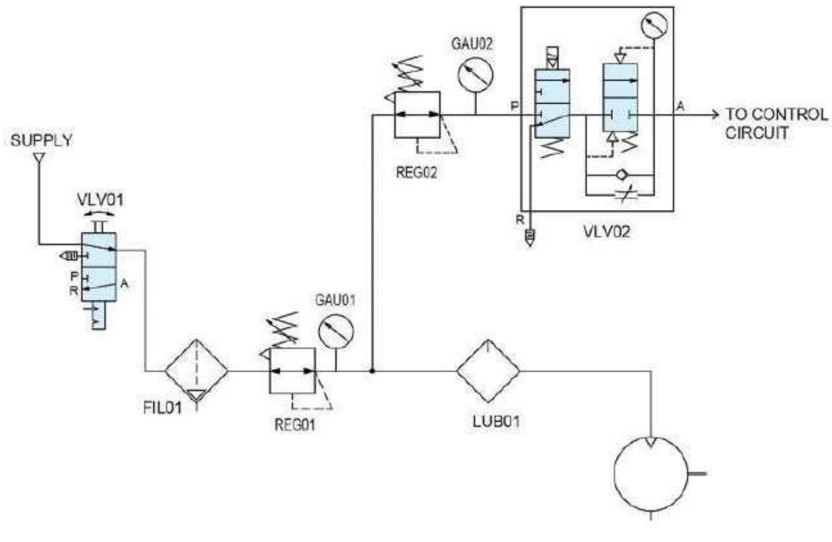
A pneumatic circuit is an interconnected set of components that convert compressed gas (usually air) into mechanical work. In the normal sense of the term, the circuit must include a compressor or compressor-fed tank.

**4 Basic Pneumatic Circuits**

The following four pneumatic circuits can be used for air preparation, double-acting cylinders, continuous cycling and hand control applications. They can also be subsystems in larger circuits.

*Air Preparation*

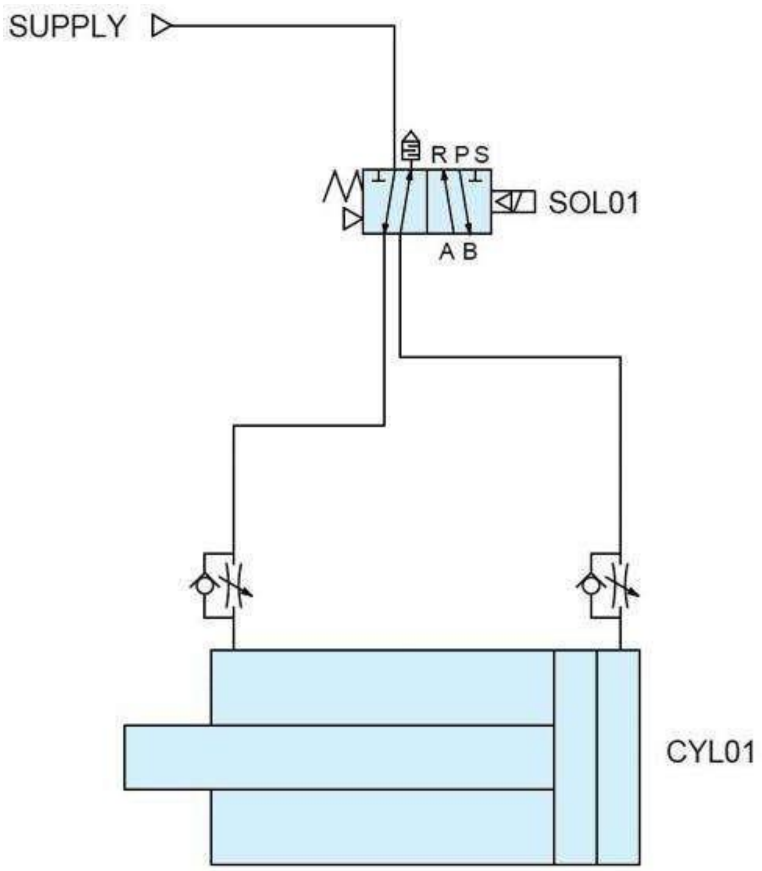
Before compressed air is used in a pneumatic device, it must be properly prepared so that it does not damage components. Here is a schematic (below) for a pneumatic device that prepares compressed air coming from a single source.




