

ENGINEERING MATHEMATICS-I

FOR DIPOLMA STUDENTS



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MATRIX

Matrix Suppose we wish to express the information that Radha has 15 notebooks. We may express it as [15] with the understanding that the number inside [] is the number of notebooks that Radha has. Now, if we have to express that Radha has 15 notebooks and 6 pens. We may express it as [15 6] with the understanding that first number inside [] is the number of notebooks while the other one is the number of pens possessed by Radha. Let us now suppose that we wish to express the information of possession of notebooks and pens by Radha and her two friends Fauzia and Simran which is as follows.

- Radha has 15 notebooks 6 pens
- Fauzia has 10 notebooks 2 pens
- Simran has 13 notebooks 5 pens

Definition

A *matrix* is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix.

Examples :
$$\begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 6 \\ 3 & 6 & 1 \end{bmatrix}$$

In the above examples, the horizontal lines of elements are said to constitute, **rows** of the matrix and the vertical lines of elements are said to constitute, **columns** of the matrix. Thus A has 3 rows and 2 columns, B has 3 rows and 3 columns while C has 2 rows and 3 columns.

Types of Matrices

In this section, we shall discuss different types of matrices.

(i) Column matrix

A matrix is said to be a *column matrix* if it has only one column.

Example:
$$\begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$$

In general, $A = [a_{ij}]_{m \times 1}$ is a column matrix of order $m \times 1$.

(ii) Row matrix

A matrix is said to be a *row matrix* if it has only one row.

Example: $[1 \quad 2 \quad 3]$

In general, $B = [b_{ij}]_{1 \times n}$ is a row matrix of order $1 \times n$.

(iii) Square matrix

A matrix in which the number of rows are equal to the number of columns, is said to be a *square matrix*. Thus an $m \times n$ matrix is said to be a square matrix if $m = n$ and is known as a square matrix of order 'n'.

Example:
$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$$

In general, $A = [a_{ij}]_{m \times m}$ is a square matrix of order m .

(iv) Diagonal matrix

A square matrix $B = [b_{ij}]_{m \times m}$ is said to be a *diagonal matrix* if all its non diagonal elements are zero, that is a matrix $B = [b_{ij}]_{m \times m}$ is said to be a diagonal matrix if $b_{ij} = 0$,

when $i \neq j$.

Example: $\begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}$

(v) Scalar matrix

A diagonal matrix is said to be a *scalar matrix* if its diagonal elements are equal, that is, a square matrix $B = [b_{ij}]_{n \times n}$ is said to be a scalar matrix if

$$b_{ij} = 0, \text{ when } i \neq j$$

$$b_{ij} = k, \text{ when } i = j, \text{ for some constant } k.$$

Example: $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

(Vi) Identity matrix

A square matrix in which elements in the diagonal are all 1 and rest are all zero is called an *identity matrix*.

Example: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(Vii) Null Matrix

A matrix where **all elements are zero**.

Example: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(viii) Zero matrix

A matrix is said to be *zero matrix* or *null matrix* if all its elements are zero.

Example: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(IX) Symmetric matrix

A **square matrix** where $A=A^T$

A symmetric matrix is a square matrix that remains the same when transposed (rows and columns are swapped). In other words, if A is a symmetric matrix, then $A = A^T$, where A^T is the transpose of A. This means that for every element a_{ij} in the matrix, $a_{ij} = a_{ji}$.

Example: $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

(x) Skew-Symmetric Matrix

If for a matrix, the transposed form of that matrix is the same as the negative of the original matrix, then that matrix is said to be a Skew-Symmetric Matrix. Let, a square matrix A of size $n \times n$ is said to be skew-symmetric if $A^t = -A$

Example- $A = \begin{bmatrix} 0 & 3 & -8 \\ -3 & 0 & 6 \\ 8 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 0 & -6 \\ -8 & 6 & 0 \end{bmatrix}$

Additon of Matrices

Two matrices can be added if they are of the same order.

Multiplication of Matrix by a Scalar

If $A = [a_{ij}]_{m \times n}$ is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by a scalar k , i.e. $kA = [ka_{ij}]_{m \times n}$

Negative of a Matrix

The negative of a matrix A is denoted by $-A$. We define $-A = (-1)A$.

Multiplication of Matrices

The multiplication of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B .

Let $A = [a_{ij}]$ be an $m \times n$ matrix and $B = [b_{jk}]$ be an $n \times p$ matrix. Then the product of the matrices A and B is the matrix C of order $m \times p$. To get the (i, k) th element c_{ik} of the matrix C , we take the i th row of A and k th column of B , multiply them elementwise and take the sum of all these products .

$C_{ik} = A_{i1} B_{1k} + A_{i2} B_{2k} + A_{i3} B_{3k} + \dots + A_{in} b_{nk}$ The matrix $C = [C_{ik}]_{m \times p}$ is the product of A and B .