- Putting the manual shut-off valve or pneumatic isolation/lockout valve first makes it easier to maintain the FRL and it protects downstream equipment when depressurizing the system for maintenance.
- For safety, operators should be able to lock the valve in the off position. If it is necessary to have clean, dry air flowing through the valve, the valve can be mounted after the FRL.

**Double-Acting Cylinder**

The schematic below shows a common automation application: using a 4-way solenoid valve (SOL01) to extend and retract a double-acting cylinder (CYL01). Triangles at each side of the symbol indicate it is a pilot-activated, single-solenoid, spring-return valve.




Double-acting cylinder circuits are common on PLC-controlled machines.

- Filtered air feeds the solenoid valve, which is usually energized by a 24 V dc PLC output. This activates the valve and lets air leave through port B and flow freely through the flow control to extend the cylinder rod and plunger to the left.
- Air on the left side of the cylinder is forced out through its flow control to the valve's port A, and then goes to port R and exits through a muffler to reduce exhaust noise.
- Pilot valves need only a small amount of air to efficiently move a large valve spool. However, valves require a minimum operating pressure, typically about 20 psi, to move the spool.
- A spring on the left side pushes the valve spool to the right to maintain its normal off or resting state. With the valve off, air flows out of port A and freely through the adjustable flow control to the left side of the cylinder (CYL01), making it retract.
- As the cylinder retracts, air on the right-side leaves through an adjustable flow-control device. As the device's check valve closes, air in the flow section can be adjusted to throttle the cylinder retraction.
- The flow-controlled air then goes through the valve's port B and leaves at port S through a muffler.

**Direct Control of Single Acting Cylinder**

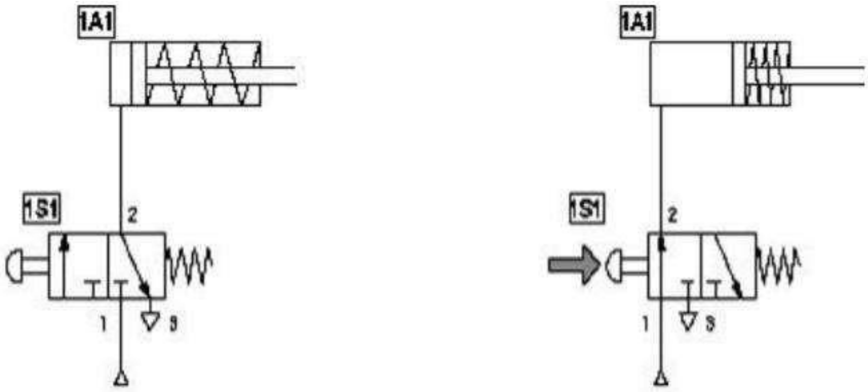



Figure 4.1 Direct Control of Single Acting Cylinder

- Pneumatic cylinders can be directly actuated by actuation of final control valve, manually or electrically in small cylinders as well as cylinders which operates at low speeds where the flow rate requirements are less.
- When the directional control valve is actuated by push button, the valve switches over to the open position, communicating working source to the cylinder volume.
- This results in the forward motion of the piston. When the push button is released, the reset spring of the valve restores the valve to the initial position [closed].
- The cylinder space is connected to the exhaust port thereby piston retracts either due to spring or supply pressure applied from the other port.

**Indirect Control of Single Acting Cylinder**

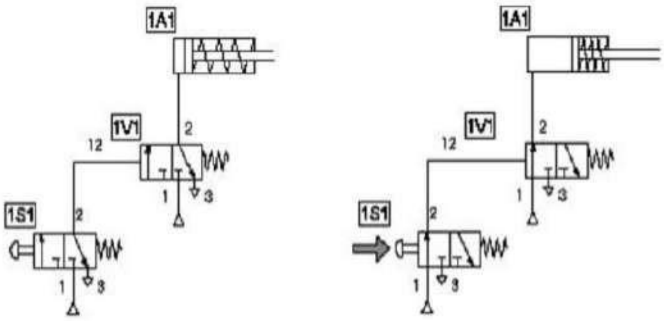



Figure 4.2: Indirect Control of Single and Double Acting Cylinders

- Large cylinders as well as cylinders operating at high speed are generally actuated indirectly as the final control valve is required to handle large quantity of air. In the case of pilot operated valves, a signal input valve [3/2 way N.C type, 1S1] either actuated manually or mechanically is used to generate the pilot signal for the final control valve.
- The signal pressure required can be around 1-1.5 bar. The working pressure passing through the final control valve depends on the force requirement [4-6 bar].
- Indirect control permits the processing of input signals. Single piloted valves are rarely used in applications where the piston has to retract immediately on taking out the set pilot signal. suitable for large single-acting cylinders.

# (HYDRAULIC CONTROL SYSTEM)

## Hydraulic system

A hydraulic system is a drive technology where a fluid is used to move the energy from e.g. an electric motor to an actuator, such as a hydraulic cylinder. The fluid is theoretically uncompressible and the fluid path can be flexible in the same way as an electric cable.

### Advantages of the hydraulic system

Talking about the advantages and disadvantages of hydraulic systems, let's first discuss the advantages. Even though it seems that the hydraulic system only relies on fluid and pressure to be distributed in all directions, this working principle is proven to be able to lift heavy materials. Here are ten advantages of a hydraulic system:

1. Any hydraulic motion is independent of the load as long as the fluid is not subject to resistance and a flow control valve can be used.
2. Precision and flexible.
3. Can move large power using only relatively small components.
4. Can move freely when lifting large loads.
5. Easy to use and friendly control system. Beginners who are just learning to operate heavy equipment with a hydraulic system will find it easy to understand.
6. The operation is smooth, does not cause unnecessary noise and vibration on the machine.
7. The design and design is simple.
8. Can be rotated in the reverse direction (reversible).
9. Make it easy to move heavy material, one lift, or move goods up to tons of weight.
10. The service life is quite long so it can save the cost of purchasing new equipment.

### Weaknesses of the hydraulic system

It is incomplete to discuss the advantages and disadvantages of hydraulic systems without showing the shortcomings of the working principle of this system. After the advantages, now we will describe the weaknesses of the hydraulic system. Even though there are shortcomings, it doesn't mean that this hydraulic system is bad. It's just that, every working principle on heavy equipment must have advantages and limitations.

1. Hydraulic systems require intensive and periodic maintenance.
2. Systems frequently require parts with a very high degree of precision.
3. A small leak in the hydraulic pipeline will be fatal to the transfer of power. This causes
4. the risk of accidents increases.
   5. The high pressure received by the fluid can also cause work accidents if the power is too high and the pipeline is unable to withstand the power delivered by the fluid.

## Hydraulic accumulators-

- A hydraulic accumulator is a pressure storage reservoir in which an incompressible hydraulic fluid is held under pressure that is applied by an external source of mechanical energy.

- The external source can be an engine, a spring, a raised weight, or a compressed gas.[note 1] An accumulator enables a hydraulic system to cope with extremes of demand using a less powerful pump, to respond more quickly to a temporary demand, and to smooth out pulsations.
- It is a type of energy storage device.
- Compressed gas accumulators, also called hydro-pneumatic accumulators, are by far the most common type.

## Pressure control valves

The valves enable the regulation of system pressure to adjust the force on a hydraulic piston rod or the torque on a hydraulic motor shaft. Pressure relief valves are used to set the maximum pressure in the circuit and protect it from overloading.