Transpose of a Matrix

1. If $A = [a_{ij}]$ be an $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A . Transpose of the matrix A is denoted by A' or (A^T) . In other words, if $A = [a_{ij}]_{m \times n}$, then $A^T = [a_{ji}]_{n \times m}$. 2. Properties of transpose of the matrices For any matrices A and B of suitable orders,

(i) $(A^T)^T = A$,

(ii) $(kA)^T = kA^T$ (where k is any constant)

(iii) $(A + B)^T = A^T + B^T$ (iv) $(AB)^T = B^T A^T$

Inverse of a Matrix using Elementary Row or Column Operations

To find A^{-1} using elementary row operations, write $A = IA$ and apply a sequence of row operations on $(A = IA)$ till we get, $I = BA$. The matrix B will be the inverse of A . Similarly, if we wish to find A^{-1} using column operations, then, write $A = AI$ and apply a sequence of column operations on $A = AI$ till we get, $I = AB$

Ex-1 (i) Find inverse of the following matrix.

Given $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$, $|A| =$

$$A^{-1} = \frac{adj A}{|A|}, |A| \neq 0$$

So the has inverse

Adj(A)

$$M_{11}=3, M_{21}=-1 \quad C_{11}=3, C_{12}=-1$$

$$M_{12}=1, M_{22}=2 \quad C_{21}=1, C_{22}=2$$

$$Adj(A) = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = A^{-1} = \frac{adj A}{|A|} = \frac{\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}}{7} = \begin{bmatrix} \frac{3}{7} & \frac{1}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{bmatrix}$$

(ii) Find the inverse of the matrix

$$\text{Let } A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \Rightarrow |A| = 3(-3+4) + 3(2-0) + 4(-2-0) = 3.1 + 6 - 8 = 1 \neq 0$$

$$M_{11} = (-3+4) = 1, C_{11} = 1$$

$$M_{12} = (2-0) = 2, C_{12} = -2$$

$$M_{13} = (-2-0) = -2, C_{13} = -2$$

$$M_{21} = (-3+4) = 1, C_{21} = -1$$

$$M_{22} = (3-0) = 3, C_{22} = 3$$

$$M_{31} = (-12+12) = 0, C_{31} = 0$$

$$M_{32} = (12-8) = 4, C_{32} = -4$$

$$M_{33} = (-9+6) = -3, C_{33} = -3$$

$$\text{Co-factor of } A = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$$

$$Adj A = \begin{bmatrix} 1 & -1 & -0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} = A^{-1} = \frac{Adj A}{|A|}$$

$$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \quad (\text{ANS})$$

Solution of a Linear Equations Using Matrix Method

Let the equations be:

$$a_1 x + b_1 y + c_1 z = d_1, a_2 x + b_2 y + c_2 z = d_2, a_3 x + b_3 y + c_3 z = d_3$$

$$\text{Where } A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$AX = B \dots \dots \dots (i)$$

A is the coefficient matrix, X is the variable matrix, and B is the constant matrix.

Multiplying (i) by A^{-1} we get

$$X = A^{-1}B = \frac{\text{adj}A}{|A|} \cdot B$$

The second method to find the solution for the system of equations is row reduction or Gaussian elimination.

DETERMINANT

To every square matrix $A = [a_{ij}]$ of order n , we can associate a number (real or complex) called determinant of the matrix A .

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A , denoted by $|A|$ (or $\det A$), is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Determinant of a matrix of order one

Let $A = [a]$ be the matrix of order 1, then determinant of A is defined to be equal to a .

Determinant of a matrix of order two

Let $A = [a_{ij}] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, be a matrix of order 2. Then the determinant of A is defined as: $\det(A) = |A| = ad - bc$

Determinant of a matrix of order three

The determinant of a matrix of order three can be determined by expressing it in terms of second order determinants which is known as expansion of a determinant along a row (or a column). There are six ways of expanding a determinant of order 3 corresponding to each of three rows (R_1 , R_2 and R_3) and three columns (C_1 , C_2 and C_3) and each way gives the same result. Consider the determinant of a square matrix $A = [a_{ij}]_{3 \times 3}$

Consider the determinant of a square matrix $A = [a_{ij}]_{3 \times 3}$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = |A| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{12}(-1)^{1+2} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}(-1)^{1+3} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Properties of Determinants

We will study some properties of determinants which simplify its evaluation by obtaining maximum number of zeros in a row or a column. These properties are true for determinants of any order. However, we shall restrict ourselves to determinants of order 3 only.

Property 1 The value of the determinant remains unchanged if its rows and columns are interchanged.

Interchange.