Pressure-control valves are found in virtually every hydraulic system, and they assist in a variety of functions, from keeping system pressures safely below a desired upper limit to maintaining a set pressure in part of a circuit. Types include relief, reducing, sequence, counterbalance, and unloading. All of these are normally closed valves, except for reducing valves, which are normally open. For most of these valves, a restriction is necessary to produce the required pressure control. One exception is the externally piloted unloading valve, which depends on an external signal for its actuation.

### Relief valves

Most fluid power systems are designed to operate within a present pressure range. This range is a function of the forces the actuators in the system must generate to do the required work. Without controlling or limiting these forces, the fluid power components (and expensive equipment) could be damaged. Relief valves avoid this hazard. They are the safeguards which limit maximum pressure in a system by diverting excess oil when pressures get too high.

**Cracking pressure and pressure override** —The pressure at which a relief valve first opens to allow fluid to flow through is known as *cracking pressure*. When the valve is bypassing its full rated flow, it is in a state of *full-flow pressure.* The difference between full-flow and cracking pressure is sometimes known as *pressure differential,* also known as *pressure override.*

### Pressure-reducing valves

The most practical components for maintaining secondary, lower pressure in a hydraulic system are pressure-reducing valves. Pressure-reducing valves are normally open, 2-way valves that close when subjected to sufficient downstream pressure. There are two types: direct acting and pilot operated.

### Fluid Power Pump-

Fluid power is the use of fluids under pressure to generate, control, and transmit power. Fluid power is subdivided into hydraulics using a liquid such as mineral oil or water, and pneumatics using a gas such as air or other gases. Compressed-air and water-pressure systems were once used to transmit power from a central source to industrial users over extended geographic areas; fluid power systems today are usually within a single building or mobile machine.

**What is a gear pump?**

A gear pump is a type of positive displacement (PD) pump. It moves a fluid by repeatedly enclosing a fixed volume using interlocking cogs or gears, transferring it mechanically using a cyclic pumping action. It delivers a smooth pulse-free flow proportional to the rotational speed of its gears.

**How does a gear pump work?**

Gear pumps use the actions of rotating cogs or gears to transfer fluids. The rotating element develops a liquid seal with the pump casing and creates suction at the pump inlet. Fluid, drawn into the pump, is enclosed within the cavities of its rotating gears and transferred to the discharge. There are two basic designs of gear pump: *external* and *internal* (Figure 1).

**External Gear Pump**

An external gear pump consists of two identical, interlocking gears supported by separate shafts. Generally, one gear is driven by a motor and this drives the other gear (the *idler*). In some cases, both shafts may be driven by motors. The shafts are supported by bearings on each side of the casing.

1. As the gears come out of mesh on the inlet side of the pump, they create an expanded volume. Liquid flows into the cavities and is trapped by the gear teeth as the gears continue to rotate against the pump casing.

2. The trapped fluid is moved from the inlet, to the discharge, around the casing.

3. As the teeth of the gears become interlocked on the discharge side of the pump, the volume is reduced and the fluid is forced out under pressure.

No fluid is transferred back through the centre, between the gears, because they are interlocked. Close tolerances between the gears and the casing allow the pump to develop suction at the inlet and prevent fluid from leaking back from the discharge side (although leakage is more likely with low viscosity liquids).

External gear pump designs can utilise spur, helical or herringbone gears.

**Internal gear pump**

An internal gear pump operates on the same principle but the two interlocking gears are of different sizes with one rotating inside the other. The larger gear (the *rotor*) is an internal gear i.e. it has the teeth projecting on the inside. Within this is a smaller external gear (the *idler* – only the rotor is driven) mounted off-centre. This is designed to interlock with the rotor such that the gear teeth engage at one point.  A pinion and bushing attached to the pump casing holds the idler in position.  A fixed crescent-shaped partition or spacer fills the void created by the off-centre mounting position of the idler and acts as a seal between the inlet and outlet ports.

1. As the gears come out of mesh on the inlet side of the pump, they create an expanded volume.  Liquid flows into the cavities and is trapped by the gear teeth as the gears continue to rotate against the pump casing and partition.

2. The trapped fluid is moved from the inlet, to the discharge, around the casing.

3. As the teeth of the gears become interlocked on the discharge side of the pump, the volume is reduced and the fluid is forced out under pressure.

Internal gear pump designs only use spur gears.

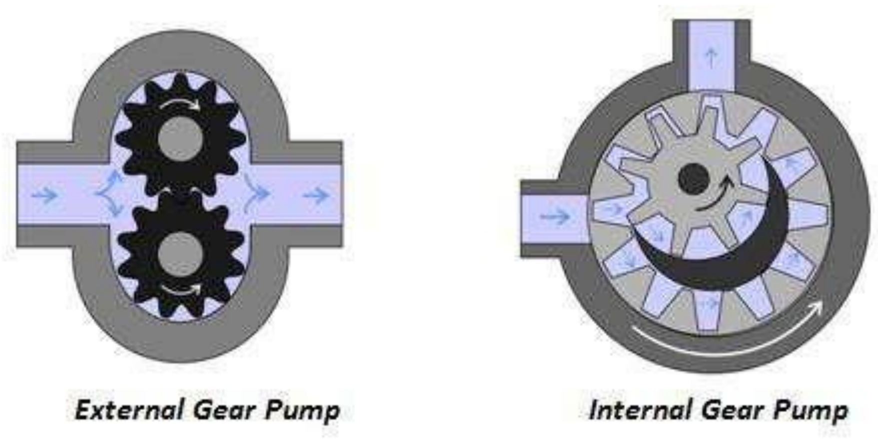
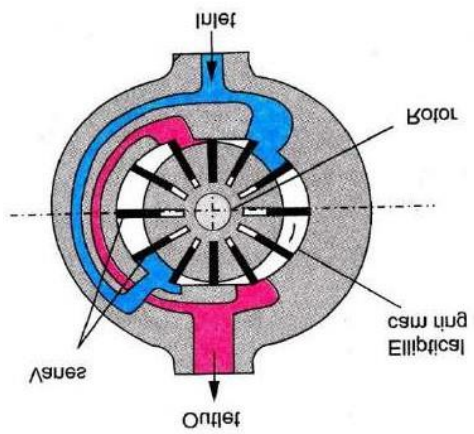



*Figure 1.    Gear pump designs (arrows indicate the direction of the pump and liquid)*

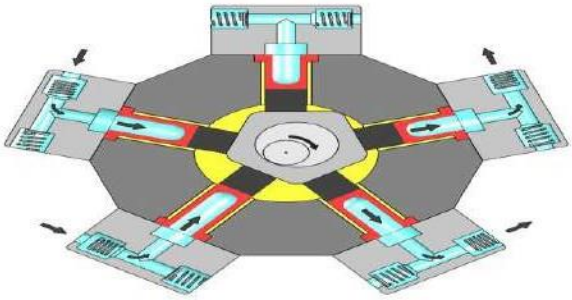
What is a vane  pump?



Vanes or blades fit within the slots of the impeller. As the rotor rotates (yellow arrow) and fluid enters the pump, centrifugal force, hydraulic pressure, and/or pushrods push the vanes to the walls of the housing.

**Radial piston pumps**

A radial piston pump is a form of hydraulic pump. The working pistons extend in a radial direction symmetrically around the drive shaft, in contrast to the axial piston pump.



These kinds of piston pumps are characterized by the following advantages:

- high efficiency
- high pressure (up to 1,000 bar or 14000psi)
- low flow and pressure ripple (due to the small dead volume in the workspace of the pumping piston)
- low noise level
- very high load at lowest speed due to the hydrostatically balanced parts possible
- no axial internal forces at the drive shaft bearing
- high reliability