Verification

$$\text{let } \Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

We are expanding along the 1st row we get

$$\Delta = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

By interchanging the rows and columns of Δ we get the determinant

$$\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\text{Expanding } \Delta_1 = a_1(b_2c_3 - c_2b_3) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

$$\Delta = \Delta_1$$

$$\text{Ex-1 verify property 1 for } \Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} \text{ due to expanding order}$$

$$\Delta = 2 \begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix} - (-3) \begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} + 5 \begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix}$$

$$= 2(0-20) + 3(-42-4) + 5(30-0)$$

$$= 2(-20) + 3(-46) + 5(30)$$

$$= -40 - 138 + 150 = -28$$

By interchanging rows and columns

$$\Delta_1 = \begin{vmatrix} 2 & 6 & 1 \\ -3 & 0 & 5 \\ 5 & 4 & -7 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 0 & 5 \\ 4 & -7 \end{vmatrix} - 6 \begin{vmatrix} -3 & 5 \\ 5 & -7 \end{vmatrix} + 1 \begin{vmatrix} -3 & 0 \\ 5 & 4 \end{vmatrix}$$

$$= 2(0-20) - 6(21-25) + 1(-12 + 0)$$

$$= 2(-20) - 6(-4) + 1(-12)$$

$$= -40 + 24 - 12$$

$$= -52 + 24 = -28 \text{ from } \Delta_1 = \Delta$$

Property 2 - If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.

$$\text{Verification let } \Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\Delta = a_1(b_2 c_3 - b_3 c_2) - a_2(b_1 c_3 - b_3 c_1) + a_3(b_1 c_2 - b_2 c_1)$$

Interchanging first and third rows the new determinant obtained is given by

$$\Delta_1 = \begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

Expanding along third row, we get

$$= a_1(c_2 b_3 - b_2 c_3) - a_2(c_1 b_3 - c_3 b_1) + a_3(b_2 c_1 - b_1 c_2)$$

$$= -[a_1(b_2 c_3 - b_3 c_2) - a_2(b_1 c_3 - b_3 c_1) + a_3(b_1 c_2 - b_2 c_1)]$$

$$\Delta_1 = -\Delta$$

We can denote the interchange of rows by $R_i \leftrightarrow R_j$ and interchange of columns by $C_i \leftrightarrow C_j$.

Property 3 - If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then value of determinant is zero.

Proof If we interchange the identical rows (or columns) of the determinant Δ then Δ does not change. However, by Property 2, it follows that Δ has changed its sign

$$\text{Therefore } \Delta = -\Delta, \Delta = 0$$

Property 4 If each element of a row (or a column) of a determinant is multiplied by a constant k , then its value gets multiplied by k .

$$\text{Verification Let } \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

So the Δ_1 be the determinant obtained by multiplying the elements of the First row by k then

$$\Delta_1 = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \Delta_1 = a_1(b_2 c_3 - b_3 c_2) - k b_1(a_2 c_3 - c_2 a_3) + k c_1(a_2 b_3 - b_2 a_3)$$

$$= k [a_1 (b_2 c_3 - b_3 c_2) - b_1 (a_2 c_3 - c_2 a_3) + c_1 (a_2 b_3 - b_2 a_3)]$$

$$= k \Delta$$

$$\begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = k \begin{vmatrix} a_1 & a_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(i) By this property, we can take out any common factor from any one row or any one column of a given determinant.

(ii) If corresponding elements of any two rows (or columns) of a determinant are proportional (in the same ratio), then its value is zero. For example

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ ka_1 & ka_2 & ka_3 \end{vmatrix} = 0$$

Property -5

If some or all elements of a row or column of a determinant are expressed as sum of two terms then determinant can be expressed as sum of two or more determinants.

$$\text{Ex- } \begin{vmatrix} a_1 + \lambda_1 & a_2 + \lambda_2 & a_3 + \lambda_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

LHS-

Expanding the determinants along the first row, we get

$$\begin{aligned} \Delta &= (a_1 + \lambda_1) (b_2 c_3 - c_2 b_3) - (a_2 + \lambda_2) (b_1 c_3 - b_3 c_1) \\ &+ (a_3 + \lambda_3) (b_1 c_2 - b_2 c_1) \\ &= a_1 (b_2 c_3 - c_2 b_3) - a_2 (b_1 c_3 - b_3 c_1) + a_3 (b_1 c_2 - b_2 c_1) + \lambda_1 (b_2 c_3 - c_2 b_3) - \lambda_2 (b_1 c_3 - b_3 c_1) + \lambda_3 (b_1 c_2 - b_2 c_1) \end{aligned}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \text{ R.H.S (Proved)}$$

Property-6

If to each element of any row or column of a determinant the equimultiples of corresponding elements of other row or column are added then value of determinant remains the same. Then value of determinant remain same if we apply the operation $.R_i \rightarrow R_i + kR_j$ or $C_i + kC_j$.