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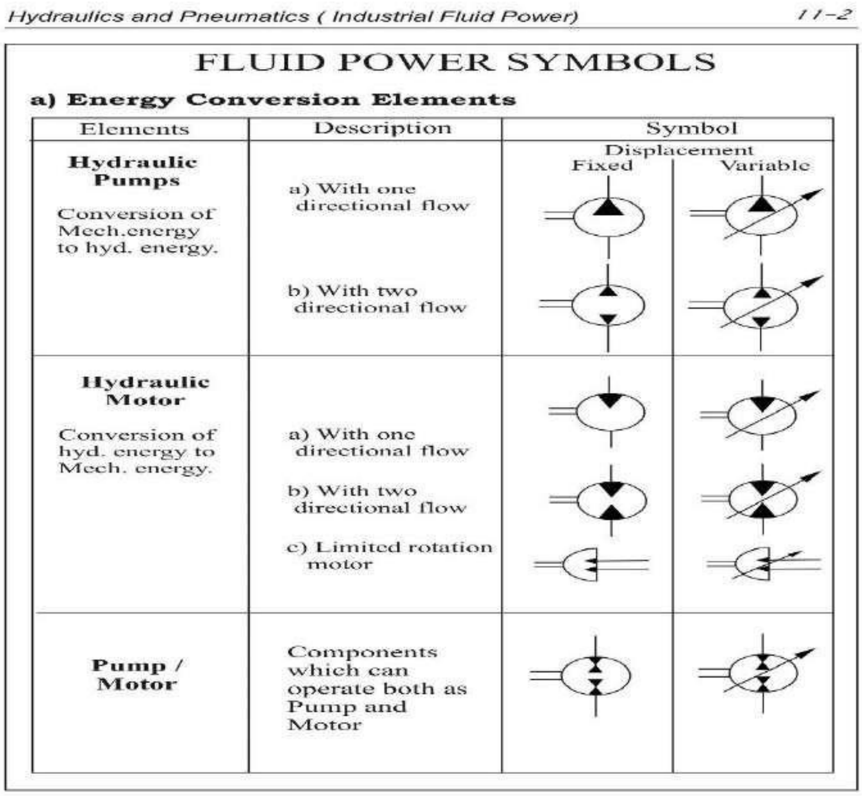
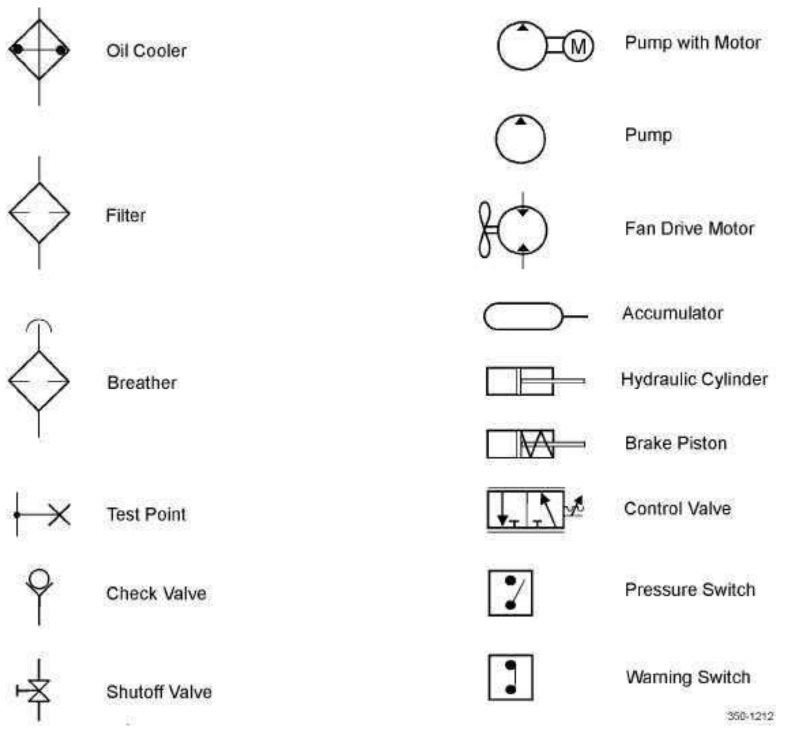
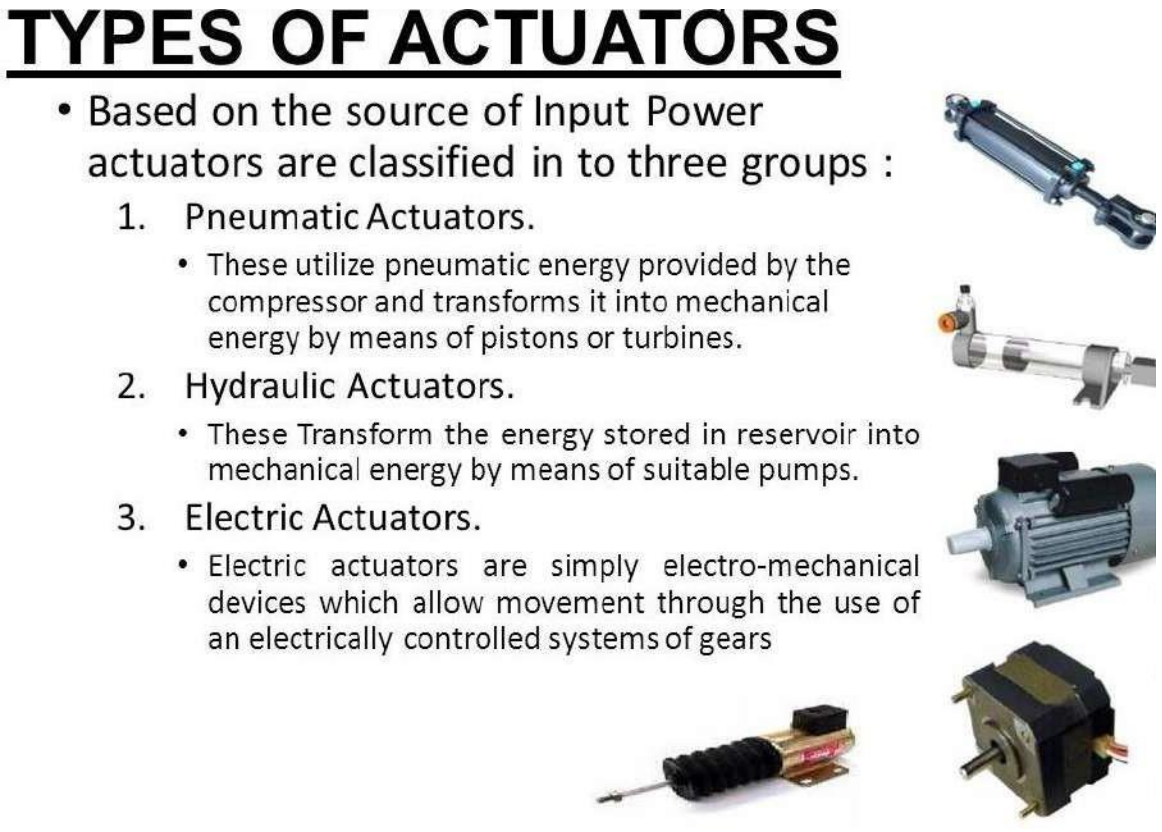
# ISO Symbols for hydraulic components