Now that

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \text{ and } \Delta_1 = \begin{vmatrix} a_1 + kc_1 & a_2 + kc_2 & a_3 + kc_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} kc_1 & kc_2 & kc_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \Delta + 0$$

$$\Delta = \Delta_1$$

(i) If Δ_1 is the determinant obtained by applying $R_i \rightarrow kR_i$ or $C_i \rightarrow kC_i$ to the determinant Δ , then $\Delta_1 = k\Delta$.

Ex-1 Evaluate

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$$\text{Sol}^n - \Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix} \quad R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1 \text{ we get}$$

$$\Delta = (b-a)(c-a) \begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 1 & -b \end{vmatrix}$$

$$= (b-a)(c-a)(-b+c)$$

$$= (a-b)(b-c)(c-a)$$

$$\text{Ex-2 If } x, y, z \text{ are different and } \Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0, \text{ then show that } 1+xyz=0$$

$\text{Sol}^n -$ we have

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$$

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} \quad (\text{property -5})$$

$$= (-1)^2 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \quad \text{using } C_3 \leftrightarrow C_2 \text{ \& } C_1 \leftrightarrow C_2$$

$$= (1+XYZ) \begin{vmatrix} 1 & X & X^2 \\ 1 & Y-X & Y^2-X^2 \\ 0 & Z-X & Z^2-X^2 \end{vmatrix} \text{ using } R_2 \rightarrow R_2-R_1 \text{ \& } R_3 \rightarrow R_3-R_1$$

So that

$$\Delta = (1+xyz) (y-x) (z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix} = (1+xyz) (y-x) (z-x) (z-y)$$

Since $\Delta = 0$ and x, y, z are all different

$x-y \neq 0$, $y-z \neq 0$, $z-x \neq 0$ so we get $1+xyz = 0$

Inverse of a matrix

Let of A be the matrix of the determinant of A be $|A|$ then the inverse is defined as

$$A^{-1} = \frac{\text{adjoint of } A}{|A|}$$

Ex-1 Find the inverse of the matrix $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$

$$\text{Given } A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$$

$$= 8+3 = 11 \neq 0$$

A^{-1} is exist

$$C_{11} = (-1)^{1+1} \cdot 4 = 4$$

$$C_{12} = (-1)^{1+2} \cdot 3 = -3$$

$$C_{21} = (-1)^{1+3} \cdot (-1) = 1$$

$$C_{22} = (-1)^{2+2} \cdot 2 = 2$$

Cofactor of the matrix

$$[C_{ij}] = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$$

$$\text{Adj } A = [C_{ij}]^T = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}^T = \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adjoint of } A}{|A|}$$

$$A^{-1} = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{11} & \frac{1}{11} \\ \frac{-3}{11} & \frac{2}{11} \end{bmatrix}$$

Ex-2 Find inverse of the Following matrices.

Step-1 Let $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$$|A| = 3(-3+4) + 3(2-0) + 4(-2-0)$$

$$= 3.1 + 6 - 8 = 1 \neq 0$$

So inverse exists

Step-2

$$C_{11} = \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -3 + 4 = 1$$

$$C_{12} = - \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} = - (2 - 4) = 2$$

$$C_{13} = \begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix} = -2 + 3 = 1$$

$$C_{21} = - \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -(-3 + 4) = 1$$

$$C_{22} = \begin{vmatrix} 3 & 4 \\ 0 & 1 \end{vmatrix} = 3 - 4 = -1$$

$$C_{23} = - \begin{vmatrix} 3 & -3 \\ 0 & -1 \end{vmatrix} = -(-3 + 3) = 0$$

$$C_{31} = \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} = -12 + 12 = 0$$

$$C_{32} = - \begin{vmatrix} 3 & 4 \\ 2 & 4 \end{vmatrix} = - (12 - 8) = -4$$

$$C_{33} = \begin{vmatrix} 3 & -3 \\ 2 & -3 \end{vmatrix} = -9 + 6 = -3$$

The cofactor of matrix is $\begin{bmatrix} 1 & 2 & 1 \\ -1 & -1 & 0 \\ 0 & -4 & -3 \end{bmatrix}$

Step-3 Calculate the adjoint of A

$$\text{Adj}(A) = C^T = \begin{bmatrix} 1 & -1 & 0 \\ 38 & -37 & -4 \\ 28 & -27 & -3 \end{bmatrix}$$

Step-4 Calculate the inverse of A

$$A^{-1} = \frac{1}{\det(A)} \text{adj}(A) = \frac{1}{1} \begin{bmatrix} 1 & -1 & 0 \\ 38 & -37 & -4 \\ 28 & -27 & -3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 38 & -37 & -4 \\ 28 & -27 & -3 \end{bmatrix}$$

So the the inverse of the matrix is

$$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 38 & -37 & -4 \\ 28 & -27 & -3 \end{bmatrix} \text{ (Ans)}$$