**Hydraulic circuits**

Hydraulic circuits transmit and control power from a mechanical input to a mechanical output by means of liquids, mostly oils. Power is transmitted hydrostatically, where high pressures make static forces dominate over dynamic forces, and energy is transmitted mostly through static pressure at low flow velocities.

**Direct control of single acting cylinder**

The single-acting cylinder is normally controlled by **a three-port valve**, for example a pneumatic solenoid valve. One port connects to the source of compressed air, the second port is used to supply/vent air to the cylinder and the third port is an exhaust port.

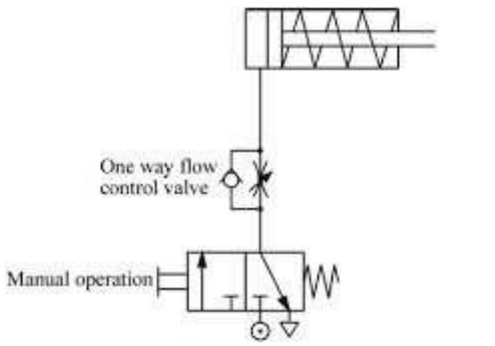
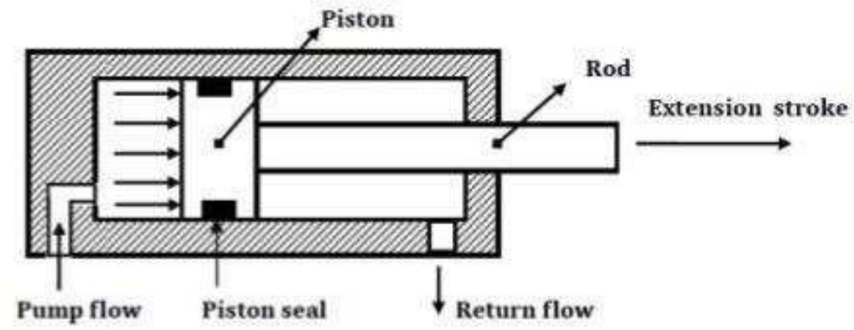
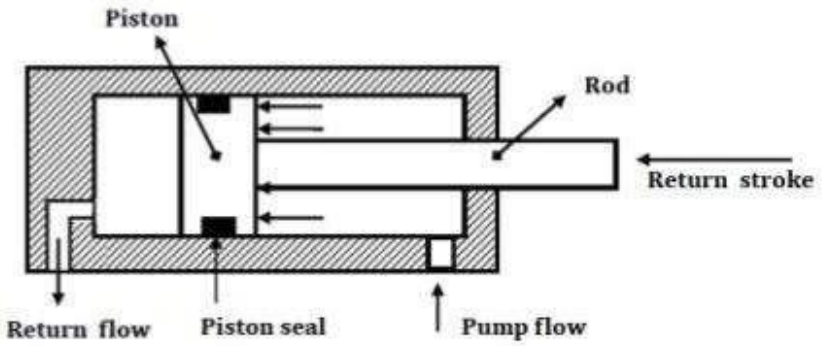



Fig. 28 Direct control of a single acting cylinder

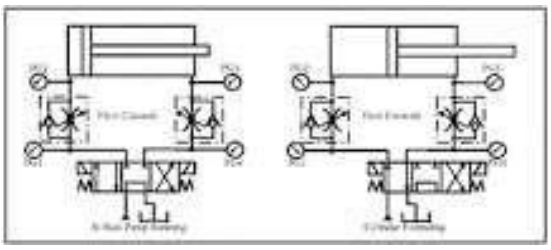
**DOUBLE ACTING CYLINDERS**




- Double-acting cylinders have a port at each end and move the piston forward and back by alternating the port that receives the high-pressure air, necessary when a load must be moved in both directions such as opening and closing a gate.
- Air pressure is applied alternately to the opposite ends of the piston.

**What is meter in and meter out circuit in hydraulics?**



Meter-in flow control circuit represents the controlling of fluid flow just before fluid enters to the actuator with the help of flow control valve We can see here the bypass check valve
that will force the fluid to flow through the adjustable orifice before fluid enters to the actuator i.e. hydraulic cylinder here.

**Comparison of hydraulic and pneumatic system Hydraulic System –**

- Fluid is used for working of system.
- Produce more power.
- Pressure produce from 100 bar up to 700 bar or even more.
- Working fluid is costly as compared to pneumatic system.

- Process is not Clean.
- System with high pressure is difficult to operate.
- Maintenance is not easier.
- Higher initial and operating cost.

**Pneumatic system –**

- Air or gas is used for working of system.
- Produce less power.
- Pressure limited upto 10 bar
- Working Fluid available in cheap rate
- This process is clean.
- Easy to operate.
- Maintenance is easier and quicker.
- Low initial and operating costs